

Strategic Insider Trading with Imperfect Information: A Trading Volume Analysis

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A model of insider trading is used to analyze the behaviour of trading volume in financial markets characterized by asymmetric information. This model extends the one in Bhattacharya and Nicodano (2001) by introducing competition among informed traders and imperfection of their private information. Contrary to the broad implications of adverse selection models and according to some empirical studies, this paper shows that trading volume is higher when the insiders are active in the market. A higher level of outsiders' risky investment, due to an improved "risk sharing" among them, leads to a higher level of trading. [JEL Codes: G14, D82, C72]

1. - Introduction

This paper extends the model of insider trading developed by Bhattacharya and Nicodano (2001) and analyzes the behaviour of trading volume in financial markets characterized by asymmetric information.

Several previous models overlook trading volume and focus either on liquidity or on agents' welfare. Yet, trading volume contributes, with liquidity, to define structure, size and efficiency of

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a financial market. Furthermore, several empirical works underline the importance of trading volume as a predictor of market trend (Lo and Wang, 2001). Even so, the main reason for our focus on volume concerns a methodological aspect. As a matter of fact, most models of insider trading show results concerning trading volume which are in contrast with empirical evidence. These differences question the validity of these theoretical models, certainly with regard to this variable, but also in general as models of asset markets under asymmetric information.

Indeed, most models of “informed trading” (Back, Cao and Willard, 2000; Foster and Viswanathan, 1996; Grossman and Stiglitz, 1980; Holden and Subrahmanyam, 1992; Kyle, 1985) attribute a portion of market trades (outsider trades) to “noise traders¹”, agents that operate in the market for exogenous reason, and to uninformed agents who are agents without private information. These models consider outsider trades as exogenous², implying no impact of insider trading on outsiders investment choices and, as a consequence, on trading volume.

Other models (George, Kaul and Nimalendran, 1994) show that any uninformed agent, as long as the only reason of trade concerns the asymmetric information, would not be willing to trade within a market where some agents has private information, consistent with “adverse selection” theory (Milgrom and Stokey, 1982). Therefore, insider trading has, according to these models, a negative effect on uninformed agents trading volume.

Empirical studies which rely on court cases relevant to insider trading show that it has a positive effect on investment levels of uninformed agents. Then, trading volume increases³ (Cornell and Sirri, 1992; Fische and Robe, 2004).

¹ They are often defined as “liquidity traders”.

² This hypothesis is justified by their objective to analyze the informational efficiency of asset prices and the conditions that aid to the transmission of private information to uninformed agents.

³ There is one study (CHUNG K.H. - JO H. - SHEFRIN H., 2003) which, using market data to infer the insider trading influence on trading volume, confirm “adverse selection” theory and show that the presence of informed agents leads market makers to widen the bid-ask spread, inducing, as a consequence, uninformed agents to reduce the volume of their trades.

This paper studies the impact of insider trading on trading volume, using, as reference model, the one developed by Bhattacharya and Nicodano (2001). In this model both noise traders and uninformed agents are modeled as agents with well-specified preferences. These agents allocate their endowments across a risky long-term and a riskless short-term investment *ex ante* when their intertemporal consumption preferences are uncertain. A shock to their preferences⁴ is then realized, inducing a subset of them (the “early-diers”) to consume by selling their risky assets in the *interim* asset market, before the payoff to their long-term investment is realized.

Contrary to the models that assume a certain flexibility of the investment choice in the interim stage (Leland, 1992), the model presented in this paper, as in Bhattacharya and Nicodano (2001), assumes instead inflexible *ex ante* aggregate investment portfolio choices by uninformed agents. Interim assets prices are influenced by a stochastic proportion of outsiders who sell and can be further modified by the presence of insider trading. The interim consumption and portfolio allocations of outsiders are clearly affected by a greater informativeness of asset prices brought about by insider trading.

This paper extends this model by allowing for the presence of two insider agents who receive imperfect signals on the future risky return. Signal imperfection and competition among insider agents affect the informativeness of asset prices⁵ and consequently also the investment choices of uninformed agents.

These two assumptions, and hence the extension of the model, are supported by evidence. Consistent with imprecise signals, there are some cases which show that informed agents gained negative profits, which means they had imperfect information about asset payoffs (Meulbroek, 1992). Several empirical works also show that there are more than one insider in many insider trading cases.

⁴ This can be interpreted as a shock to their other incomes.

⁵ Asset prices could convey a wrong information if the signals differ from future risky return.

As in Bhattacharya and Nicodano (2001), also the model in this paper has been worked out numerically, for reason of tractability in the face of possibly binding interim liquidity constraints or “corner solutions”.

This paper is set out as follow. The main features of the model and the solution method for it are described in Section 2, beginning with the equilibrium with outsiders only (subsection 2.1) and describing the insider trading equilibrium (subsection 2.2). Section 3 analyzes the effect of signal imperfection and of oligopoly of private information on informativeness of asset prices. The analysis of numerical results concerning trading volume is carried out in Section 4. Section 5 concludes. Appendix 1 gathers all Proofs of Lemmas, Appendix 2 analyzes the effect of signal imperfection on insiders’ expected profits and Appendix 3 shows an analytical development of trading volume.

2. - The Model

There are three time points $t = 0, 1, 2$ and two types of agents, born at $t = 0$: “outsiders” (uninformed agents) and “insiders” (informed agents).

As regards the outsiders, endowments can be invested either in a risky technology paying off only at $t = 2$ or in a riskless storage technology paying off at $t = 1$ and, if reinvested at $t = 1$, at $t = 2$. Risky technology can, however, be traded in a “secondary market” at $t = 1$ (“interim stage”), with selling by agents who wish to consume early.

The storage technology has unit gross returns while the risky technology has final payoffs per unit investment of $\tilde{\theta}$ distributed as:

$$(1) \quad \tilde{\theta} = \begin{cases} \theta_L & \text{with prob. } \pi \quad (\equiv \pi_L) \\ \theta_H & \text{with prob. } 1 - \pi \quad (\equiv \pi_H) \end{cases}$$

with: $\theta_L < 1 < \theta_H$

It is assumed that π is common knowledge among all the agents and so is the (unconditional) expected return on the risky asset:

$$(2) \quad E(\tilde{\theta}) = \pi\theta_L + (1 - \pi)\theta_H$$

that must be > 1 in order to sustain positive risky investment.

The outsider agents' intertemporal preferences for consumption can be described as follows. There are two "aggregate liquidity states" $\{l, h\}$ and associated conditional probabilities ($0 < \alpha_l < \alpha_h < 1$) such that each agent's utility function for consumption at time $t = 1$ and $t = 2$ is an independently identically distributed random variable:

$$(3) \quad U(C^1, C^2) = \begin{cases} U(C^1) & \text{with prob. } \{\alpha_l \text{ or } \alpha_h\} \\ U(C^2) & \text{with prob. } \{(1 - \alpha_l) \text{ or } (1 - \alpha_h)\} \end{cases}$$

Therefore, an "outsider" will consume early ($t = 1$), because of a liquidity shock, with probability α_i for $i \in 2 \{l, h\}$, or late ($t = 2$) with probability $(1 - \alpha_i)$. These aggregate liquidity states $\{l, h\}$ are assumed to arise with *ex ante* probabilities $q \equiv q_l$ e $(1 - q) \equiv q_h$. It is assumed that $\{q, \alpha_l, \alpha_h\}$ are common knowledge, but that each uninformed agent only knows her own realized utility function $U(C^1, C^2)$, but not the aggregate state. Moreover, we assume the asset payoffs are independent of the liquidity states.

These *ex ante* random interim preferences, coupled with their aggregate variability, have effects on interim asset prices similar to those arising from noise trading.

Outsiders make *per capita* investment choices across two technologies. They invest K in the "short-term" asset and $(1 - K)$ in the "long-term" asset at $t = 0$. Agents who wish to consume at $t = 1$ and those who wish instead to postpone their consumption until $t = 2$ can trade their risky assets, thus determining equilibrium prices $P(K, \theta_j, \alpha_j)^6$, $j \in \{L, H\}$, $i \in \{l, h\}$ per unit investment.

⁶ $P(K, \theta_j, \alpha_j)$ is the rational expectation equilibrium price mapping the underlying aggregate state, which includes the equilibrium investment choice K at $t = 0$.

The two “insiders”, instead, have an exogenous endowment of the risky technology⁷, from which they could choose to sell an amount and reinvest the proceeds in the riskless technology at $t = 1$. Since they are risk neutral, they maximize expected profits conditional on their private information. At the “interim stage” they receive two imperfect signals, \tilde{S}^1 and \tilde{S}^2 , which present the same distribution:

$$(4) \quad \tilde{S} = S_k \text{ with prob. } \sum_{j=L,H} \Pr(\theta_j) \Pr(S_k | \theta_j)$$

for $k \in \{L, H\}$

or:

$$(5) \quad \tilde{S} = \begin{cases} S_H \text{ with prob.} & \pi_H \Pr(S_H | \theta_H) + \pi_L \Pr(S_H | \theta_L) \\ S_L \text{ with prob.} & \pi_L \Pr(S_L | \theta_L) + \pi_H \Pr(S_L | \theta_H) \end{cases}$$

where S_k is the event in which the “insider” receives a signal which indicates a k -type future return. In particular, S_H is the set of signal realizations such that $\tilde{S} \geq \theta_H$ while S_L is the set of signal realizations such that $\tilde{S} \leq \theta_L$.

The conditional expected value of the risky return based on the signal, is:

$$(6) \quad E(\tilde{\theta} | S_k) = \sum_{j=L,H} \theta_j \Pr(\theta_j | S_k)$$

where, given Bayes's Theorem:

$$(7) \quad \Pr(\theta_j | S_k) = \frac{\Pr(\theta_j) \Pr(S_k | \theta_j)}{\sum_{j=L,H} \Pr(\theta_j) \Pr(S_k | \theta_j)}$$

It is important to notice that:

$$(8) \quad E(\tilde{\theta} | S_k) \neq E(\tilde{\theta}) \text{ if } \Pr(\theta_j | S_k) \neq \Pr(\theta_j)$$

⁷ If n_1 and n_2 are the endowments (units of the risky technology) of the two “insiders”, we assume also that $n_z \geq [\alpha_n - \alpha_j] \forall z \in \{1, 2\}$.

$$(9) \quad E(\tilde{\theta} | S_k) = E(\tilde{\theta}) \text{ if } \Pr(\theta_j | S_k) = \Pr(\theta_j)$$

Hence, for higher levels of $\Pr(\theta_j | S_j)$ the conditional expected value of the future risky return tends to the effective value. In particular, if the signal were perfect, $\Pr(\theta_j | S_j) = 1$, then:

$$(10) \quad E(\tilde{\theta} | S_j) = \theta_j \quad \forall j \in \{L, H\}$$

Now, we can define the correlation coefficient between the signals and an index of signal precision, two key concepts that will help us in deriving the equilibrium strategies of the two informed agents. Since the two signals are characterized by the same distribution (and so by the same probabilities) it means that if the signals were perfect, $\Pr(\theta_j | S_j) = 1 \quad \forall j \in \{L, H\}$, the correlation coefficient would be equal to 1, $\text{Corr}(\tilde{S}^1, \tilde{S}^2) \equiv \rho_S = 1$. Therefore, the lower is the above mentioned probability, the lower is the correlation between the two signals.

