

Promotion and Relegation in Sporting Contests[◇]

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The conventional model of a team sports league is based on the North American major leagues which have a fixed number of members, entry is rare and only granted by permission of the incumbents (the closed system). European soccer leagues operate a system of promotion and relegation, effectively permitting entry on merit to all-comers (the open system). This paper examines the impact of openness on the incentive of teams to invest (expend effort) and share resources (redistribution) in the context of a Tullock contest. The main conclusion of the paper is that openness tends to enhance effort incentives, but diminishes the incentive to share income [JEL Codes: L83, P51].

L'analisi economica tradizionale dei campionati sportivi a squadre si basa sul modello "chiuso" nord-americano, dove vi è un numero prefissato di squadre in lizza per il titolo. Il modello europeo è invece "aperto" in quanto consente l'entrata di nuove squadre tramite un sistema di promozione e retrocessione. Questo lavoro propone un confronto tra un campionato "chiuso" e uno "aperto" utilizzando modelli di tornei à la Tullock. I due sistemi sono confrontati in base alle loro capacità di fornire alle squadre incentivi agli investimenti (ad esempio nel talento dei giocatori) e alla redistribuzione delle risorse tra i partecipanti al torneo. Le principali conclusioni sono due. Rispetto ad un sistema "chiuso", un campionato "aperto" fornisce maggiori incentivi ad investire mentre ha una propensione più bassa alla redistribuzione delle risorse.

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1. - Introduction

The structure, conduct and performance of professional sports leagues have been the subject of vehement criticism on both sides of the Atlantic in recent years. In the US the major leagues in baseball, basketball and American Football have been described as “classic, even textbook, examples of business cartels”¹ and several articles have enumerated abuses of local monopoly power, in particular the extraction of public subsidies for the construction of stadiums and their facilities (see e.g. Noll and Zimbalist, 1997; Siegfried and Zimbalist, 2000). A key factor in these abuses is the monopoly nature of the dominant league and the failure of entry by rival leagues which have either folded (possibly due to predation) or been co-opted (see e.g. Quirk and Fort, 1992). One solution is the enforced break-up of the majors into competing leagues, proposed by, *inter alia*, Ross (1989) and Quirk and Fort (1999). An alternative is to adopt a system that generates entry, not at the level of the league, but at the level of the team. Noll (2002) analyses in detail the European system of promotion and relegation, by which the worst performing teams at the end of each season are demoted to the immediately junior league, to be replaced by the best performing teams in the junior league. Ross and Szymanski (2002) go further, and argue that the promotion and relegation system would be welfare enhancing for US consumers and taxpayers.

At the present time, however, the dominant European soccer leagues that have long operated the system of promotion and relegation are not in the best of health. Several teams in England have fallen into administration, the UK equivalent of Chapter 11². So frequent has this become that the league authorities have introduced penalties for teams that go into administration³. In

¹ FORT R.- QUIRK J. (1995, p. 1265).

² Barnsley, Bournemouth, Bradford, Carlisle, Chesterfield, Crystal Palace, Ipswich, Leicester, Lincoln, Luton, Millwall, Notts County, Oxford, QPR, Swansea and Swindon have all gone into administration in the last 5 years. <http://football.guardian.co.uk/clubsincrisis/story/0,11737,816711,00.html>.

³ <http://www.terrier-bytes.com/articles.php/news/145>.

Germany the government has agreed to underwrite the losses of the leading clubs due to losses of broadcast income⁴ and in Italy the lower house of the Parliament passed the *salva calcio* law enabling the clubs to write off losses over a longer period than is available to ordinary corporations⁵. Critics of the soccer administrators who have overseen the financial crisis in Europe point to the extent to which redistributive measures, so common in the US majors, are lacking in Europe⁶. In the European leagues there is no reserve clause, no draft, no roster limit no salary cap, no luxury tax, no gate sharing and no collective merchandising agreement. The only form of revenue sharing in European leagues relates to the collective selling of broadcast rights, but even this is absent in many leagues (e.g. Italy and Spain) and limited in others (e.g. only 50% of broadcast income is shared in England). These restraints, claim the owners of franchises in the US majors, are desirable precisely because they promote a degree of competitive balance in league competition and prevent rival teams from falling into bankruptcy. Even critics in the US who complain that these restraints are unnecessarily restrictive accept that sports teams are special kinds of businesses in that the bankruptcy of rivals, so welcome in most lines of business, is in fact harmful to the remaining teams in the league⁷.

