Optimal Rationing in IPOs with Risk Averse Institutional Investors

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Using a mechanism design approach, we derive endogenously the optimal IPO mechanism when institutional investors are risk averse. We show that the optimal allocation rule is such that all the institutional investors with sufficiently good information are allocated a positive quantity of shares which is increasing in the quality of the their information. Additionally, we also derive the optimal rationing scheme which is uniform, i.e. all institutional investors are rationed by the same amount of shares [JEL Classification: D8, G2].

In questo articolo, usando un approccio di mechanism design, *deriviamo endogenamente il meccanismo di offerta pubblica ottimale quando gli investitori istituzionali sono avversi al rischio. Il meccanismo è tale per cui tutti gli investitori istituzionali con un'informazione sufficientemente buona ricevono una quantità positiva di azioni, che risulta inoltre crescente nella qualità dell'informazione. Infine, deriviamo anche lo schema di razionamento ottimale che è uniforme cioè tutti gli investitori istituzionali sono razionati per un uguale numero di azioni.*

1. - Introduction

The extensive literature on Initial Public Offerings (IPOs) of new issues has so far mainly focused on the explanation of some apparent market pathologies such as underpricing, oversubscription and long-term under-performances (Rock, 1986, Allen and Faulhaber, 1989; Benveniste and Spindt, 1989; Cornelli and Goldreich, 2001). Relatively few papers have instead investigated the optimal design of IPOs, namely the choice of optimal pricing and

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rationing rules. This paper belongs to this less popular strand of the IPO literature. In particular, we investigate the characteristics of the optimal IPO, focusing on the allocation and rationing rule, when institutional investors are risk averse. Institutional investors are typically considered regular participants in this kind of market operations and also endowed with private information about the market value of the issue on sale. The standard IPO set up in the literature always assumes institutional investors to be risk neutral. This assumption has a couple of drawbacks. First, in order to avoid trivial outcomes (see Bennouri and Falconieri, 2005*b*) it must always be joint to the assumption of cash constrained retail investors, which is not necessarily true in practice because retail investors may be individually cash constrained but not in the aggregate (see Ellul and Pagano, 2006). Secondly, from the market microstructure and exchange rate market literature (see O'Hara, 1995; Lyons, 2003) we know that even institutional investors might exhibit risk aversion to the "inventory risk", i.e. the risk associated with the composition of their portfolio. Therefore we think it is interesting to investigate how the optimal IPO would change when we introduce risk aversion on the institutional investors' side while assuming that retail investors are not subject to cash constraints. The question is relevant because in the presence of risk aversion the principal (the selling firm) faces a trade-off between risk insurance and incentives.[1] The objective of the paper is to see how the allocation rule and thus possibly the need for rationing changes in this case. In a previous paper (Bennouri and Falconieri, 2005*b*), we already show that with risk neutral institutional investors the seller always optimally allocate all the shares to the retail investors when they are not subject to cash constraint. We show here that this is no longer the case with risk averse institutional investors. Specifically, we derive the optimal allocation and rationing rules in this context. Rationing, in our IPO, occurs when there is an oversubscription of the issue. Then, all the shares are allotted to the institutional investors but to an amount smaller than the demanded one.

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[1] This is a very well known fact in the literature on optimal auction (see e.g. Maskin E. - Riley J., 1984 and Esö P., 2005).

The paper also provides a methodological contribution since we use an optimal auction approach to describe the IPO process. IPOs are a very natural example of an optimal auction design problem because of their peculiar informational structure which exhibits a number of agency problems and because the currently used IPO mechanisms are either auction or auction-like mechanisms (first price auctions, bookbuilding, mise en vente). As a matter of fact, the idea of using an optimal mechanism design approach to describe initial offerings is not new in the literature. Biais, Bossaerts and Rochet (2002) employ the same approach to show that fixed price offering is not an optimal selling procedure in presence of asymmetries of information. They derive the optimal selling mechanism that enables the seller to extract all the information from the coalition and find that this mechanism specifies a price schedule decreasing in the quantity allocated to retail investors. This price schedule eliminates the winner's curse problem faced by the uninformed investors since, when the stock of shares they are allotted is quite large the price is set at a relatively low level. They also find that the optimal mechanism exhibits underpricing to institutional investors. This underpricing represents the informational rents left by the selling firm to the informed investors. The main difference between their model and this one is on the role of the intermediary in the IPO. Biais, Bossaerts and Rochet assume that, because of their long-term relationship, the intermediary forms a (perfect) coalition with the informed investors. We instead assume that the intermediary in charge with the marketing of the issue serves the interests of the issuing firm, in line with most of the IPO literature. We denote this coalition firm/intermediary as the seller. The role an intermediary plays in an IPO depends on the financial market we refer to. The assumption of a coalition between the intermediary and the institutional investors can be realistic in European financial markets where firms and intermediaries usually interact only occasionally for the initial offering whereas the intermediary and the professional investors are often engaged in long term and repeated relationships. However, the same assumption is not reasonable in the American financial markets where, on the contrary, firms rely on intermediaries for a number of other financial operations besides the issuing procedures. This makes their relationship much more stable over time.

A similar approach is also used by Biais and Faugeron-Crouzet (2002) and Maksimovic and Pichler (2002). Biais and Faugeron-Crouzet (2002) compare the performances of alternative IPO procedures and find that in general auction-like mechanisms such as Mise en Vente and Bookbuilding are more informationally efficient and can lead to price discovery. They also show that pure auction mechanisms might lead to tacit collusion among the bidders and, consequently, to inefficient outcomes.