As concerns the index of (relative) signal precision (Ψ), it is defined as follow:

$$(11) \quad \Psi = 1 - \left(\frac{\text{Var}(\tilde{\theta} | S_k)}{\text{Var}(\tilde{\theta})} \right)$$

Analogously, if the signals were perfect, the index of relative signal precision would assume value 1, since the conditional variance would be equal to 0. Again, the lower is the probability $\Pr(\theta_j | S_j) \quad \forall j \in \{L, H\}$, the lower is the index of relative signal precision. Therefore, there is a positive relation between ρ_S and Ψ .

Let θ_j represents the event in which the future return is j -type, S_k^1 the event in which the signal received by "insider" 1 indicates a k -type future return, and S_s^2 the event in which the signal received by "insider" 2 indicates a s -type future return. Then, we can define the following joint probabilities:

$$(12) \quad \Pr(\theta_j, S_k^1, S_s^2) = \Pr(\theta_j) \Pr(S_k^1 | \theta_j) \Pr(S_s^2 | \theta_j, S_k^1) \\ \forall j, k, s \in \{L, H\}$$

So, “insiders” strategic action will depend on these probabilities, which determine P_S (and Ψ). The higher the signal correlation coefficient, the higher the probability that the two informed agents will act in the same direction.

2.1 Traded Equilibria without Insider Trading

Defining $i \in \{l, h\}$ as the index of the liquidity state and $j \in \{L, H\}$ as the index of payoffs, the consumption levels of “early-diers” $C_j^1(i)$ and “late-diers” $C_j^2(i)$ are:

$$(13) \quad C_j^1(i) = [K + (1 - K)P_j(i)]$$

$$(14) \quad C_j^2(i) = [(K - P_j(i)X_j(i)) + \theta_j(1 - K) + \theta_j X_j(i)]$$

where $X_j(i)$ is the net amount of the risky asset that every “late-dier” is inclined to buy at $t = 1$, paying the price $P_j(i)$. Since in an equilibrium without insider trading, $P_j(i)$ and $X_j(i)$ depend only on the liquidity state i , we can omit the index j .

The following “market clearing” condition must be satisfied:

$$(15) \quad (1 - \alpha_i)X(i) = \alpha_i(1 - K)$$

and since we assume that “late-diers”, wishing to consume only at $t = 2$, can not borrow at $t = 1$ from “early-diers”, we need a “no-borrowing” condition:

$$(16) \quad K - P(i)X(i) \geq 0$$

Equations (15) and (16) together imply the “aggregate liquidity” constraint on market-clearing prices:

$$(17) \quad P(i)\alpha_i(1 - K) \leq (1 - \alpha_i)K$$

At $t = 0$, in their *ex ante* choice of K , “outsiders” maximize their *ex ante* expected utility:

$$(18) \quad \max_{K, X(i)} \sum_{i=l, h} q_i \pi_i \left[\alpha_i U(C^1(i)) + (1 - \alpha_i) U(C^2(i)) \right]$$

At $t = 1$, given $P(i)$, which in equilibrium will only reveal liquidity state $\{l, h\}$ and no private information about the future risky return θ_j , “late-diers” choose $X(i)$ in order to:

$$(19) \quad \max_{X(i)} \sum_{j=L,H} \pi_j U(C_j^e(i)) | P(i)$$

Using the first-order conditions for the maximization problem in (19) and the equation (15), we determine possible interim equilibrium prices $P(i, K)$ for a given K . These prices are found from among the positive real roots, considering the aggregate liquidity constraints in equation (17). The implied *ex ante* choice of K is then derived using the maximization program in equation (18) and taking the interim prices $P(i)$ and trades $X(i)$ as being given by the earlier set of calculations, and iterate until convergence in K .

2.2 Noisy REE with Insider Trading

“Insiders” aim to maximize their expected profits conditional on their private information. This requires to mask their presence⁸ in order not to reveal their private information (as in Kyle, 1985 and in Gorton and Pennacchi, 1990). However, here each “insider” must conjecture the other “insider’s” action because they can mask their presence only if the aggregate quantity sold by them is such that “late-diers” do not know whether they are buying from “early-diers” or from the “insiders”.

DEFINITION 1

Insiders’ expected profit conditional on their private information can be defined as follow:

$$(20) \quad E(\Pi^I | \mathcal{S}_k^I) = \sum_{i=l,h} \sum_{j=L,H} \sum_{s=L,H} q_i \left[(P_{ks}(i) - \theta_j) Q_k^I \cdot \Pr(\theta_j, \mathcal{S}_s^e | \mathcal{S}_k^I) \right]$$

⁸ “Early-diers outsiders” supply their “long-term” asset inelastically and hence the “insiders” can mimic their sales only via (many small) market orders.

$$(21) \quad E(\Pi^2 | S_s^2) = \sum_{i=l,h} \sum_{j=L,H} \sum_{s=L,H} q_i \left[(P_{ks}(i) - \theta_j) Q_s^2 \cdot \Pr(\theta_j, S_k^1 | S_s^2) \right]$$

where Q_k^1 and Q_s^2 represent the amount of risky asset sold respectively by insider 1 and insider 2, conditional on the type of signal they receive.

So, the conditional expected profit of each "insider" is a direct function of the quantity she has sold and indirect function of the quantity sold by the other, the precision of the signals and their correlation.

Since "insiders" will enter the market only if their signal have indicated a L -type future return,

$$(22) \quad Q_k^1 = \begin{cases} Q^1 & \text{if } k = L \\ 0 & \text{if } k = H \end{cases}$$

and:

$$(23) \quad Q_s^2 = \begin{cases} Q^2 & \text{if } s = L \\ 0 & \text{if } s = H \end{cases}$$

Therefore, if we consider "insider" 1, her expected profit conditional on her private information can be rewritten as follow:

$$(24) \quad E(\Pi^1 | S_k^1) = \begin{cases} \varpi^1(Q^1) & \text{if } k = L \\ 0 & \text{if } k = H \end{cases}$$

where:

$$(25) \quad \varpi^1(Q^1) = \sum_{i=l,h} \sum_{j=L,H} \sum_{s=L,H} q_i \left[(P_{Ls}(i) - \theta_j) Q^1 \cdot \Pr(\theta_j, S_s^2 | S_L^1) \right]$$

The same holds for "insider" 2.

DEFINITION 2

The equilibrium strategies, Q^{1*} and Q^{2*} , must satisfy the following conditions:

$$(26) \quad E[\Pi^1(Q^{1*}, Q^2)] \geq E[\Pi^1(Q^1, Q^2)]$$

$$(27) \quad E[\Pi^2(Q^2, Q^1)] \geq E[\Pi^2(Q^2, Q^1)]$$

We can now solve for the overall equilibrium. The “outsiders’” consumptions are defined as follow:

$$(28) \quad C_{ks}^1(i) = K + (1 - K) P_{ks}(i)$$

$$(29) \quad C_{jks}^2(i) = (K - P_{ks}(i)X_{ks}(i)) + ((1 - K) + X_{ks}(i))\theta_j \\ \forall i \in \{l, h\} \text{ and } \forall j, k, s \in \{L, H\}$$

where $P_{ks}(i)$ and $X_{ks}(i)$ are respectively the price and the net amount of the “long-term” asset bought per unit of “late-diers”, when the liquidity shock is i -type, “insider” 1 observes S_k^1 and “insider” 2 observes S_s^2 . Equation (28) shows how “outsider early-diers” consumption does not depend on the future risky return.

“Outsiders” trades at $t = 1$ must satisfy the “no-borrowing” constraint:

$$(30) \quad P_{ks}(i)X_{ks}(i) \leq K \quad \forall i, k, s$$

while the “aggregate liquidity” constraint incorporates “insiders” supply of risky assets:

$$(31) \quad P_{ks}(i)[\alpha_i(1 - K) + (Q_k^1 + Q_s^2)] \leq (1 - \alpha_i) K$$

In the aggregate states $\{iks\}$ ⁹ for $i \in \{l, h\}$ and $k, s \in \{L, H\}$, equilibrium prices $P_{ks}(i)$ and beliefs on $\hat{\theta}$ conditioned to asset prices $(\hat{\pi}_j | P_{ks}(i))$ must satisfy the following market clearing condition:

$$(32) \quad \alpha_i(1 - K) + (Q_k^1 + Q_s^2) = (1 - \alpha_i)X_{ks}(i)$$

At $t = 0$, in their *ex ante* choice of K , every “outsider” maximizes his *ex ante* expected utility:

⁹ We do not make distinction between the states $\{iHL\}$ and $\{iLH\}$.

$$(33) \quad \max_{K, X_{ks}(i)} \sum_{i=l,h} \sum_{j=L,H} \sum_{k=L,H} \sum_{s=L,H} q_i \left\{ \alpha_i \left[\Pr(S_k^1, S_s^2) U(C_{ks}^1(i)) \right] + (1 - \alpha_i) \left[\Pr(\theta_j, S_k^1, S_s^2) U(C_{jks}^2(i)) \right] \right\}$$

At $t = 1$, the “outsider late-diers” will choose the amount $X_{ks}(i)$ to buy in the interim market in order to maximize their conditional expected utility:

$$(34) \quad \max_{X_{ks}(i)} \sum_{j=L,H} \sum_{k=L,H} \sum_{s=L,H} \hat{\Pr}(\theta_j, S_k^1, S_s^2) U(C_{jks}^2(i)) P_{ks}(i)$$

and “insiders” maximize their expected profits conditional on their private information, $E(\Pi^1 | S_k^1)$ and $E(\Pi^2 | S_s^2)$

$$(35) \quad \max_{Q^1} \sum_{i=l,h} \sum_{j=L,H} \sum_{s=L,H} q_i \left[(P_{Ls}(i) - \theta_j) Q^1 \cdot \Pr(\theta_j, S_s^2 | S_k^1) \right]$$

$$(36) \quad \max_{Q^2} \sum_{i=l,h} \sum_{j=L,H} \sum_{k=L,H} q_i \left[(P_{kL}(i) - \theta_j) Q^2 \cdot \Pr(\theta_j, S_k^1 | S_s^2) \right]$$

PROPOSITION 1

Given the two subset of values of the signal precision index, if $\tilde{S}^z = S_z^z$,

$$(37) \quad Q^{z*} = \begin{cases} \frac{Q^M}{2} & \text{if } \psi \geq \hat{\psi} \quad \forall z \in \{1, 2\} \\ 0 & \text{if } \psi < \hat{\psi} \end{cases}$$

represents the pure strategy Nash equilibrium of the strategic game, which is the strategy that satisfies the equilibrium conditions in Definition 2, where Q^M is the quantity which a monopolist would trade in the market.