In this paper we examine the relationship between rent dissipation and the incentive to share revenues in closed and open (i.e. open to promotion and relegation) league structures⁸. We

⁴ <http://football.guardian.co.uk/clubsincrisis/story/0,11737,679178,00.html>.

⁵ <http://news.bbc.co.uk/2/hi/europe/2749673.stm>.

⁶ See e.g. HAMIL S. *et AL.* (1999).

⁷ This observation, first noted in the economics literature by NEALE W. (1964), has been acknowledged by all writers on sports economics since. Indeed, it is arguable that the sports economics as a distinct field of research rests primarily on this point.

⁸ THE EUROPEAN COMMISSION (1998) has identified the promotion and relegation system as "one of the key features of the European model of sport". In the European system, all teams belong to a governing body that oversees a strictly defined hierarchy of divisional competitions. At all levels a limited number of the worst performing teams (usually between one and four) in any given division are demoted at the end of each season to the immediately junior division, to be replaced by the equivalent number of top performing teams from the junior division. This seamless hierarchy connects the lowest levels of amateur competition to the highest levels of European competition.

model league competition in the context of an infinitely repeated logit (Tullock, 1980) contest⁹. The standard contest model involves competition for prize every period (season) among a fixed number of teams (as in a closed league). We extend this model to include a penalty for coming last: relegation. In our model the teams are divided into two groups (division one and division two). The teams in division one compete in the current period for the main prize as in the closed league (only with half as many competitors) while the teams in division two compete for the opportunity to compete in division one in the next period. In the next period the winner of division two replaces the worst performing team in division one in the current period (and this team then competes in division two).

Using this model we examine two main issues: firstly, which system gives the greatest incentive to invest in effort, and secondly which system provides the greatest incentive to promote competitive balance, meaning the closeness of the competition between the teams in the league. We find that under plausible conditions open leagues with promotion and relegation tend to promote more effort than closed leagues, but undermine incentives to share resources. Both of these issues lie at the heart of public policy in relation to professional sports leagues. Antitrust authorities have traditionally adopted a very lenient approach to collective agreements between league members on the grounds that a competitive balance is in the interest of the fans. However, collective agreements can also undermine effort incentives and this can be a particular problem in closed leagues¹⁰. On the other hand, the cost of extracting higher effort through promotion and

⁹ Models based on Tullock contest success functions are standard to analyse the properties and design of alternative contest structures (e.g. GRADSTEIN M. - KONRAD K.A., 1999). See SZYMANSKI S. (2003) for a review of the application of contest theory to sporting competition.

¹⁰ TAYLOR B. - TROGDON J. (2002) provide evidence that weak teams in the NBA have in fact attempted to lose matches toward the end of the season, since losing offers the chance of a better draft pick. Note that not only does the threat of relegation makes this strategy unrealistic for weak teams, but also that agreement to implement a draft system is less likely to be feasible under promotion and relegation. It is worth noting that there is some loss of incentives to compete in a promotion and relegation system if the bottom teams are not allowed to enter a play-off for relegation - we are grateful for the editor for this point.

relegation may be a reduction in competitive balance compared to a closed league.

Aside from its significance for the organization of sports leagues, the idea of promotion and relegation has implications for the optimal design of tournaments in general, e.g. procurement auctions. It is well known that in most contest structures individual effort is decreasing in the number of contestants (see e.g. Fullerton and McAfee, 1999), but reducing the number of eligible bidders makes collusion easier. If a principal holds regular auctions (as in the case with government agencies) it may make sense to create an "A" list and a reserve "B" list and to allow promotion and relegation between the two. This can ensure that bidders supply optimal effort while minimizing the incentive to collude.