Maksimovic and Pichler (2002) instead look at the optimal amount of information gathering by comparing private and public offering with a particular attention to the underpricing determinants. They find that in absence of any allocation constraint and under risk neutrality, the seller is able to design a selling mechanism with zero underpricing (no rents to the informed investors). We prove a similar result in a particular subcase of our model. Taking this result as a benchmark, the authors then link the existence and magnitude of underpricing to various constraint eventually imposed on the bookbuilding procedure. Both of them however consider risk neutral institutional investors, which implies a very different solution for the optimal IPO.

A closer paper to ours is the one by Rajan and Parlour (2005) in which they specifically look at the rationing in IPOs. They compare different rationing rules and they show that it may be indeed optimal, i.e. revenue-enhancing, for the selling firm to ration the investors instead of choosing the market clearing price. The reason is twofold. On the one hand, rationing weakens the winner's curse problem and, on the other hand, rationing allows for discretionary allocations, which may benefit the seller in the long term. Our paper differs from theirs because we do not restrict the choice to some specific rationing rule, both the optimal allocation rule and rationing scheme is endogenously determined. In this paper we are not interested in deriving the optimal pricing rule, however we can show (Bennouri and Falconieri, 2005b) that the optimal pricing rule is a uniform price for all investors.

In our IPO a firm wants to sell a given amount of shares to two possible class of investors: n institutional investors and, a group of atomistic investors (*retail investors*) who only occasionally participate in the offering. Each institutional investor receives a signal about the market value of the asset. Signals are i.i.d. and the true value of the asset is given by the average of all signals (common value auction). Institutional investors are risk averse. Retail investors have no information about the value of the asset on sale. They are characterized by an initial endowment W they are willing to invest in the new shares which is common knowledge in the IPO. The seller tries to elicit the information from institutional investors by offering them an optimal contract which fixes the quantity and the price investors have to pay for each share. We consider linear price schedule. Institutional investors' preferences are described by a concave utility function to capture their risk aversion. We first compute the optimal quantities for both institutional and retail investors, which indicates that the seller rewards better information received by the informed investors. Additionally, whenever an oversubscription of shares occurs, the optimal IPO is such that *a)* all the issues are assigned to institutional investors; and *b)* institutional investors are *uniformly* rationed, in the sense that they are rationed by the same amount of shares.

The paper is organized as follows. In the next section we set up the model and the mechanism design problem for the seller. In Section 3, we derive the optimal quantities from the relaxed problem in which the monotonicity constraint is ignored and checked ex-post. Section 4 analyses the case of an oversubscription of shares and derives the rationing scheme. Section 5 links our theoretical result up to the empirical evidence on current IPO mechanisms.

The last section summarizes and concludes.

2. - The Model

2.1 *Agents*

Consider a firm offering a fixed amount of shares Q in an

IPO, which, without loss of generality, is normalized to 1.[2] An intermediary is in charge of marketing the issues. He serves the firm's interests, so, hereafter, we denote the coalition firm-intermediary as the *seller*.[3] The seller's objective is to maximize the proceeds from the sale.

There are two classes of investors participating into the IPO, institutional and retail investors. There is a set N of $n > 2$ institutional investors. Their preferences are described by the following utility function:

$$u_i(p_i, q_i, v) = q_i\left[\left(\alpha v - \frac{\delta}{2} q_i\right) - p_i\right] \quad \text{with } i = 1, 2, \ldots n \text{ and } \alpha > 1$$

where q_i is the quantity assigned to investor i and p_i is the price per share investor i has to pay. We denote by $T_i = p_i q_i$ the payment from investor i to the seller for the whole quantity q_i. Notice that, the above utility function is linear in the transfer T_i.[4] Also, define $z(q_i, v) = q_i(\alpha v - \delta/2\, q_i)$, which is concave in the quantity q_i. Notice that, by the concavity of $z(.)$, institutional investors exhibit an increasing risk aversion in the quantity they receive. A possible reason for this kind of preferences is that, investors are averse to the *inventory risk*, that is the risk associated to the composition of their portfolio. Furthermore, the function z looks very much like a Mean-Variance utility function with the only difference that the expected value v is weighed more than the transfer T_i. The parameter α measures the regular investor's aggressiveness: the higher α is, the more aggressive the investor is in the IPO procedure. To see it differently. Assuming that $\alpha > 1$ has an intuitive justification in our setting. For $q_i \to 0$, the risk exposure

[2] The quantity to be sold is not always fixed in reality. In American IPOs, for instance, the seller can exploit the so-called over allotment option: he can increase the quantity announced at the beginning of the IPO by at most 10%. For the sake of simplicity we do not allow this option. It is likely however that introducing the possibility to vary the quantity during the auction would allow the design of more efficient mechanisms.

[3] The need for an intermediary is often due to the fact that the firm cannot reach directly the investors.

[4] This kind of utility functions exhibiting linearity in the total payment, are very common in the auction literature. (See for instance CRÈMER J. - MCLEAN R.P., 1988).

of informed bidders becomes equal to that of uninformed, i.e. they are both risk neutral for small quantities of shares. If $\alpha = 1$, informed investors would not have any specific role since they would behave exactly in the same way as the uninformed. The assumption $\alpha > 1$ implies that, given the same risk exposure, they will however behave more aggressively than uninformed investors in the offering due to their superior information. We think this is a very reasonable assumption.