We now explain why there is not a continuum of equilibria, which is the logic underlying proposition 1.

The strategies $Q^* \neq \{Q^M/2, Q^M\}$ can be eliminated because only the following two scenarios are possible: both the “insiders” enter the market, only one “insider” enters the market. As a matter of fact, since the informed agents want to enter the market avoiding to reveal their private information, any other strategy would lead to a revelation of their information in all aggregate states. According to the first scenario the optimal strategy would be $Q^* = Q^M/2$, while according to the second $Q^* = Q^M$.

In order to eliminate the dominated strategy, it is necessary to compare the aggregate states in which the “insiders” are able to mask their presence, according to the two possible equilibrium strategies. If they supply $Q^M/2$, “late-diers” can not distinguish between the aggregate states $\{hHH\}$ and $\{lLL\}$; hence, the private information is not revealed with *ex ante* probability equal to $[\Pr(S_L^1, S_L^2) \cdot q_l]$. Supplying Q^M , instead, “late-diers” can not distinguish between the aggregate states $\{hHH\}$ and $\{lHL\}$ (or $\{lLH\}$) and so the private information is not revealed with *ex ante* probability equal to $[\Pr(S_{Hh}^1, S_L^2) \cdot q_l]$ (that is also equal to $[\Pr(S_L^1, S_H^2) \cdot q_l]$).

When the signal precision Ψ is low, the signals correlation coefficient is also low and this would mean that the probability the two “insiders” do not enter the market together is high. However, for low values of the correlation coefficient the “insiders” do not enter the market since their expected profits are negative. It can be shown that, for those values of Ψ that induce the “insiders” to enter the market ($\Psi \geq \hat{\Psi}$), the above mentioned probability $\Pr(S_L^1, S_L^2)$ is close to 1, while $\Pr(S_L^1, S_H^2)$ is close to 0. Hence, we can eliminate the strategy $Q^* = Q^M$.

Finally, given the “insiders’” equilibrium strategies, the “market clearing” conditions can be rewritten as follow:

$$\begin{aligned} \alpha_f(1 - K) &= (1 - \alpha_j)X_e \\ \alpha_f(1 - K) + Q &= (1 - \alpha_j)X_d \\ \alpha_h(1 - K) &= (1 - \alpha_h)X_c \\ \alpha_f(1 - K) + 2Q &= (1 - \alpha_j)X_c \end{aligned}$$

$$\alpha_h(1 - K) + Q = (1 - \alpha_h)X_b$$

$$\alpha_h(1 - K) + 2Q = (1 - \alpha_h)X_a$$

where:

$$X_{HH}(l) \equiv X_e, X_{HL}(l) \text{ or } X_{LH}(l) \equiv X_d, X_{HH}(h) \equiv X_c$$

$$X_{LL}(l) \equiv X_c, X_{HL}(h) \text{ or } X_{LH}(h) \equiv X_b, X_{LL}(h) \equiv X_a$$

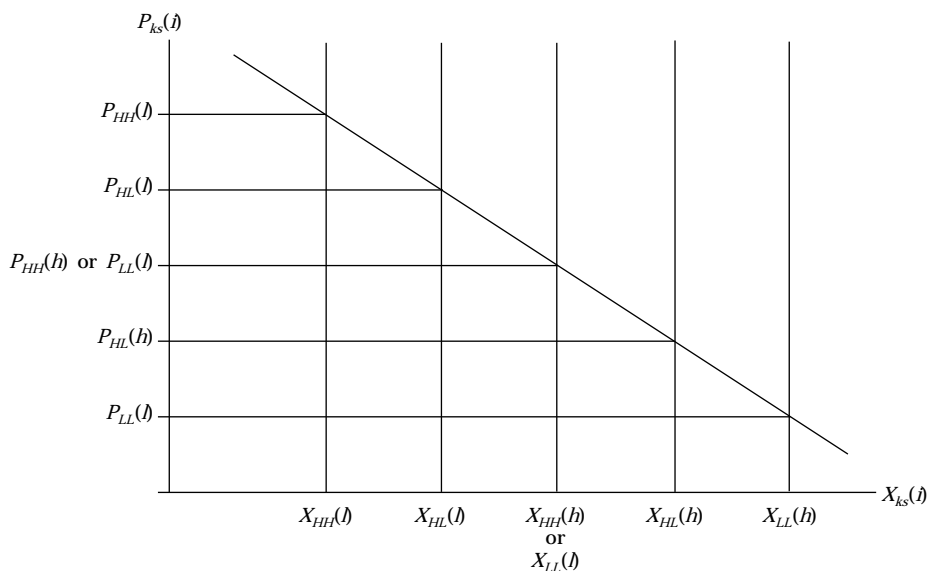
3. - Informativeness of Asset Prices

In Bhattacharya and Nicodano (2001) the asset payoff is not revealed by asset prices only when the “insider” masks her presence in the market. Hence, in all the other aggregate states the informativeness of asset prices is perfect.

In this model, as in Bhattacharya and Nicodano (2001) (one insider with perfect signal), the information of the signals is not revealed only when the “insiders” succeed in masking their pres-

GRAPH 1

SECONDARY MARKET OF THE RISKY ASSET WITH INSIDER TRADING



ence. However, even when the information is revealed, the uninformed agents can only infer through prices the type of signal but not the asset payoff because of the imperfection of the signals. They will never know for certain if $\tilde{\theta}$ is higher or lower than 1.

Graph 1 shows the partitions of the aggregate states in the interim market. We can notice that only in one of these aggregate states, $\{ILL\}$, the “insiders” succeed in masking their presence. However, the possibility for the informed agents to receive a wrong signal prevents “outsiders” from knowing through prices the future risky payoff for certain. In particular, when the “insiders” receive different signals, uninformed agents have the same probability ($= 1/2$) to guess the future return.

In Appendix 2 we show that “insiders” expected profits are lower when signal imprecision is higher. This occurs because, when Ψ (index of signals precision) decreases, the expected gain rises, because of the greater uncertainty transferred to the “outsiders”, but by a rate of growth lower than the expected loss rate.

Moreover, another important contribution of this model deals with the presence of two “insiders”, that is an oligopoly of private information. As a matter of fact, the possibility for the two informed agents to act not always in the same direction creates two further partitions of the aggregate states. These allow a better distribution of “outsiders” uncertainty among these states. The presence of two intermediate aggregate states reduces the probability to be in one of the farthest aggregate states inconsistently with the future risky return. Therefore, we can conclude that, for the same signal imperfection, the presence of an additional informed agent leads to a lower “outsiders” uncertainty, and then to a higher risky investment, if compared to the case of monopoly of private information.

4. - Trading Volume

In this Section we carry out the analysis of numerical results concerning trading volume. This analysis is based on the model discussed in the previous Section with a particular choice for the characterization of the signals.

$$(38) \quad \tilde{S}^1 = \tilde{\theta} + \varepsilon_1$$

$$(39) \quad \tilde{S}^2 = \tilde{\theta} + \varepsilon_2$$

The two error terms have the same distribution:

$$(40) \quad \hat{\varepsilon}_z = \begin{cases} x & \text{with prob. } \gamma \\ 0 & \text{with prob. } \beta \text{ for } z=1,2 \\ -x & \text{with prob. } \gamma \end{cases}$$

where:

$$(41) \quad x = \theta_H - \theta_L$$

In this way we assume that $E(\tilde{\varepsilon}) = 0$ and then the expected value of the signal is equal to the (unconditional) expected return on the risky asset ($E(\tilde{S}) = E(\tilde{\theta})$).

Assuming that each error term of the signals is independent of the other and of the future risky return:

$$(42) \quad \text{Corr}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2) = 0 \text{ and } \text{Corr}(\tilde{\varepsilon}_z, \tilde{\theta}) = 0 \quad \forall z$$

the signals distribution is:

$$(43) \quad \tilde{S} = \begin{cases} S_H & \text{with prob. } \pi_H(\gamma + \beta) + \pi_L(\gamma) \\ S_L & \text{with prob. } \pi_L(\gamma + \beta) + \pi_H(\gamma) \end{cases}$$

So, the conditional expected value of the risky return is:

$$(44) \quad E(\tilde{\theta} | S_k) = \begin{cases} \theta_H \left(\frac{\pi_H(\gamma + \beta)}{\pi_H(\gamma + \beta) + \pi_L\gamma} \right) + \theta_L \left(\frac{\pi_L(\gamma)}{\pi_H(\gamma + \beta) + \pi_L\gamma} \right) & \text{per } k = H \\ \theta_L \left(\frac{\pi_L(\gamma + \beta)}{\pi_L(\gamma + \beta) + \pi_H\gamma} \right) + \theta_H \left(\frac{\pi_H(\gamma)}{\pi_L(\gamma + \beta) + \pi_H\gamma} \right) & \text{per } k = L \end{cases}$$

We can, then, define the joint probabilities as follow:

$$(45) \quad \Pr(\theta_j, S_k^1, S_s^2) = \begin{cases} \pi_j(\gamma + \beta)^2 & \text{if } k = j, s = j \\ \pi_j(\gamma)^2 & \text{if } k \neq j, s \neq j \quad \forall j \\ \pi_j(\gamma + \beta)(\gamma) & \text{if } k = j, s \neq j \\ \pi_j(\gamma)(\gamma + \beta) & \text{if } k \neq j, s = j \end{cases}$$

LEMMA 1

Given the independence assumption of signal error terms, the signal correlation coefficient is equal to:

$$(46) \quad \text{Corr}(\tilde{S}^1, \tilde{S}^2) = \frac{\text{Cov}(\tilde{S}^1, \tilde{S}^2)}{\sigma_{\tilde{S}^1} \sigma_{\tilde{S}^2}} = \frac{\text{Var}(\tilde{\theta})}{\text{Var}(\tilde{S})}$$

The signals correlation coefficient can be expressed as a function of the probabilities that characterize the two stochastic components of the signals ($\tilde{\theta}$ e $\tilde{\varepsilon}$):

$$(47) \quad \text{Corr}(\tilde{S}^1, \tilde{S}^2) = G(\pi, \beta)$$

The assumption of equiprobability *ex ante* of the risky return distribution ($\pi = 1/2$ and then $\pi_H = \pi_L$) allows to express the signal correlation as a function only of the error term distribution¹⁰:

$$(48) \quad \text{Corr}(\tilde{S}^1, \tilde{S}^2) = g(\beta)$$

Given the initial assumption on the signals error terms, it follows that the signal precision of the two informed agents is the same. When $\beta = 1$ and $\gamma = 0$ equation (44) reveals that $E(\tilde{\theta}|S_k) = \theta_k$, therefore the conditional variance is equal to 0 while the index of relative precision measure is equal to 1. On the contrary, the conditional variance is equal to the unconditional variance when $\beta = 0$: in this case “insiders” would not be better informed than “outsiders”. Therefore, if $\beta = 0$ the capacity of the signal to explain the risky return variability is null, while if $\beta = 1$ it is perfect.