In the next section we discuss in more detail some comparisons between Major League Baseball and English Soccer, the archetypal North American and European sports leagues. Section 3 analyses effort incentives in a symmetric model and Section 4 examines revenue sharing in asymmetric contests. Section 5 concludes.

2. - Some Comparisons between Major League Baseball and English Soccer

The National League of baseball, founded in 1876, is the oldest surviving team sports league in the world, and the English Football League, founded in 1888, is the oldest surviving soccer league. Each of these leagues became the template for the organization professional team sports on their continent and while rival leagues and team sports have generated organizational innovations and differences, the similarities within North America and Europe are much greater than those between the two continents¹¹. North American leagues have many mechanisms for the maintenance of competitive balance (e.g. roster limits, draft

¹¹ See HOEHN T. - SZYMANSKI S. (1999) for a comparison of the main institutional differences.

rules, salary caps, luxury taxes, gate and broadcast revenue sharing), most of which are either unused or are implemented in less egalitarian ways in Europe. By contrast, European leagues promote rivalry not only through competition for the championship, but also through competition to avoid relegation.

This can make comparisons difficult. For example, the most widely used measure of competitive balance in North America is the standard deviation of winning percentage (wpc) relative to the idealized standard deviation (see e.g. Fort and Quirk, 1995)¹². On this basis European Leagues can in fact look more balanced than their American counterparts. Graph 1 compares the standard deviation ratios for National and American Leagues (the two leagues that comprise Major League Baseball and whose champions contest the World Series) with the English Premier League¹³ over the period 1980-1999.

In fifteen out of the twenty seasons the Premier League had a lower standard deviation ratio than either the National or American Leagues, suggesting that competition within the season was more balanced. Comparing the means, the average standard deviation ratios for the National and American Leagues were 1.68 and 1.70 respectively, while that of the Premier League was 1.43, significantly lower than either baseball league at the 1% level.

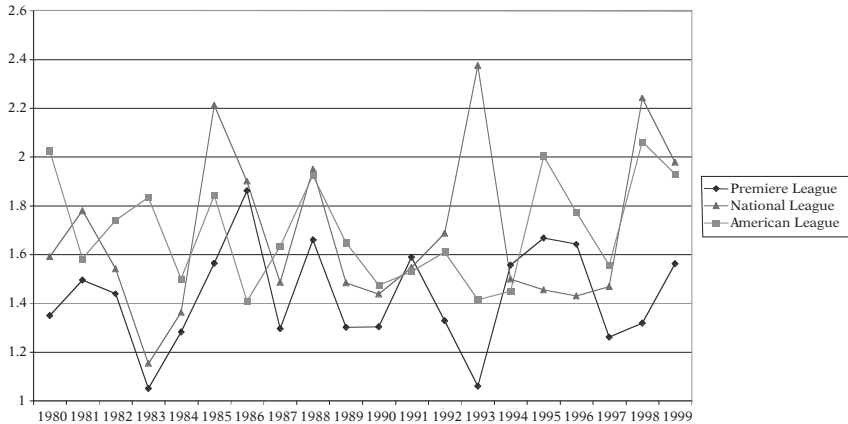
However, this measure tells us little about the dominance of particular teams across seasons. Buzzacchi *et Al.* (2003) examine the theoretical number teams that would be expected to reach a given rank at least once by the end of a given number of seasons and compare it to the actual numbers. For example, in any one year only one team can have the highest winning percentage, but

¹² The idealised standard deviation is calculated on the assumption that each team has an equal chance of winning, and is therefore equal to $.5/\sqrt{m}$ where m is the number of matches played by each team in the season. The extent to which the actual standard deviation exceeds the idealised value thus gives some indication of the extent of competitive imbalance during the season.

¹³ The Premier League is the top division of English Soccer; it consists of twenty teams and was formed by a breakaway from the English Football League in 1992. Crucially, however, it retained the promotion and relegation relationship with the First Division of the surviving English Football League. Currently three teams are relegated and promoted between the two divisions each season. Seasons up until 1992 refer to the old First Division of the Football League.