After the intermediary has completed his investigation of the issuing firm, the information acquired is made public in the prospectus. Some additional information about the future market value of the asset may however become available to institutional investors. In particular, we assume that each institutional investors receive a signal si about the future market value of the shares on the stock market. The value of the shares v is given by the average of all the signals, that is

$$v = \frac{1}{n}\sum_i s_i$$

Signals are i.i.d. according to a uniform distribution defined on $\Omega_i = [\underline{s}; \bar{s}]$, so the cumulative distribution function is

$$F_i(s_i) = \frac{s_i - s}{\Delta s}$$

and the density function

$$f_i(s_i) = \frac{1}{\Delta s}$$

Let us also denote by $f(s)$ the joint density function so that

$$f(s) = f(s_i, s_{-i}) = \prod_i f_i(s_i) \quad \text{with} \quad s = (s_i, s_{-i}) \in \Omega = \underset{i}{\otimes}\Omega_i = \left[\underline{s}, \bar{s}\right]^n$$

We assume a continuum of competitive, risk neutral small or retail bidders. The total mass of these retail bidders is normalized to one. Each of them has a maximum amount of wealth W he is willing to invest in the issued asset. We set $W \geq \bar{s}$ so that retail investors are never cash constrained, even when they are assigned the whole quantity. We denote by q_R the quantity they receive in equilibrium; by p_R the price per unit of share they are asked and by $T_R = q_R p_R$ the total transfer to the seller.

The seller wants to maximize the proceeds of the sale, computed in expected value on Ω. Its objective function is thus,

$$U_F = E_s\left[\sum_i T_i + T_R\right]$$

where T_i is the transfer paid by the *ith* informed bidders and T_R is the transfer paid by retail bidders.

Retail investors share the same information as the seller. They do not receive any signal about the market value of the asset and only observe the density $f_i(s_i)$ of signals.

In order to extract the information from the institutional investors, the seller offers a contract specifying the quantity and the price per share to pay to each institutional investor and to retail investors. By using the *revelation principle*, we can focus on *direct mechanisms* in which the firm asks each investor to announce his signal and then fixes the quantity and the price as a function of their announcements in such a way to induce them to truthfully reveal their information (see Fudenberg and Tirole, 1991).

The contract the seller proposes can thus be written in the following way:

$$\Gamma = \left\{p_i(\hat{s}_i, \hat{s}_{-i}), q_i(\hat{s}_i, \hat{s}_{-i}); p_R(\hat{s}_i, \hat{s}_{-i}), q_R(\hat{s}_i, \hat{s}_{-i})\right\} \quad \forall i$$

with $\hat{s}_i$ being institutional investor i´s announcement and $\hat{s}_{-i}$ the vector of the announcements of all the other investors.

We assume that all the shares issued must be allocated between institutional and retail investors.[5] Consequently, by choosing the vector $\{q_i\}_i$ the firm implicitly determines the number of shares to allocate to retail investors, which is given by:

$$q_R = 1 - \sum_i q_i$$

2.2 *The IPO Design Problem*

The optimal IPO mechanism for the firm, denoted by $\Gamma^* = \{p_i^* (\hat{s}_i, \hat{s}_{-i}), q_i^* (\hat{s}_i, \hat{s}_{-i}), \forall_i; p_R^* (\hat{s}_i, \hat{s}_{-i})\}$, is the solution to the following optimization program:

$$\max_{q_i} U_F = E_s\left[\sum_i T_i(\hat{s}_i, \hat{s}_{-i}) + T_R(\hat{s}_i, \hat{s}_{-i})\right]$$

subject to the standard constraints (see Fudenberg and Tirole, 1991):

- *Retail Investors' Participation Constraint (RPC):*

$$E_S\ [q_R(s_i, s_{-i})(v - p_R(s_i, s_{-i})] \geq 0$$

which requires their expected payoff to be larger than their reservation utility which, without loss of generality, may be set equal to zero.

- *Institutional Investors' Participation Constraint (IPC):*

$$U_i(s_i, s_i) = E_{s-i}\left\{q_i(s_i, s_{-i})\left[\left(\alpha v - \frac{\delta}{2} q_i(s_i, s_{-i})\right) - p_i(s_i, s_{-i})\right]\right\} \geq 0 \quad \forall s_i, \forall_i$$

which has the same meaning as the RPC.

[5] This means that we do not allow the so-called *firm-commitment contract* of underwriting widely used in practice, in particular in the US financial market. With this kind of contract, the underwriter (the intermediary) commits to buy all the shares not sold during the initial offering (see RITTER J.R., 1987 and BIAIS B. - FAUGERON-CROUZET A.M., 2002, for more details respectively on the US and the European markets).

• *Institutional Investors' Incentive Compatibility Constraint (IIC):*

$$U_i(\hat{s}_i, s_i) \leq U_i(s_i, s_i) \quad \forall s_i, \forall \hat{s}_i \text{ and } \forall_i$$

with

$$U_i(\hat{s}_i, s_i) = E_{s-i}\left\{q_i(\hat{s}_i, s_{-i})\left[\left(\alpha v - \frac{\delta}{2} q_i(\hat{s}_i, s_{-i})\right) - p_i(\hat{s}_i, s_{-i})\right]\right\}$$

which ensures that institutional investors do not have incentive to mis-represent their type — the signal they receive — to the firm.