So, the index of relative precision of the signals can be conveniently expressed as a function of the probabilities that characterize the two stochastic components of the signals ($\tilde{\theta}$ and $\tilde{\varepsilon}$):

$$(49) \quad \Psi = F(\pi, \beta)$$

¹⁰ It is also possible to prove that, for values of $\beta < 1$, the signal correlation coefficient $\text{Corr}(\tilde{S}^1, \tilde{S}^2)$ reaches its maximum (in the interval $[0, 1]$) for $\pi = 1/2$. This result comes also from Graph 2.

When $\pi = 1/2$ (and then $\pi_H = \pi_L$) the relative precision index is a function only of the error term distribution¹¹:

$$(50) \quad \Psi = f(\beta)$$

LEMMA 2

If $\pi_H = \pi_L$, then the relative precision index is equal to the square of β :

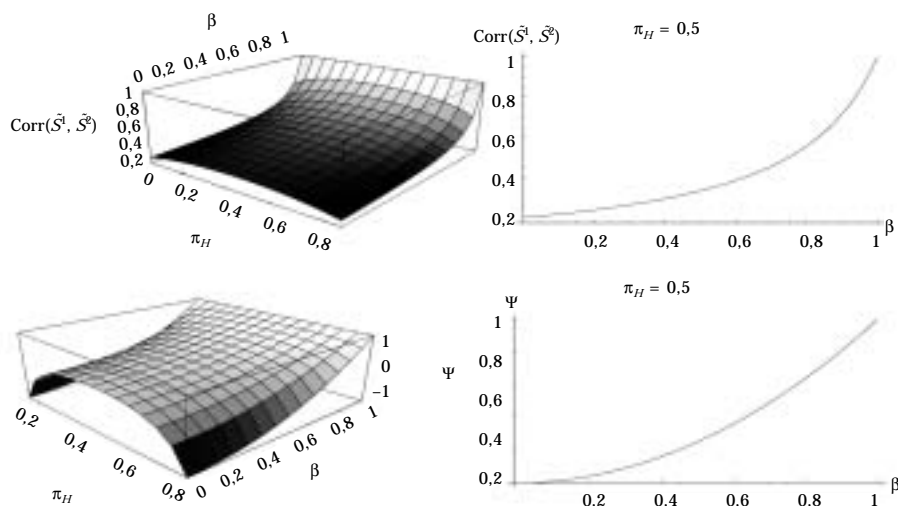
$$(51) \quad \tilde{\Psi} = \beta^2$$

The figures in Graph 2 represent equations (47), (48), (49) and (50).

Furthermore, Graph 3 shows exactly what we argued in proposition 1: the strategy to offer half of the monopolistic amount, given the assumptions on the signals error terms, dominates the strat-

GRAPH 2

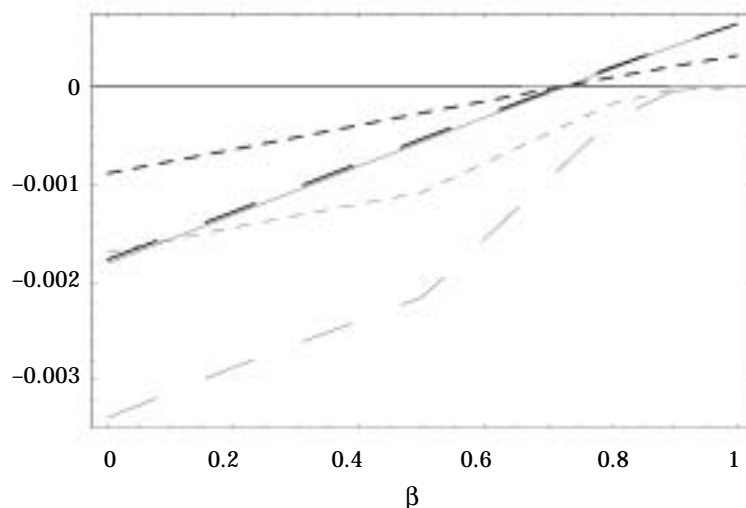
SIGNAL CORRELATION COEFFICIENT AND RELATIVE PRECISION INDEX



¹¹ It is also possible to prove that, for values of $\beta < 1$, the signals relative precision index $\tilde{\Psi}$ reaches its maximum for $\pi = 1/2$. This result comes also from Graph 2.

GRAPH 3

EXPECTED PROFIT OF AN OLIGOPOLISTIC INSIDER
ACCORDING TO DIFFERENT STRATEGIC ACTIONS AND
SIGNAL IMPERFECTION DEGREES



egy to offer the monopolistic amount. The broken curve with small dashes represents the single “insider” expected profit if she decides to offer Q^M , while the bold broken curve with small dashes represents her expected profit if she decides to offer $Q^M/2$.

The broken curve with big dashes and the bold broken curve with big dashes, instead, represent the “insiders” aggregate expected profit respectively in the two previous cases. The continuous curve represents the monopolist expected profit. It is possible to notice that the aggregate expected profit ($E(\Pi^1) + E(\Pi^2)$), when the strategic action is $Q^M/2$ for both the “insiders”, is equal to monopolist expected profit¹² ($E(\Pi^M)$).

We have computed¹³ equilibrium allocations for the grid of parameter values below:

¹² The bold broken line with big dashes and the continuous line do not coincide exactly because of not significant errors which characterize numerical resolution.

¹³ The model has been worked out numerically using the software *MATHEMATICA*®.

$$\begin{aligned}
\{\pi, q\} &= \{1/2, 1/2\} \\
\{\alpha_l, \alpha_h\} &= \{0.1, 0.15\}, \{0.9, 0.95\}, \{0.48, 0.53\}, \{0.45, 0.55\}, \{0.4, 0.6\} \\
\{\theta_l\} &\in \{0.75, 0.8, 0.85, 0.9, 0.95\} \\
\{\theta_H\} &= \{1.25, 1.3, 1.35, 1.4, 1.45, 1.5\} \\
U(C) &= -C^2
\end{aligned}$$

In the next Sections, we will adopt the following notation: K_T is the investment choice without insider trading, K_I the investment choice with insider trading; $\tilde{\phi}$ is the total trading volume with insider trading, $\tilde{\zeta}$ the net trading volume with insider trading, while $\tilde{\varphi}$ the trading volume without insider trading.

4.1 Total Trading Volume

Trading volume is equal to the amount of risky asset offered by the “outsider early-diers” plus the possible aggregate supply of the two “insiders” $\alpha_i(1 - K) + (Q_k^1 + Q_s^2)$, that is, given the “market clearing” condition, equal to the amount demanded by the “outsider late-diers” $(1 - \alpha_i)X_{ks}(i)$.

$$(52) \quad \tilde{\phi} = (1 - \alpha_i) X_{ks}(i) \quad \text{with prob. } q_i \Pr(S_k^1, S_s^2) \\
\forall i \in \{l, h\} \text{ and } \forall k, s \in \{L, H\}$$

Expected trading volume with insider trading is therefore equal to:

$$(53) \quad E(\tilde{\phi}) = \sum_{i=l,h} \sum_{k=L,H} \sum_{s=L,H} q_i \Pr(S_k^1, S_s^2) [(1 - \alpha_i) X_{ks}(i)]$$

Expected trading volume without insider trading, instead, is equal to:

$$(54) \quad E(\tilde{\varphi}) = \sum_{i=l,h} q_i [(1 - \alpha_i) X(i)]$$

TABLE 1

TOTAL TRADING VOLUME
 $(\alpha_l = 0.48, \alpha_h = 0.53, \beta = 0.9)$

θ_H/θ_L	0.75	0.80	0.85	0.90	0.95
<i>Average trading volume without insider trading - $E(\tilde{\varphi})$</i>					
1.25	$-1.74754 \cdot 10^{-8}$	0.041367	0.108205	0.231043	0.242763
1.30	0.0276199	0.0682261	0.132585	0.238161	0.243581
1.35	0.047003	0.0865377	0.14831	0.238688	0.244194
1.40	0.0610873	0.0994388	0.158698	0.239036	0.244638
1.45	0.0715835	0.108732	0.16562	0.239264	0.244947
1.50	0.0795511	0.115523	0.1702	0.239397	0.245145
<i>Average total trading volume with insider trading - $E(\tilde{\phi})$</i>					
1.25	$9.06054 \cdot 10^{-7}$	0.0455224	0.137218	0.256038	0.260949
1.30	0.0286356	0.0840548	0.173621	0.25741	0.260757
1.35	0.0563092	0.110861	0.198001	0.257397	0.260575
1.40	0.0767099	0.130219	0.214813	0.25728	0.2604
1.45	0.0921857	0.144561	0.226574	0.257081	0.260234
1.50	0.104175	0.155373	0.234594	0.256817	0.260075
<i>Differences between total trading volumes - $E(\tilde{\phi}) - E(\tilde{\varphi})$</i>					
1.25	$9.2553 \cdot 10^{-7}$	0.0041554	0.0290138	0.0249946	0.0181854
1.30	0.00101573	0.0158286	0.0410354	0.0192494	0.017176
1.35	0.00930625	0.0243234	0.0496909	0.0187089	0.016381
1.40	0.0156226	0.0307804	0.0561147	0.0182442	0.0157618
1.45	0.0206021	0.0358285	0.0609549	0.0178162	0.0152868
1.50	0.0246242	0.0398497	0.0643938	0.0174204	0.0149298

GRAPH 4

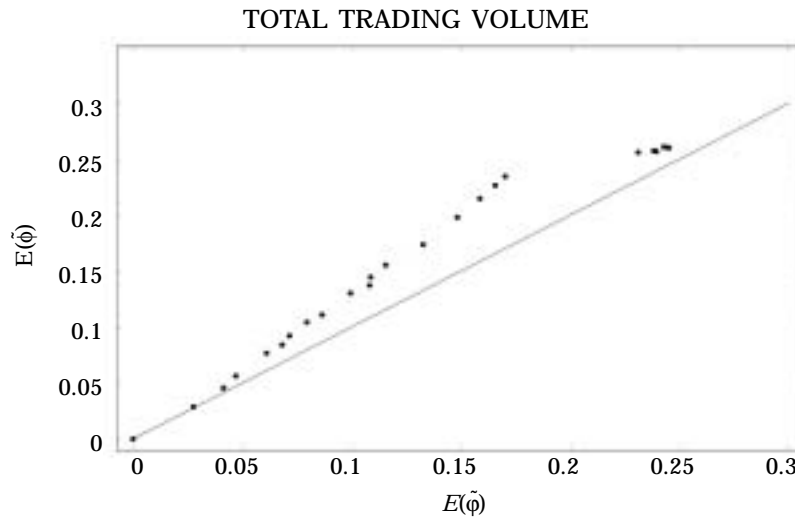


Table 1 shows the average trading volume according to different combinations of beliefs on the risky return, for the two cases of absence and presence of insider trading.