GRAPH 1

RATIO OF ACTUAL STANDARD DEVIATION OF WPC
TO IDEALIZED, 1980-1999, NATIONAL LEAGUE,
AMERICAN LEAGUE AND PREMIER LEAGUE

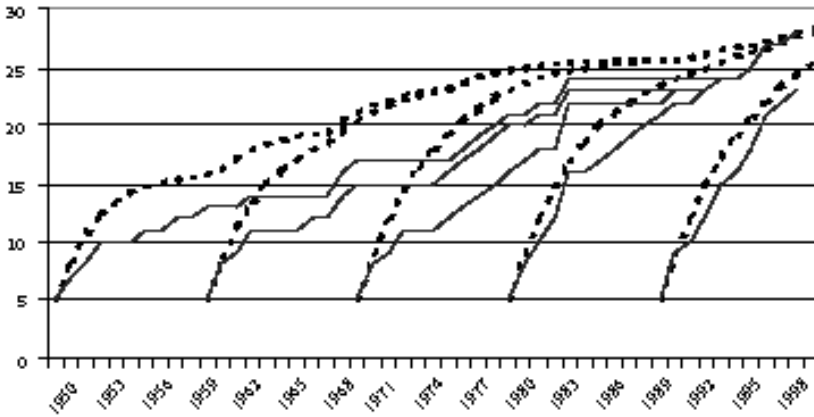


in a perfectly balanced repeated contest among a fixed number of teams the expected number of teams reaching this rank expands, until eventually all teams will be expected to have reached it at least once. In an open league with promotion and relegation this number expands quite rapidly over time, given that more and more teams have the opportunity to compete. Buzzacchi *et Al.* therefore calculated these expectations for Major League Baseball, taking account of franchise expansion, and for the English Premier League taking account of the rules of promotion and relegation, for a database covering the period 1950-2000. Graphs 2 and 3 compare the results.

The dotted lines in these figures tell us the expected number of teams that would have ever entered the top five ranks of winning percentage under perfect balance, starting from five different arbitrary dates, while the unbroken lines show the actual numbers. In the case of baseball, these lines are quite close together, indicating that almost every team that could have reached the highest ranks has actually done so, even if we consider the most recent period, starting from 1990.

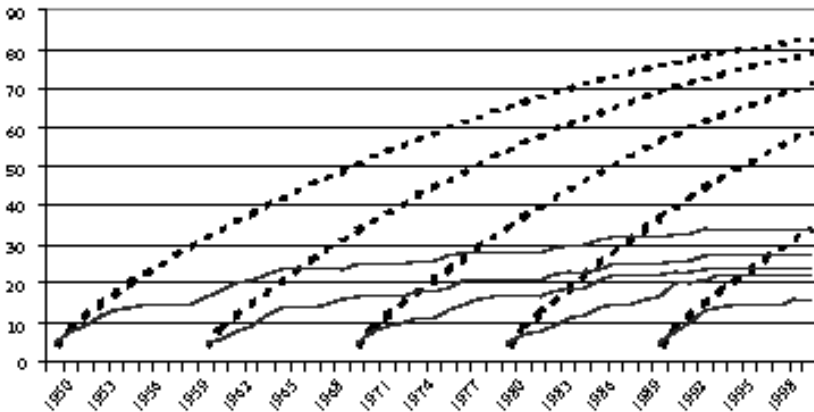
GRAPH 2

EXPECTED (DOTTED LINE) AND ACTUAL (UNBROKEN LINE) NUMBER OF TEAMS EVER ENTERING THE TOP FIVE RANKS OF WINNING PERCENTAGE IN MAJOR LEAGUE BASEBALL STARTING FROM 1950, 1960, 1970, 1980 AND 1990



GRAPH 3

EXPECTED (DOTTED LINE) AND ACTUAL (UNBROKEN LINE) NUMBER OF TEAMS EVER ENTERING THE TOP FIVE RANKS OF WINNING PERCENTAGE IN ENGLISH PREMIER LEAGUE STARTING FROM 1950, 1960, 1970, 1980 AND 1990



If we compare the English Premier League, the gap between statistical expectation based on equal chances and actual performance is much greater. While similar numbers of teams have entered the top five ranks as in baseball, openness through promotion means that many more teams would have entered these ranks if competition was truly balanced. For example, since 1950 over eighty teams would have achieved a top five placing in the Premier League at least once, compared to just over thirty that have in fact done so. Buzzacchi *et Al.* show that a similar pattern is observed in other North American major leagues and other European soccer leagues.