• Last, the *Full Allocation Constraint* (FAC):

$$\sum_i q_i(s_i, s_{-i}) + q_R(s_i, s_{-i}) = 1$$

and the *feasibility constraint:*

$$q_i(s_i, s_{-i}) \geq 0 \quad \forall_i, \forall s_i \text{ and } s_{-i}$$

Before turning to the solution of this optimization problem, we make the following assumptions on the function $z(q_i, v) = q_i(\alpha v - \delta/2\, q_i)$ and on the hazard rate:

• **A.1** z is increasing in q_i and v thus $z_1 = \alpha v - \delta q_i > 0$ and $z_2 = \alpha q_i > 0$;
this assumption holds under the condition that $\underline{s} > \delta/\alpha$ which we assume hereafter.

• **A.2** $z_{11} = -\delta < 0$;
which implies that informed buyers take into account the effect of the inventory risk when evaluating the quantity of shares they are assigned.

• **A.3** $z(0, v) = 0$ for all v;

• **A.4** $z_{12} = \alpha > 0$;
this is the usual single-crossing condition used in the mechanism design literature. It states that the agent's type affects the marginal rate of substitution between the allocations and the payment

in a monotonic way. The single-crossing condition is a necessary condition for the mechanism to be implementable (see Fudenberg and Tirole, 1991).

- **A.5** $z_{122} = z_{112} = 0$;

this assumption puts a restriction on the marginal rate of substitution between the quantity and the asset value which must be non-increasing in v and non-decreasing in q.

- **A.6** $z_{12}(0, v) = \alpha \geq 1$

this assumption is made for technical reasons. It allows to characterize implementable mechanisms as those satisfying the monotonicity condition (Guesnerie and Laffont, 1984 and Fudenberg and Tirole, 1991). It implies that the marginal rate of substitution between allocations and the transfers does not increase too fast when the transfer goes to infinity.

- **A.7**: Monotonic Hazard Rate:

$$\frac{\partial}{\partial s_i}\left[\frac{f_i(s_i)}{1-F_i(s_i)}\right] \geq 0$$

in our setting, the hazard rate of the uniform distribution function is

$$\frac{1}{(\bar{s}-s_i)}$$

and clearly verifies the monotonicity condition. This assumption allows us to focus on the relaxed problem, that is to check the SOC — the monotonicity constraint — ex-post (see the next section).

By the RPC and the maximand we know that the seller's profit is increasing in the retail investors' payments, therefore at the optimum he will make the RPC binding. We can then rewrite the RPC in the following way:

$$\text{(1)} \quad \int_{\Omega} P_R(s) q_R(s) f(s) ds = \int_{\Omega} v q_R(s) f(s) ds = \int_{\Omega} v \left(1 - \sum_i q_i(s)\right) f(s) ds$$

where we have put $s = (s_i, s_{-i})$ and replaced to $q_R(s)$ from the FAC.

Now, let us consider the IPC which can be rewritten as follows:

$$(2)\quad \int_{\Omega_{-i}} p_i(s)q_i(s)f_{-i}(s_{-i})ds_{-i} = \int_{\Omega_{-i}} z(q_i(s),v)f_{-i}(s_{-i})ds_{-i} - U_i(s_i,s_i) \geq 0$$

for all s_i and i

by integrating over Ω_i (that is taking the expectations over s_i) we find:

$$(3)\quad \int_{\Omega} p_i(s)q_i(s)f(s)ds = \int_{\Omega} z(q_i(s),v)f(s)ds - \int_{\Omega i} U_i(s_i,s_i)f_i(s_i)ds_i$$

From the IIC, after some computations (see the Appendix for the details), we can write the following equation:

$$(4)\quad \int_{\Omega i} U_i(s_i,s_i)f_i(s_i)ds_i = \frac{1}{n}\int_{\Omega} \alpha(\bar{s} - s_i)q_i(s_i,s_{-i})f(s)ds$$

where $(\bar{s} - s_i)$ is the inverse of the *hazard rate*.

By replacing equation *(4)* into equation *(3)* and using equation *(1)*, the firm's optimization program can be rewritten in the following way:

$$\max_{\{q_i\}_1^n} \int_{\Omega} \left\{ v + \sum_i \left[q_i(s)\left((\alpha - 1)v - \frac{\delta}{2} q_i(s) - \frac{\alpha}{n}(\bar{s} - s_i) \right) \right] \right\} f(s)ds$$

$$s.t:$$

$$\frac{\partial q_i(s)}{\partial s_i} \geq 0 \quad \text{for all } i \text{ and all } s$$

$$q_i(s) \geq 0 \quad \text{for all } i \text{ and all } s$$

$$\sum_i q_i(s) \leq 1 \text{ for all } s$$

where the first constraint is the *monotonicity constraint* which ensures that the SOC are met and the mechanism is implementable.[6]

In the next section, the optimal quantities are derived. We consider the relaxed problem in which we drop the monotonicity constraint and check it ex post. We will then compute the optimal price schedules and the rationing rate.

3. - The Optimal Quantities

The relaxed problem is set by ignoring the monotonicity condition. The objective function becomes then an ordinary maximand with the constraints defined in each point and can be maximized pointwise on Ω.