Average trading volume is higher if the market is characterized by insider trading activity, for the same belief on the risky return. This result is mirrored in Graph 4, which reports on the horizontal axis the average trading volume with insider trading and on the vertical axis the average trading volume without: all the points are above the bisector, confirming this result. Furthermore, this result holds true when the signals are perfect ($\beta = 1$).

Therefore, the total average trading volume is higher than the one which would characterize a market without insider trading, for any degree of precision of the signals. This is consistent with results presented in Cornell and Sirri (1992) and in Fische and Robe (2004).

Appendix 3 presents an analytical formulation of the total trading volume, in order to understand analytically the conditions which guarantee the result showed in this Section.

4.2 Trading Volume Net of Insider Trading

In the previous subsection we dealt with total trading volume, that is the volume traded by both the “outsiders” and the “insiders”. We want now to analyze the volume traded only by the uninformed agents. This analysis allows us to understand how the presence of agents endowed with private information could affect the “outsiders” trading choices, and then their trading volume.

The distribution of trading volume net of insiders sales can be defined as follow:

$$(55) \quad \tilde{\zeta} = (1 - \alpha_j) X_{ks}(j) - (Q_k^1 + Q_s^2) \quad \text{with prob. } q_i \Pr(S_k^1, S_s^2) \\ \forall i \in \{l, h\} \text{ and } \forall k, s \in \{L, H\}$$

Hence, given “market clearing” conditions, expected net trading volume is equal to:

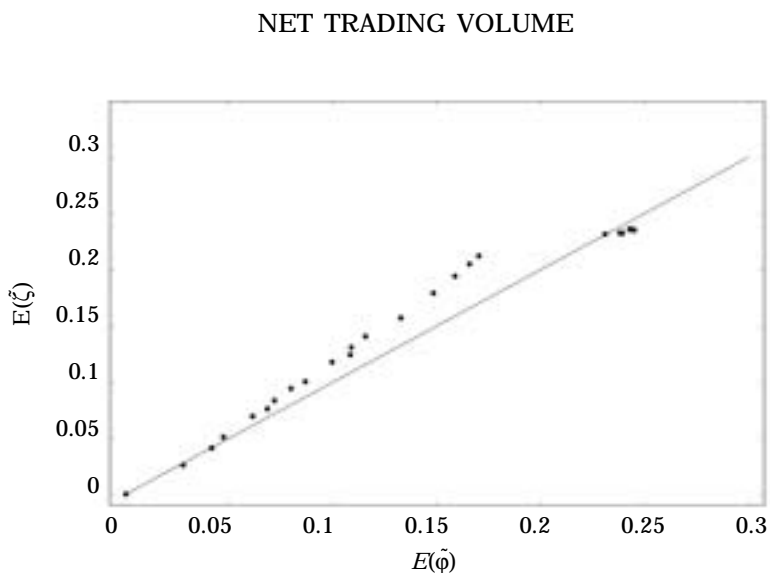
$$(56) \quad E(\tilde{\zeta}) = (1 - K_T) E(\tilde{\alpha})$$

that is equal to the average supply of risky asset by the “outsider early-diers”. Since $E(\tilde{\varphi}) = (1 - K_T) E(\tilde{\alpha})$, the difference between the two volumes is simply equal to the difference between the average supplies of risky asset.

$$(57) \quad E(\tilde{\zeta}) - E(\tilde{\varphi}) = [\Delta(1 - K)] E(\tilde{\alpha})$$

Therefore, $E(\tilde{\zeta})$ will be higher than $E(\tilde{\varphi})$ only if “outsiders” risky investment choice with insider trading is higher than the choice without insider trading. Graph 5 shows that the difference between net trading volumes is positive in most cases.

GRAPH 5



Contrary to the results presented in Appendix 3, we can state that:

1) it is sufficient and necessary that the following equation is realized

$$(58) \quad \Delta (1 - K) > 0$$

for the difference between net trading volumes to be positive, $E(\tilde{\zeta}) > E(\tilde{\varphi})$.

4.3 Net Trading Volume and Risk Sharing

Since we demonstrated that net trading volume increase is due to a higher “outsiders” investment in the risky asset, in this subsection we will analyze its non-univocal behavior. Table 2 shows that: *a)* for low values of θ_L the difference between net trading volumes is positive ($E(\tilde{\zeta}) - E(\tilde{\varphi}) > 0$); *b)* for high values of θ_L the difference between net trading volumes is negative ($E(\tilde{\zeta}) - E(\tilde{\varphi}) < 0$).

The explanation of this behaviour lies in the uninformed

TABLE 2

NET TRADING VOLUME ($\alpha_l = 0.48, \alpha_h = 0.53, \beta = 0.9$)					
θ_H/θ_L	0.75	0.80	0.85	0.90	0.95
$E(\tilde{\zeta}) - E(\tilde{\varphi})$					
1.25	8.3719·10 ⁻⁷	-0.000182544	0.0159379	0.000596159	-0.00668108
1.30	-0.00171303	0.00781884	0.0244906	-0.00527989	-0.00767224
1.35	0.0039404	0.0137592	0.0308229	-0.00581917	-0.0084498
1.40	0.00831271	0.0183715	0.0356447	-0.00627268	-0.00905238
1.45	0.0118175	0.0220529	0.0393641	-0.00668166	-0.00951148
1.50	0.014697	0.0250438	0.0420388	-0.00705238	-0.00985346
$\Delta (1 - K)$					
1.25	1.6578·10 ⁻⁶	-0.000361474	0.0315602	0.00118051	-0.0132299
1.30	-0.00339214	0.0154829	0.0484963	-0.0104552	-0.0151926
1.35	0.00780277	0.0272459	0.0610354	-0.0115231	-0.0167323
1.40	0.0164608	0.0363791	0.0705835	-0.0124211	-0.0179255
1.45	0.0234011	0.0436692	0.0779487	-0.013231	-0.0188346
1.50	0.0291031	0.0495917	0.0832451	-0.0139651	-0.0195118

agents “risk sharing”. If the risk sharing improves with insider trading, then the risky investment will increase and, as a consequence, net trading volume will be higher as well. Still, we have to explain why the risk sharing improves and why this happens only under some circumstances.

First of all, the risk sharing improves if the information in the market is greater and above all more precise. If we initially consider the model with perfect information, we can state that the future risky return is rightly revealed in a subset of the aggregate states. Without insider trading, since the private information can't be revealed in any aggregate states, “outsider late-diers” would buy the “long term” asset paying to “outsider early-diers” the same asset price independently from its future return. With insider trading, instead, they would pay a higher price if the information revealed a “High” future return and a lower price if it revealed a “Low” return. The improved “outsiders” risk sharing¹⁴, due to the presence of insider trading, means that at $t = 1$ “late-diers” agents will pay a price coherent with the future risky return and then with the consumption at $t = 2$. Without asymmetric information they would pay a price too low if $\tilde{\theta} = \theta_H$ or too high if $\tilde{\theta} = \theta_L$.

Introducing now signal imperfection, risk sharing will improve only if this imperfection does not lead to an uncertainty for the uninformed agents such that the positive effect above described is offset. If the signals were very imprecise, the “outsiders” would risk to pay a high price¹⁵ receiving then a “Low” future return. Therefore, signals imperfection reduces the benefit to pay a price coherent with the future risky return.

In this Section we have shown that for β values approximately lower than 0.75 the two “insiders” would not be disposed to trade because their expected profit would be negative. Therefore, it is possible to maintain that signal imperfection reduces the positive effect of “risk sharing” but it does not offset it entirely because for very imprecise signals informed agents would not enter the market. Table 3 points out the trend of the difference between

¹⁴ We can define it as “non-zero sum sharing”.

¹⁵ And not an intermediate price which would be paid without insider trading.

TABLE 3

RISKY INVESTMENT AND SIGNALS PRECISION INDEX
($\alpha_l = 0.48$, $\alpha_h = 0.53$)

θ_H/θ_L	0.75	0.80	0.85	0.90	0.95
$\Delta (1 - K) - (\beta = 0.9)$					
1.25	$1.6578 \cdot 10^{-6}$	-0.000361474	0.0315602	0.00118051	-0.0132299
1.30	-0.00339214	0.0154829	0.0484963	-0.0104552	-0.0151926
1.35	0.00780277	0.0272459	0.0610354	-0.0115231	-0.0167323
1.40	0.0164608	0.0363791	0.0705835	-0.0124211	-0.0179255
1.45	0.0234011	0.0436692	0.0779487	-0.013231	-0.0188346
1.50	0.0291031	0.0495917	0.0832451	-0.0139651	-0.0195118
$\Delta (1 - K) - (\beta = 0.75)$					
1.25	$4.72377 \cdot 10^{-6}$	-0.00181671	0.0205797	-0.00234768	-0.0140501
1.30	-0.00368648	0.00957682	0.0325673	-0.0120245	-0.0157694
1.35	0.00436931	0.0179802	0.0412361	-0.0129251	-0.0171822
1.40	0.0105801	0.0244264	0.0476295	-0.0136905	-0.0182743
1.45	0.0155227	0.0294851	0.0523801	-0.0143787	-0.0191044
1.50	0.0195411	0.0335047	0.0558867	-0.0150008	-0.019721
$\Delta (1 - K) - (\beta = 0.5)$					
1.25	0.0000168313	-0.00453575	0.00320091	-0.00796403	-0.0169672
1.30	-0.00452359	0.000054263	0.00788671	-0.0186949	-0.0170916
1.35	-0.00128368	0.00340929	0.0111084	-0.0172023	-0.0175579
1.40	0.00121538	0.0059151	0.0133537	-0.0160331	-0.0184745
1.45	0.00317657	0.00780947	0.0149136	-0.00152129	-0.0191661
1.50	0.00473203	0.00925065	0.015993	-0.0154842	-0.0196735

risky investment in the two cases of presence and absence of insider trading, according to different signals imprecision degrees.