We now derive the optimal quantity $q_i(s)$ for each i and each s (hereafter we drop the "*" to denote the optimal mechanism), that is the amount of shares the firm must assign to investor i in order to elicit his information. $q_i(s)$ is the solution of the following maximization problem, for each $s \in \Omega$:

$$\max_{\{q_i\}_{i=1,\dots n}} \sum_i \left[q_i(s)\left((\alpha-1)v - \frac{\delta}{2} q_i(s) - \frac{\alpha}{n}(\bar{s} - s_i) \right) \right]$$

$$s.t:$$

$$U_i(\underline{s}, \underline{s}) = 0 \qquad \forall_i$$

$$q_i(s) \geq 0 \quad \text{and} \quad \sum_i q_i(s) \leq 1 \qquad \forall i, \forall s$$

[6] This monotonicity condition is stronger than what is sufficient for the implementability of the optimal mechanism. By the second derivatives of $U_i(s_i, s_i)$, it would be sufficient the following inequality to hold,

$$\frac{\alpha}{n} E_{s-i}\left[\frac{\partial q_i(s)}{\partial s_i} \right] \geq 0$$

for each i and s_i.

The Kuhn-Tucker conditions for this maximization program are the following:

$$\text{(5)}\quad \begin{cases} (\alpha - 1)v - \delta q_i - \dfrac{\alpha}{n}(\bar{s} - s_i) + \lambda_i(s) - \beta(s) = 0, \quad \text{for all } i \\ \lambda_i(s) q_i(s) = 0 \\ \beta(s)\left[1 - \sum_i q_i(s)\right] = 0 \end{cases}$$

with λ_i and β are the Kuhn-Tucker multipliers associated respectively to the feasibility constraint and to the FAC.

Now let us denote by $H(q, v)$ our objective function, that is:

$$H(q,v) = \sum_i \left[q_i(s)\left((\alpha - 1)v - \frac{\delta}{2} q_i(s) - \frac{\alpha}{n}(\bar{s} - s_i) \right) \right]$$

with $q_i \in [0, 1]$ and $s \in \Omega = [\underline{s}, \bar{s}]^n$. This function is concave in q_i for all i, since

$$\frac{\partial^2 H}{\partial q_1^2} \leq 0$$

by assumptions (A.2) and (A.5).

If $q_i(s) \neq 0$, then $\lambda_i(s) = 0$, so from equation *(5)* it is:

$$\frac{\partial H}{\partial q_i} = (\alpha - 1)v - \delta q_i - \frac{\alpha}{n}(\bar{s} - s_i) = \beta(s) \geq 0$$

given the sign of the Kuhn-Tucker multiplier. Let us denote by $h_i(s)$ the first derivative of H w.r.t. q_i, i.e.

$$\frac{\partial H}{\partial q_{i}}\bigg|_{q_i = 0}$$

A sufficient condition for the inequality above to hold is that $h_i(s) > 0$. That is:

$$h_i(s) = (\alpha - 1)v - \frac{\alpha}{n}(\bar{s} - s_i) =$$

(6)

$$= \frac{1}{n}\left[(\alpha - 1)v_{-i} + (2\alpha - 1)s_i - \alpha\bar{s}\right] > 0$$

where we have put

$$v_{-i} = \sum_{j \neq i} s_i$$ [7]

And, consequently, $q_i(s) = 0$ if $h_i(s) \leq 0$.

The solution of the optimization program depends on the sign of the function $h_i(s)$. We have to distinguish the two possible cases:

- $h_i(s) \leq$ if and only if

$$\frac{1}{n}\left[(\alpha - 1)v_{-i} + (2\alpha - 1)s_i - \alpha\bar{s}\right] \leq 0 \Leftrightarrow$$

$$s_i \leq \frac{\alpha\bar{s} - (\alpha - 1)v_{-i}}{(2\alpha - 1)} = s^{o}_{i}(v_{-i})$$

so the optimal quantity for all i such that $s_i \leq s_i^{o}(v_{-i})$ would be zero. Consequently,

- $h_i(s) > 0$ for $s_i > s_i^{o}(v_{-i})$ and the optimal quantity for all i for which this condition holds, would be positive.

Notice also that for values of

$$v_{-i} > \frac{\alpha\bar{s} - \underline{s}(2\alpha - 1)}{(\alpha - 1)} = v^{o}$$

[7] This result uses the fact that given a function Ψ continuous and concave on an interval $[a, b]$, if $\Psi'(a) \leq 0$, then Ψ is decreasing all over on $[a, b]$. In our case, this means that if $h_i(s) \leq 0$, the first derivative of H with respect to q_i, $\partial H/\partial q_i$, is also negative and so is the Kuhn-Tucker multiplier $\beta(s)$ what is impossible. By contradiction, it must be that $q_i(s) = 0$ if $h_i(s) \leq 0$.

it is $s^o_i(v_{-i}) < s$ and then $s_i > s^o_i(v_{-i})$ for all i and consequently the optimal quantity is positive for all institutional investors.[8]

Furthermore, to ensure that $q_i(\underline{s}, s_{-i}) = 0$ we assume that $h_i(\underline{s}, s_{-i}) \leq 0$ which is true if $v_{-i} \leq v^o$ for all s_{-i}.[9]

The next lemma states a sufficient condition for this to be true:

Lemma 1: A sufficient condition for $v_{-i} \leq v^o$ *t*o hold for all s_{-i}, is that:

$$\bar{s} \leq \frac{2\alpha - 1}{n(\alpha - 1)} \Delta s$$

Proof: (See the Appendix).