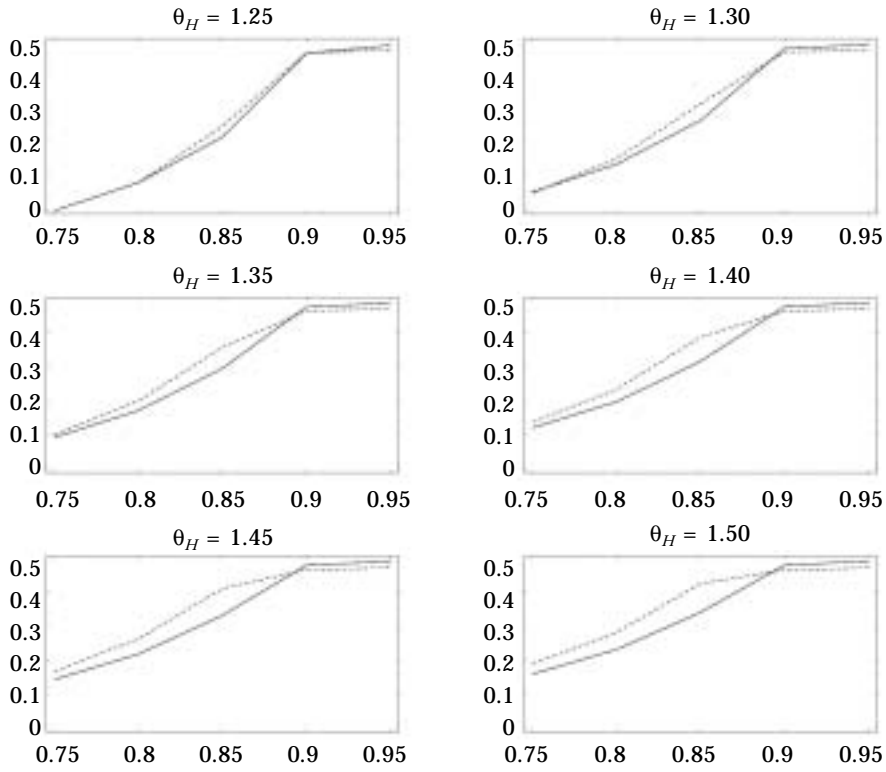
Now, we will focus on the reason why the net trading volume decreases only for high values of θ_L . At this regard it is necessary, also through a graphic analysis, to examine the behaviour of the risky investment choice in the two already mentioned cases.

The figures in Graph 6 show, for the same θ_H , the trend of $(1 - K_T)$ (continuous curve) and of $(1 - K_I)$ (broken curve) according to increasing values of θ_L . Of course, this analysis is valid for all those values of β such that "insiders" have incentive to enter the market.

Without insider trading, the rise of θ_L leads to increasing rises of the risky investment, but, for values of θ_L next to 1, the investment growing rate decreases considerably. As a matter of fact, for high values of θ_L the risky investment has already reached very high levels and then any further increase of θ_L would lead to a

GRAPH 6

RISKY INVESTMENT CHOICES ($\beta = 0.9$)



positive but decreasing benefit. The figures in Graph 6 clearly show this “turning point” coming out for $\theta_L \geq 0.9$.

Therefore, we can state that:

$$(59) \quad \frac{\partial(1 - K_T)}{\partial\theta_L} \Big|_{\forall\theta_L} > 0$$

$$(60) \quad \left[\frac{\partial(1 - K_T)}{\partial\theta_L} \Big|_{\theta_L < x} \right] > \left[\frac{\partial(1 - K_T)}{\partial\theta_L} \Big|_{\theta_L \geq x} \right]$$

A higher probability for “late-diers” agents to pay an asset price coherent with the future risky return (and then coherent with the

consumption at $t = 2$) leads to a greater risky investment. Nevertheless, this effect decreases considerably for high values of θ_L .

As a matter of fact, if θ_L increases, for the same θ_H and if the future return were "Low" (< 1), then uninformed agents would have a high probability to pay a price coherent with this return but closer and closer to the price they would have paid without insider trading¹⁶. Therefore, for high values of θ_L the positive effect of the "risk sharing" on the risky investment is definitely weaker.

Moreover, thanks to an improved risk sharing, the risky investment will reach before those high values for which any further increase of θ_L leads to a positive but decreasing benefit¹⁷. So, the "turning point" with insider trading will happen for a value of θ_L lower than the one characterizing the case without insider trading. As a consequence, according to different values of θ_L , we can formulate the following relations:

$$(61) \quad \frac{\partial(1 - K_I)}{\partial\theta_L} \Big|_{\forall\theta_L} > 0$$

$$(62) \quad \left[\frac{\partial(1 - K_I)}{\partial\theta_L} \Big|_{\theta_L < y} \right] > \left[\frac{\partial(1 - K_T)}{\partial\theta_L} \Big|_{\theta_L < y} \right] \text{ per } y < x$$

$$(63) \quad \left[\frac{\partial(1 - K_I)}{\partial\theta_L} \Big|_{y < \theta_L < x} \right] < \left[\frac{\partial(1 - K_T)}{\partial\theta_L} \Big|_{y < \theta_L < x} \right]$$

$$(64) \quad \left[\frac{\partial(1 - K_I)}{\partial\theta_L} \Big|_{\theta_L > x} \right] \cong \left[\frac{\partial(1 - K_T)}{\partial\theta_L} \Big|_{\theta_L > x} \right]$$

Finally, the evidence whereby $(1 - K_T)$ is higher than $(1 - K_I)$ for values of $\theta > x$ means that in the interval $y < \theta_L < x$, the risky investment without insider trading not only has grown more than the one with insider trading, but it has exceeded it.

¹⁶ The lower θ_L , the greater benefit increase.

¹⁷ This effect is as greater as higher θ_H

This analysis pointed out, therefore, that the “risk sharing” is able to explain the reason why net trading volume is higher with insider trading, despite imperfect information.

4.4 *Trading Volume and Signal Imperfection*

This last subsection shows the effect of signal imperfection on total and net trading volume.

4.4.1 Net Trading Volume

As far as the net trading volume is concerned we demonstrated that the difference between this volume and the one without insider trading is equal to the difference between the average supply of risky asset.

$$(65) \quad E(\tilde{\zeta}) - E(\tilde{\varphi}) = [\Delta(1 - K)] E(\tilde{\alpha})$$

Moreover, in the previous subsection we showed that a more imperfect signal reduces the risky investment with insider trading and the difference between this investment and the one without insider trading:

$$(66) \quad \frac{\partial \Delta(1 - K)}{\partial \beta} > 0$$

This, as already argued, is due to the greater uncertainty for uninformed agents, who face asset prices which could also incorporate a wrong information on the future risky return. The greater the signal imperfection, the higher the probability that asset prices reveal a wrong information.

Since $E(\tilde{\alpha})$ is not affected by the signal precision, we can conclude that:

$$(67) \quad \frac{\partial [E(\tilde{\xi}) - E(\tilde{\phi})]}{\partial \beta} > 0$$

The difference between net trading volumes will be as greater as higher the signals precision will be.

4.4.2 Total Trading Volume

Concerning total trading volume, it is opportune to anticipate equation (113), developed in Appendix 3, which represents the necessary and sufficient condition for the difference between total trading volumes to be positive $E(\tilde{\phi}) - E(\tilde{\varphi}) > 0$.

$$(68) \quad (1 - K_I) > (1 - K_T) \cdot \left(\frac{1 + \Omega}{\Omega} \right) \text{ with: } 0 < \frac{1 + \Omega}{\Omega} < 1$$

Maintaining the assumption of equiprobability of the future risky return ($\pi_H = \pi_L$), it follows:

$$(69) \quad \frac{\partial \Omega}{\partial \beta} = 0$$

Therefore, knowing that:

$$(70) \quad \frac{\partial (1 - K_I)}{\partial \beta} > 0$$

we can conclude that the rise in signal imperfection reduces the difference between the total trading volume in the two cases of presence and absence of insider trading.

$$(71) \quad \frac{\partial [E(\tilde{\phi}) - E(\tilde{\varphi})]}{\partial \beta} > 0$$

Analyzing numerical results we notice that the above mentioned difference, even though it decreases, remains positive for

every value of β . Therefore, the total amount of risky asset traded in the market is always higher (for deeply imperfect signals too) if this market is characterized by insider trading.

5. - Concluding Remarks

A model of insider trading is used to analyze the behaviour of trading volume in financial markets characterized by asymmetric information. This model extends the one in Bhattacharya and Nicodano (2001) by introducing competition among informed traders and imperfection of their private information.

As far as total trading volume is concerned, this is always higher than the volume traded absent insider trading. Moreover, the possibility to model uninformed agent as agents with well-specified preferences allows to understand if the presence of insider trading leads them to trade more. In the paper it is demonstrated that net trading volume is equal to early-diers average supply of risky asset. Since this is proportional to risky investment by outsiders, we sought for the conditions which justify a rise in the risky investment by uninformed agents.

Contrary to implications of adverse selection models, and consistent with empirical evidence, in most cases risky investment is higher if the market is characterized by asymmetric information. It is slightly lower only for high values of the “Low” future return. There is a clear explanation, based on an improved “risk sharing” among uninformed agents, for these results.

The presence of insider trading leads to a better “risk sharing” because the information revealed by prices to market participants is greater and more precise. As a matter of fact, in a subset of aggregate states, future risky return is correctly revealed, if the signals are perfect, and has a high probability to be correctly revealed, if the signals are imperfect. Therefore, with insider trading the “late-diers” would pay a price for the risky asset which is consistent with the future risky payoff. Without insider trading, instead, they would pay the same price independently from the future risky payoff.