Now, assume that

$$\bar{s} \leq \frac{2\alpha - 1}{n(\alpha - 1)} \Delta s$$

Then, for any $s \in \Omega = [\underline{s}; \bar{s}]^n$, we can split the set N into two sub-sets

$$N^-(s) \ = \{ i \in N | s_i \leq s^o{}_i(v_{-i}) \}$$

$$N^+(s) = \{ i \in N | s_i > s^o{}_i(v_{-i}) \}$$

such that $N(s) = N^+(s) \cup N^-(s)$. This partition of the set N is related to the characteristics of the investors' utility functions which in turn depend on the value of the signal they receive. This becomes clear if we look at risk neutral institutional investors. In this case, in fact, the set $N^+(s)$ is empty.

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[8] It is instead always true that $s^o_i(v_{-i}) < \bar{s}$, since $s^o_i(v_{-i}) > \bar{s}$ if and only if

$$\alpha\bar{s} - (\alpha - 1)v_{-i} > (2\alpha - 1)\bar{s} \Leftrightarrow$$
$$v_{-i} < -\bar{s}$$

which is impossible for $s > 0$. This proves the result.

[9] This condition is sligthly more restrictive than what we need, but it considerably simplifies the analysis.

We can then state the following result:

Proposition 1: Assume that institutional investors' utility function is $u_i(q_i, p_i, v) = q_i[(\alpha v - \delta/2q_i) - p_i]$ with $\alpha > 1$ and that signals are i.i.d. according to a uniform distribution with support $\Omega_i = [\underline{s}; \bar{s}]$. Then, for any $s \in \Omega = [\underline{s}; \bar{s}]^n$ the optimal mechanism is such that:

- for any $i \in N^-(s)$, the optimal quantity is $q_i(s) = 0$;
- for any $i \in N^+(s)$ we can compute the quantity $\tilde{q}_i(s) > 0$ which is equal to:

(7)
$$\tilde{q}_i = \frac{(\alpha - 1)nv - \alpha(\bar{s} - s_i)}{n\delta}$$

then,

a) if

$$\sum_{i \in N^+(s)} \tilde{q}_i(s) < 1$$

the optimal quantity each institutional investor receives is exactly $\tilde{q}_i(s)$;

b) if, instead,

$$\sum_{i \in N^+(s)} \tilde{q}_i(s) > 1$$

the optimal quantity is $q_i(s) = \hat{q}_i(s) < \tilde{q}_i(s)$ solving the following system of equations:

(8)
$$\begin{cases} \hat{q}_i(s)\left[(\alpha - 1)v - \delta\hat{q}_i(s) - \dfrac{\alpha}{n}(\bar{s} - s_i) - \beta(s)\right] = 0 \\ \displaystyle\sum_{i \in N^+(s)} \hat{q}_i(s) = 1; \quad \beta(s) > 0 \end{cases}$$

where $\beta(s)$ is as usual the Kuhn-Tucker multiplier.

Proof: (see the Appendix).

Notice that, *a)* the optimal quantity $\tilde{q}_i(s)$ is increasing in the signal s_i. The seller rewards good signals with a larger quantity. *b)* The case in which

$$\sum_{i \in N^+(s)} \tilde{q}_i(s) > 1$$

corresponds to an *oversubscription* of shares and implies that institutional investors are rationed in equilibrium.

The next step is to prove that the optimal mechanism obtained by solving the relaxed problem is implementable, that is it satisfies the monotonicity condition. The result is stated in the next proposition. We omit the proof, which is quite long and complicated.[10]

Proposition 2: The optimal mechanism derived in proposition 1 is implementable, that is the optimal quantity $q_i(s)$ satisfies the following monotonicity condition:

$$\frac{\partial q_i(s)}{\partial s_i} \geq 0$$

for all i and s_i.

4. - Oversubscription and Rationing

The case

$$\sum_{i \in N^+(s)} \tilde{q}_i(s) > 1$$

corresponds to a situation of oversubscription of shares, because institutional investors demand overall more shares than the amount on sale. Consequently, in equilibrium, they do not receive their optimal quantity $\tilde{q}_i(s)$, but instead, each institutional investor is assigned the lower amount $\hat{q}_i(s)$ determined by the system of equation *(8)*. Retail investors receive nothing in equilibrium.

[10] See BENNOURI M. and FALCONIERI S. (2005*a*) for a detailed proof in a general model of auction design.

Proposition 3: In the case of oversubscription of the new shares,

$$\sum_{i\in N^+(s)} \tilde{q}_i(s) > 1$$

in equilibrium, institutional investors will be uniformely rationed. The quantity they are assigned is

$$\hat{q}_i(s) = \tilde{q}_i(s) - \beta(s)/\delta$$

where, the Kuhn-Tucker multiplier is given by

$$\beta(s) = \frac{\delta}{Card^+}\left(\sum_{i\in N^+(s)} \tilde{q}_i(s) - 1\right)$$

Proof: By the system of equations *(8)*, the quantity each institutional investor is assigned, $\hat{q}_i(s)$, is

$$\hat{q}_i(s) = \frac{(\alpha-1)v - \frac{\alpha}{n}(\bar{s} - s_i) - \beta(s)}{\delta} = \tag{9}$$

$$= \tilde{q}_i(s) - \beta(s)/\delta$$

where $\hat{q}_i(s) < \tilde{q}_i(s)$, being $\beta(s) > 0$.

The value of the Kuhn-Tucker multiplier, $\beta(s)$, can be explicitly computed by using the condition

$$\sum_{i\in N^+(s)} \hat{q}_i(s) = 1$$

By equation *(9)*, it is

$$\sum_{i\in N^+(s)} \hat{q}_i(s) = \sum_{i\in N^+(s)} \left[\tilde{q}_i(s) + \beta(s)/\delta\right] = 1$$

Define now $Card^+ \equiv Card[N^+(s)]$, re-arranging the above equality then yields

$$\beta(s) = \frac{\delta}{Card^+}\left(\sum_{i \in N^+(s)} \tilde{q}_i(s) - 1\right)$$

Notice that, the rationing scheme is *uniform* in the sense that, in equilibrium, the quantity assigned to each institutional investor is reduced of the same amount with respect to what would be otherwise optimal for them. This is consistent with monotonicity of the optimal quantity schedule according to which the seller assigns more shares to the investors with the better signals.

Nonetheless, the rationing rate as defined by the ratio $R_i = \hat{q}_i / \tilde{q}_i < 1$, varies across investors and, by replacing from equation *(9)*, is equal to

$$R_i = 1 - \frac{\beta(s)}{\delta \cdot \tilde{q}_i}$$

which implies that better signals, lower rationing. This is consistent with what is observed in the bookbuilding procedure, where investment banks reward investors who convey more information through their bids (e.g. limit orders) (Cornelli and Goldreich, 2001). Here, the seller rewards the better information by applying a lower rationing rate.

From a normative standpoint, our result suggests that the current regulation of IPOs, which prevents the selling firms from price discriminating, is not inefficient, since rationing alone is enough to achieve optimal performances. This is consistent with what we observe in practice in the current IPO mechanisms.

In particular, our result seems to be confirmed by the evidence from the bookbuilding procedure. Investment banks exhibit a strong attitude to strategically use rationing as a reward to investors who transmit more and better information during the opening of the book (e.g. limit orders). Because of that, investment banks often set the price sufficiently low to induce oversubscription and, consequently, to have complete discretion in how to ration shares among the investors. (Cornelli and Goldreich, 2001).

Our result is also consistent with the literature both empirical and theoretical indicating auction mechanisms as the most efficient ones to sell new shares.

With regard to this, interesting evidence comes from the Israeli IPOs which are conducted as non-discriminatory (uniform price) auctions. Kandel and Sarig (1999) find that Israeli IPOs exhibit a very low underpricing (4,5%) compared to other IPO mechanisms and only on the first day of trading. No significant abnormal return is found beyond the first trading day. Derrien and Womack (2003) use French IPO data for the 1992-1998 period to compare the efficiency of three different IPO selling procedures: fixed priced, auction and bookbuilding.[11] They find that the auction mechanism results in less underpricing as well as in a lower variability of the underpricing in hot versus cold market conditions.

In line with the empirical literature, Biais and Faugeron-Crouzet (2002) prove, in their paper, that auction-like mechanisms perform better than others, the reason being that auction procedures are more informationally efficient, i.e. they allow to elicit and incorporate more information from the market as well as from investors into the pricing of IPOs.

Finally, in the auction literature, other papers have shows that the choice of the proper allocation and possibly rationing rule is crucial in determining the seller's revenues. For instance, Kremer and Nyborg (2004) show how appropriate allocation rules when there is an excess of demand may be revenue-enhancing by avoiding low-prices equilibria. Similarly, Gresik (2001) shows that the choice of the allocation rule affects the auction performances by shaping the bidder's bidding functions.

5 - Conclusions

The Initial Public Offerings of shares can be described as a

[11] Respectively, *Offre a prix ferme, Mise en Vente* and *Placement Garanti.*

very special kind of auction in which the auctioneer faces two groups of bidders with different information sets. In particular, fully uninformed bidders coexist with informed bidders. In the IPO context, the uninformed bidders are represented by the retail investors who only occasionally take part to the offering, whereas the informed bidders are institutional investors who are professional investors regularly taking part into the offerings.

In this paper, we use the tools of optimal auction theory to analyze the functioning of initial offerings with the focus on the choice by the seller of the optimal allocation and rationing rule when institutional investors are risk averse, which is a case unexplored in the IPO literature. Our results show that *a)* the optimal quantity is increasing in the quality of the information transmitted by informed investors; *b)* when there is an oversubscription of shares, retail investors get zero and institutional investors are rationed. We derive the optimal rationing scheme and show that it *uniform* across investors, i.e all investors are rationed by the same quantity.

In this paper we abstract from the computation of the optimal price but we can show (see Bennouri and Falconieri, 2005*b*) that the optimal IPO can be implemented by a uniform price schedule across all investors.

Our model can be alternatively interpreted as a model of optimal auctions. In this respect, it can be applied to all kind of auctions exhibiting the same informational structure as an IPO, namely, the auctioneer facing two groups of bidders with different information sets. The main insight we can derive from our result is that the existence of a group of uninformed bidders allows the auctioneer to extract the information from the informed bidders at a lower cost. The reason relies upon the more discretion the auctioneer enjoys in deciding to whom to allocate the good which makes the informational problem less severe.