As already mentioned, signal imperfection implies a lower risky investment if compared with the case of perfect private information. This is due to a greater uncertainty for uninformed agents, who could infer from prices the wrong future risky return since insiders receive imperfect signals.

Another important result deals with risky investment and net trading volume in the cases of monopoly (one only insider) and oligopoly (two insiders), for the same signals imperfection. Numerical results show that *ex-ante* risky investment and net trading volume are higher if there are two rather than one insider in the market. This is due to the higher number of possible aggregate states, which reduces the probability for “late-diers” to pay a very high price when the future risky return will be *L*-type and a very low price when the risky return will be *H*-type. So, the higher is the number of informed agents, the lower is the probability for “late-diers” to be in one of the farthest aggregate states inconsistently with the future risky return. Indeed, with two insiders who receive imperfect signals there are two more aggregate states, which take place when only one of them enters the market.

This model suffers from the assumption of risky neutrality which characterizes the informed agents; therefore, removing this assumption we would avoid that they were at the same time insiders and insurers. Moreover, a promising development of this research could consist in modifying outsiders' preferences as in Bhattacharya and Gale (1987).

1. - Proofs

1.1 *Proof of Lemma 1*

$$(72) \quad \text{Cov}(XY) = E(XY) - E(X)E(Y)$$

$$(73) \quad \text{Var}(X) = E(X^2) - E(X)^2$$

Then the covariance between the two signals will be equal to:

$$(74) \quad \text{Cov}(\tilde{S}^1 \tilde{S}^2) = E(\tilde{S}^1 \tilde{S}^2) - E(\tilde{S}^1) E(\tilde{S}^2)$$

Making $E(\tilde{S}^1 \tilde{S}^2)$ explicit, we obtain that:

$$(75) \quad E(\tilde{S}^1 \tilde{S}^2) = E(\tilde{\theta}^2) + E(\tilde{\theta})E(\tilde{\epsilon}_1) + E(\tilde{\theta})E(\tilde{\epsilon}_2) + E(\tilde{\epsilon}_1)E(\tilde{\epsilon}_2)$$

Since:

$$(76) \quad E(\tilde{\epsilon}) = 0$$

$$(77) \quad E(\tilde{S}) = E(\tilde{\theta})$$

then:

$$(78) \quad E(\tilde{S}^1 \tilde{S}^2) = E(\tilde{\theta}^2)$$

$$(79) \quad E(\tilde{S}^1)E(\tilde{S}^2) = E(\tilde{\theta})^2$$

Therefore:

$$(80) \quad \text{Cov}(\tilde{S}^1 \tilde{S}^2) = E(\tilde{\theta}^2) - E(\tilde{\theta})^2$$

$$(81) \quad \text{Cov}(\tilde{S}^1 \tilde{S}^2) = \text{Var}(\tilde{\theta})$$

Moreover, given $\sigma_{\tilde{S}^1} = \sigma_{\tilde{S}^2}$,

$$(82) \quad \sigma_{\tilde{S}^1} = \sigma_{\tilde{S}^2} = \text{Var}(\tilde{S})$$

So, we have proved that:

$$(83) \quad \text{Corr}(\tilde{S}^1, \tilde{S}^2) = \frac{\text{Var}(\tilde{\theta})}{\text{Var}(\tilde{S})}$$

with:

$$(84) \quad \text{Var}(\tilde{S}) = \text{Var}(\tilde{\theta}) + \text{Var}(\tilde{\varepsilon})$$

1.2 Proof of Lemma 2

Given the equiprobability assumption of the risky asset distribution ($\pi = 1/2$):

$$(85) \quad E(\tilde{\theta}) = \frac{1}{2} \sum_{j=L,H} \theta_j$$

$$(86) \quad E(\tilde{\theta}|S_k) = \sum_{j=L,H} \theta_j \Pr(\theta_j|S_k)$$

$$(87) \quad \begin{aligned} \text{Var}(\tilde{\theta}) &= \frac{1}{2} \sum_{j=L,H} (\theta_j - E(\tilde{\theta}))^2 \\ &= \frac{1}{4} (\theta_H - \theta_L)^2 \end{aligned}$$

$$(88) \quad \begin{aligned} \text{Var}(\tilde{\theta}|S_k) &= \sum_{j=L,H} (\theta_j - E(\tilde{\theta}|S_k))^2 \Pr(\theta_j|S_k) \\ &= \frac{1}{4} (1 - \beta^2) (\theta_H - \theta_L)^2 \end{aligned}$$

$$\Psi = 1 - \left(\frac{\text{Var}(\tilde{\theta}|S_k)}{\text{Var}(\tilde{\theta})} \right)$$

$$\begin{aligned}
 (89) \quad &= 1 - \left(\frac{\frac{1}{4}(1 - \beta^2)(\theta_H - \theta_L)^2}{\frac{1}{4}(\theta_H - \theta_L)^2} \right) \\
 &= \beta^2
 \end{aligned}$$

2. - Insiders' Expected Profit

It is necessary to understand if and how greater “outsiders” uncertainty could benefit the informed agents. An “insider’s” expected profit can be divided in two parts, the expected gain ($E(\Gamma)$), that is obtained if the signal is consistent with the future return, and the expected loss ($E(Y)$), that is suffered if the signal reveals a future return different from the real one. Because of the symmetry of the signals, we can focus our analysis just on “insider 1” and then consider the same results for “insider 2”.

$$(90) \quad E(\Pi^1) = E(\Gamma^1) - E(Y^1)$$

where:

$$(91) \quad E(\Gamma^1) = \sum_{i=l,h} \sum_{s=L,H} q_i \left[(P_{Ls}(i) - \theta_L) Q^1 \cdot \Pr(\theta_L, S_L^1, S_s^2) \right]$$

$$(92) \quad E(Y^1) = \sum_{i=l,h} \sum_{s=L,H} q_i \left[(P_{Ls}(i) - \theta_H) Q^1 \cdot \Pr(\theta_H, S_L^1, S_s^2) \right]$$

As regards the same model with perfect information, $\Psi = 1$, (denoted with p):

$$(93) \quad E(\Pi_p^1) = E(\Gamma_p^1) - E(Y_p^1)$$

where:

$$(94) \quad E(\Gamma_p^1) = \sum_{i=l,h} q_i \left[(P_{LL}(i)_p - \theta_L) Q_p^1 \cdot \Pr(\theta_L, S_L^1, S_L^2) \right]$$

$$(95) \quad E(Y_p^1) = 0$$

Analyzing numerical results of both the two above mentioned models, the following relations hold for both “insiders”:

$$(96) \quad E(\Gamma) > E(\Pi_p)$$

$$(97) \quad E(\Pi) < E(\Pi_p)$$

The expected gain with imperfect information is higher than the expected profit with perfect information¹⁸, when the signal is right, but the expected loss, when the signal is wrong, offsets and exceeds the expected gain, thus making the expected profit always lower than the one with perfect information. The possibility that the private information could be revealed wrongly represents an advantage for every “insider” because it increases her expected gain, but, on the other side, the signal imperfection leads to the possibility to have positive expected losses.

We compare below the expected gain, loss and profit, obtained by an oligopolistic “insider” with an imperfect signal. We consider different signal imperfection degrees and use as “benchmark” the expected profit of an oligopolistic “insider” endowed with perfect information. When the signal imperfection increases (i.e. when β decreases) the expected gain rises, because of the greater uncertainty transferred to the outsiders, but by a rate of growth lower than the one which characterizes the expected loss.

$$(98) \quad \frac{\partial E(\Gamma)}{\partial \beta} < 0; \quad \frac{\partial E(Y)}{\partial \beta} < 0$$

$$(99) \quad \left| \frac{\partial E(\Gamma)}{\partial \beta} \right| < \left| \frac{\partial E(Y)}{\partial \beta} \right|$$

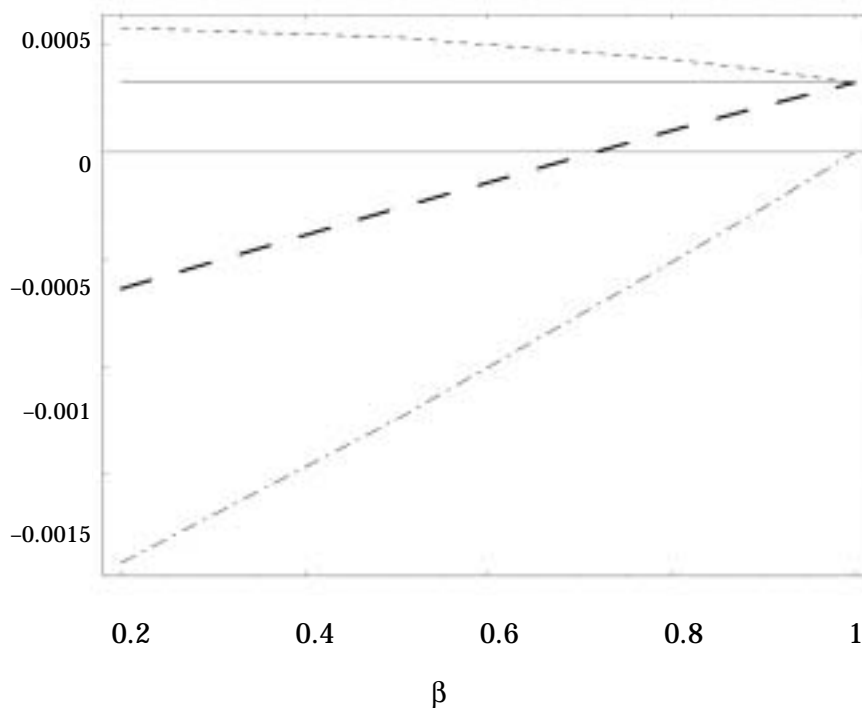
¹⁸ The expected gain with imperfect information is also higher than the expected gain with perfect information because, if the signal is perfect, the expected loss is always equal to 0.

$$(100) \quad \frac{\partial E(\Pi)}{\partial \beta} > 0$$

Graph 7 shows that, under the same conditions, the expected profit of an oligopolistic “insider” reaches its maximum when the signal is perfect ($\Psi = 1$). The “benchmark” is represented by the horizontal continuous line, the expected gain, loss and profit respectively by the broken curve with small dashes, the broken curve with points and dashes and the bold broken curve with big dashes.