APPENDIX

Proof of Equation *(4)*:

By applying the envelope theorem to the IIC, we have:

$$U_i^{'}(s_i, s_i) = \frac{1}{n}\int_{\Omega-1} \alpha q_i(s) f_{-i}(s_{-i}) ds_{-i}$$

thus,

$$U_i(s_i, s_i) = U_i(\underline{s}, \underline{s}) + \int_{\underline{s}}^{s_i} \left\{ \frac{1}{n}\int_{\Omega-i} \alpha q_i(\tilde{s}_i, s_{-i}) f_{-i}(s_{-i}) ds_{-i} \right\} d\tilde{s}_i$$

For the IPC to be satisfied we can simply set $U_i(\underline{s}, \underline{s}) = 0$. This assumption means that the seller will leave no rents to the informed traders with the lowest evaluations.

Applying Fubini's theorem to the last equation yields:

$$U_i(s_i, s_i) = \frac{1}{n}\int_{\Omega-i} \left\{ \int_{\underline{s}}^{s_i} \alpha q_i(\tilde{s}_i, s_{-i}) d\tilde{s}_i \right\} f_{-i}(s_{-i}) ds_{-i}$$

Integrate over Ω_i, and apply again Fubini's theorem. We get the following

$$\int_{\Omega_i} U_i(s_i, s_i) f_i(s_i) ds_i = \frac{1}{n}\int_{\Omega_{-i}} \left\{ \int_{\Omega_i} \left[\int_{\underline{s}}^{s_i} \alpha q_i(\tilde{s}_i, s_{-i}) d\tilde{s}_i \right] f_i(s_i) ds_i \right\} f_{-i}(s_{-i}) ds_{-i}$$

Now, after having integrated by parts the integral

$$\int_{\Omega_i} \left[\int_{\underline{s}}^{s_i} \alpha q_i(\tilde{s}_i, s_{-i}) d\tilde{s}_i \right] f_i(s_i) ds_i$$

yields the following equation:

$$\int_{\Omega_i}\left[\int_{\underline{s}}^{s_i} \alpha q_i(\tilde{s}_i, s_{-i})d\tilde{s}_i\right]f_i(s_i)ds_i =$$

$$\left[\left\{\int_{\underline{s}}^{s_i} \alpha q_i(\tilde{s}_i, s_{-i})d\tilde{s}_i\right\}(F_i(s_i)-1)\right]_{\underline{s}}^{\bar{s}} - \int_{\Omega_i} \alpha q_i(s_i, s_{-i})(F_i(s_i)-1)ds_i$$

from which, Equation *(4)* is straightforward.

Proof of *Lemma 1:* A sufficient condition to have

$$v_{-i} \leq \frac{\alpha\bar{s} - \underline{s}(2\alpha - 1)}{(\alpha - 1)} = v^{\circ} \qquad \forall s_{-i}$$

is that the following holds:

$$v_{-i} \leq (n-1)(\alpha-1)\bar{s} \leq \alpha\bar{s} - \underline{s}(2\alpha - 1) \qquad \forall s_{-i}$$

By re-arranging, the above inequality can be re-interpreted as a (sufficient) condition on the support of the signals,

$$n(\alpha-1)\bar{s} \leq (2\alpha-1)\Delta s \Rightarrow \bar{s} \leq \frac{2\alpha-1}{n(\alpha-1)}\Delta s$$

Proof of *Proposition 1:* By the definition of the two sets $N^-(s)$ and $N^+(s)$, we know that for each $i \in N^-(s)$ it must be $q_i(s) = 0$.

For each $i \in N^+(s)$, define the quantity $\tilde{q}_i$ as the one solving the equation below,

$$(\alpha-1)v - \delta\tilde{q}_i(s) - \frac{\alpha}{n}(\bar{s} - s_i) = 0$$

By the concavity of function H (with respect to q_i), $\tilde{q}_i(s)$ is strictly positive for each $i \in N^+(s)$. Now, if

$$\sum_{i \in N^+(s)} \tilde{q}_i(s) \leq 1$$

the quantity $\tilde{q}_i(s)$ is the solution of our mechanism.[12]

$$\sum_{i \in N^+(s)} \tilde{q}_i(s) > 1$$

this case corresponds to a situation of oversubscription of the new shares. The quantity $\tilde{q}_i(s)$ cannot thus be the optimal solution because it violates the FAC, so the optimal mechanism is given by the solution of the following system of equations that we denote by $\hat{q}_i(s)$:

$$\begin{cases} \hat{q}_i(s)\left[(\alpha - 1)v - \delta\hat{q}_i(s) - \dfrac{\alpha}{n}(\bar{s} - s_i) - \beta(s)\right] = 0 \\ \displaystyle\sum_{i \in N^+(s)} \hat{q}_i(s) = 1; \quad \beta(s) > 0 \end{cases}$$

which implies that $\hat{q}_i(s)$ is either zero or is positive and solves the following equation

$$(\alpha - 1)v - \delta\hat{q}_i(s) - \frac{\alpha}{n}(\bar{s} - s_i) - \beta(s) = 0$$

Notice that, in this case, all the share are allotted to institutional investors. Retail investors get nothing in equilibrium.

[12] In this case, the equation above defines the FOC of our objective function H, since the Kuhn-Tucker multipliers, $\lambda_i(s)$ and $\beta(s)$ are both zero.

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