GRAPH 7

COMPONENTS OF THE EXPECTED PROFIT OF AN OLIGOPOLISTIC INSIDER ACCORDING TO DIFFERENT SIGNAL IMPERFECTION DEGREES



3. - Trading Volume Analytical Formulation

An analytical formulation allows to confirm and better interpret results obtained from the model's numerical resolution. Using equations (54) and (53) we obtain the following equations.

$$(101) \quad E(\tilde{\varphi}) = (1 - K_T) E(\tilde{\alpha})$$

$$(102) \quad E(\tilde{\phi}) = (1 - K_I) E(\tilde{\alpha}) + E(\tilde{Q}_A)$$

where Q_A is the aggregate amount of risky asset sold by the two "insiders": $Q_A = Q_k^1 + Q_s^2 \forall k, s \in \{L, H\}$. The average trading volume with insider trading is equal to "early-diers" and "insider" average supplies of risky asset¹⁹.

Expressing trading volume with insider trading as a function only of "outsiders" risky investment, we obtain:

$$(103) \quad E(\tilde{\phi}) = (1 - K_I) \left[\alpha_h + \left(\frac{(\beta\pi_L + \gamma)(\beta + 2\gamma)}{(1 - \alpha_h)} - q_I \right) (\alpha_h - \alpha_l) \right]$$

with:

$$Q_A = (1 - K_I) \frac{(\alpha_h - \alpha_l)}{(1 - \alpha_h)}$$

Then, considering the partial derivative of trading volume, we can conclude that:

$$(104) \quad \frac{\partial E(\tilde{\phi})}{\partial(1 - K_I)} > 0$$

$$(105) \quad \frac{\partial E(\tilde{\phi})}{\partial(1 - K_I)} > \frac{\partial E(\tilde{\varphi})}{\partial(1 - K_T)}$$

Now, it is necessary to analyze the difference between trad-

¹⁹ "Insiders" average supply of risky asset, $E(\tilde{Q}_A)$, is equal to: $(Q^1 + Q^2) \cdot \Pr(S_L^1, S_H^2) + Q^1 \Pr(S_L^1, S_H^1) + Q^2 \Pr(S_H^1, S_H^2) + 0 \cdot \Pr(S_H^1, S_H^1)$.

ing volume in the two cases of presence and absence of insider trading.

$$\begin{aligned}
 E(\tilde{\phi}) - E(\tilde{\varphi}) &= (1 - K_I) \left[\alpha_h + \left(\frac{(\beta\pi_L + \gamma)(\beta + 2\gamma)}{(1 - \alpha_h)} - q_I \right) (\alpha_h - \alpha_I) \right] + \\
 (106) \quad &\quad - (1 - K_T) [\alpha_h - q_I (\alpha_h - \alpha_I)] \\
 &= (1 - K_T) \left[\frac{(\beta\pi_L + \gamma)(\beta + 2\gamma)}{(1 - \alpha_h)} (\alpha_h - \alpha_I) \right] + \\
 &\quad + \Delta(1 - K) \left[\alpha_h + \left(\frac{(\beta\pi_L + \gamma)(\beta + 2\gamma)}{(1 - \alpha_h)} - q_I \right) (\alpha_h - \alpha_I) \right]
 \end{aligned}$$

The following remarks rise from equation (106).

1) if $\Delta(1 - K) = 0$, then trading volume with insider trading is always higher than trading volume without;

2) if $\Delta(1 - K) > 0$, then trading volume with insider trading is always higher than trading volume without;

3) if $\Delta(1 - K) < 0$, then the difference between trading volume will be positive ($E(\tilde{\phi}) > E(\tilde{\varphi})$) if:

$$(107) \quad \Delta(1 - K) > (1 - K_T) \left[- \frac{(1 - \alpha_h)(\alpha_h - q_I(\alpha_h - \alpha_I))}{(\alpha_h - \alpha_I)(\beta\pi_L + \gamma)(\beta + 2\gamma)} - 1 \right]^{-1}$$

and negative ($E(\tilde{\varphi}) > E(\tilde{\phi})$) if:

$$(108) \quad \Delta(1 - K) < (1 - K_T) \left[- \frac{(1 - \alpha_h)(\alpha_h - q_I(\alpha_h - \alpha_I))}{(\alpha_h - \alpha_I)(\beta\pi_L + \gamma)(\beta + 2\gamma)} - 1 \right]^{-1}$$

Dwelling upon the third case and considering

$$(109) \quad \Omega \equiv \left[- \frac{(1 - \alpha_h)(\alpha_h - q_I(\alpha_h - \alpha_I))}{(\alpha_h - \alpha_I)(\beta\pi_L + \gamma)(\beta + 2\gamma)} - 1 \right]$$

it is important to precise that Ω is always negative (and ≤ -1) conditionally to the three constraints on α_i , $i \in \{l, h\}$:

$$\alpha_l > 0; \alpha_h > 0; \alpha_h > \alpha_l$$

Moreover, we can observe that the following derivatives of the difference between trading volumes are both positive:

$$(110) \quad \frac{\partial [E(\tilde{\phi}) - E(\tilde{\varphi})]}{\partial(1 - K_T)} = \frac{(\beta\pi_L + \gamma)(\beta + 2\gamma)}{(1 - \alpha_h)} (\alpha_h - \alpha_l) > 0$$

$$(111) \quad \frac{\partial [E(\tilde{\phi}) - E(\tilde{\varphi})]}{\partial\Delta(1 - K)} = \left[\alpha_h + \left(\frac{(\beta\pi_L + \gamma)(\beta + 2\gamma)}{(1 - \alpha_h)} - q_l \right) (\alpha_h - \alpha_l) \right] > 0$$

Table 4 and the figures in Graph 8 report the difference between trading volumes (continuous curve) and the differences between investment levels of the risky asset (broken curve).

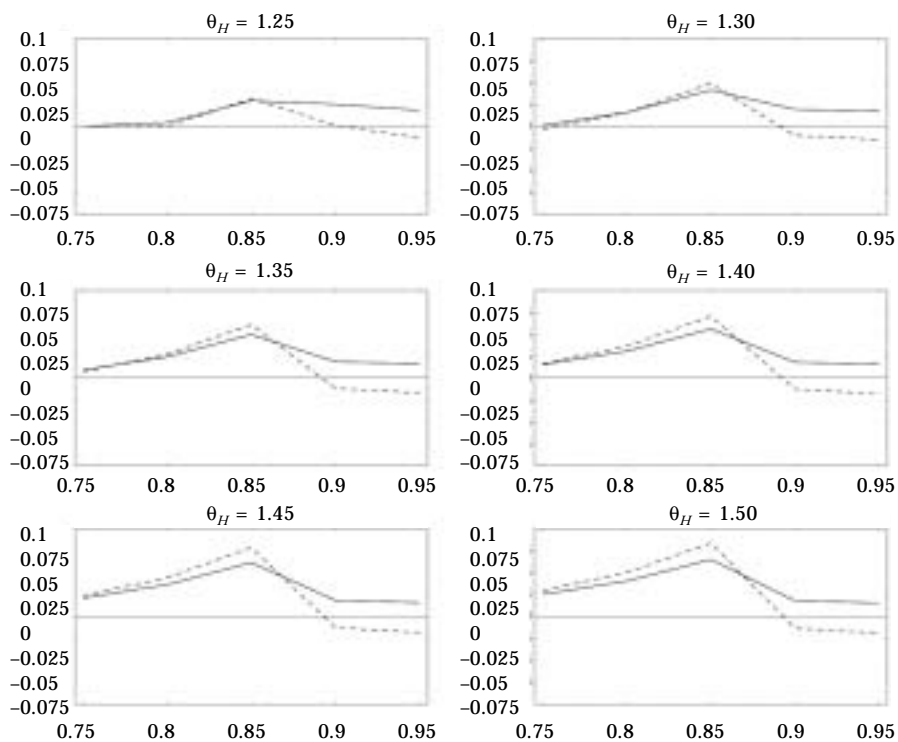
TABLE 4

TRADING VOLUME AND RISKY INVESTMENT
($\alpha_l = 0.48, \alpha_h = 0.53, \beta = 0.9$)

θ_H/θ_L	0.75	0.80	0.85	0.90	0.95
<i>Differences between total trading volumes - $E(\tilde{\phi}) - E(\tilde{\varphi})$</i>					
1.25	9.2353·10 ⁻⁷	0.0041554	0.0290138	0.0249946	0.0181854
1.30	0.00101573	0.0158286	0.0410354	0.0192494	0.017176
1.35	0.00930625	0.0243234	0.0496909	0.0187089	0.016381
1.40	0.0156226	0.0307804	0.0561147	0.0182442	0.0157618
1.45	0.0206021	0.0358285	0.0609549	0.0178162	0.0152868
1.50	0.0246242	0.0398497	0.0643938	0.0174204	0.0149298
<i>Risk investment levels without insider trading - $(1 - K_T)$</i>					
1.25	-3.46048·10 ⁻⁸	0.0819148	0.214266	0.457511	0.48072
1.30	0.0546928	0.135101	0.262545	0.471606	0.482339
1.35	0.0930752	0.171362	0.293684	0.47265	0.483552
1.40	0.120965	0.196909	0.314254	0.473338	0.484432
1.45	0.14175	0.215311	0.327959	0.473791	0.485043
1.50	0.157527	0.228759	0.33703	0.474053	0.485437
<i>Differences between risk investments - $\Delta(1 - K)$</i>					
1.25	1.6578·10 ⁻⁶	-0.000361474	0.0315602	0.00118051	-0.0132299
1.30	-0.00339214	0.0154829	0.0484963	-0.0104552	-0.0151926
1.35	0.00780277	0.0272459	0.0610354	-0.0115231	-0.0167323
1.40	0.0164608	0.0363791	0.0705835	-0.0124211	-0.0179255
1.45	0.0234011	0.0436692	0.0779487	-0.013231	-0.0188346
1.50	0.0291031	0.0495917	0.0832451	-0.0139651	-0.0195118

GRAPH 8

DIFFERENCE BETWEEN TRADING VOLUMES AND
DIFFERENCE BETWEEN RISKY INVESTMENTS



Through this analysis of the components which affect trading volume, we attain to the following conclusions:

1) it is sufficient but not necessary, for the difference between total trading volumes to be positive, $E(\tilde{\phi}) > E(\tilde{\phi})$, that the following equation is realized:

$$(112) \quad \Delta(1 - K) \geq 0$$

2) it is sufficient and necessary, for the difference between

total trading volumes to be positive, $E(\tilde{\phi}) > E(\tilde{\phi})$, that the following equation is realized:

$$(113) \quad (1 - K_I) > (1 - K_T) \cdot \left(\frac{1 + \Omega}{\Omega} \right) \quad \text{with} \quad 0 < \left(\frac{1 + \Omega}{\Omega} \right) < 1$$

These results, however, can not be expressed as functions of the exogenous variables of the model because of its numerical resolution.

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