One way in which we can account for these findings is that the threat of relegation makes teams compete much more intensively throughout the season, even if they are out of contention for the title leading to a smaller ratio of standard deviations within the season. However, over the longer term only a small group of teams have access to the resources necessary to mount a credible challenge for the title. Redistributive measures in the major leagues ensure that more teams have the potential to reach the highest levels, but for some reason European soccer leagues are unable to implement such redistributive measures.

To explore further the question of access to resources it is useful to look at some economic financial performance data. Table 1 provides data for Major League Baseball teams for the 1999 season on wpc, attendance, payroll, revenues and estimates of franchise values. One indicator that captures both the relative inequality of resources and the struggle of the weaker teams to survive under promotion and relegation is the share of income devoted to payroll. The three teams with the poorest winning records in both the American and National Leagues spent less than the league average of 54% of total revenues on the payroll. In the Premier League only one out of the seven worst performing teams spent less than the league average of 60% on salaries. Blackburn Rovers who were in fact relegated in this season, spent more than 100% of their income on payroll. Perhaps most striking is the following contrast: the top 15 clubs in baseball by wpc devoted 58% of their aggregate income to payroll, compared to

TABLE 1

MAJOR LEAGUE BASEBALL 1999

<i>Team</i>	<i>Wpc</i>	<i>Attendance m</i>	<i>Player Payroll \$m</i>	<i>Revenues \$m</i>	<i>Franchise Value \$m</i>
Atlanta Braves	0.63	3.28	79.8	128.3	357
Arizona Diamondbacks	0.61	3.02	70.2	102.8	291
New York Yankees	0.6	3.29	92.4	177.9	491
Cleveland Indians	0.59	3.47	73.3	136.8	359
New York Mets	0.59	2.73	72.5	140.6	249
Houston Astros	0.59	2.71	58.1	78.1	239
Texas Rangers	0.58	2.77	81.7	109.3	281
Boston Red Sox	0.58	2.45	75.3	117.1	256
Cincinnati Reds	0.58	2.06	38.9	68.4	163
San Francisco Giants	0.53	2.08	46.0	74.7	213
Oakland Athletics	0.53	1.43	24.6	62.6	125
Toronto Blue Jays	0.51	2.16	50.0	73.8	162
Baltimore Orioles	0.48	3.43	78.9	123.6	351
Seattle Mariners	0.48	2.92	47.0	114.2	236
Pittsburgh Pirates	0.48	1.64	24.5	63.2	145
Los Angeles Dodgers	0.47	3.10	76.6	114.2	270
Philadelphia Phillies	0.47	1.83	32.1	77.2	145
St Louis Cardinals	0.46	3.24	46.3	101.8	205
Chicago White Sox	0.46	1.35	24.5	79.5	178
Milwaukee Brewers	0.46	1.70	43.6	63.6	155
San Diego Padres	0.45	2.52	46.5	79.6	205
Colorado Rockies	0.44	3.24	72.5	102.8	311
Anaheim Angels	0.43	2.25	53.3	86.1	195
Tampa Bay Devil Rays	0.42	1.75	37.9	75.5	225
Detroit Tigers	0.42	2.03	37.0	78.1	152
Montreal Expos	0.42	0.77	18.1	48.8	84
Chicago Cubs	0.41	2.81	55.5	106.0	224
Florida Marlins	0.39	1.37	16.4	72.9	153
Kansas City Royals	0.39	1.51	17.4	63.6	96
Minnesota Twins	0.39	1.20	15.8	52.6	89
Total	0.50	70.10	1507.0	2773.7	6605

Payroll and revenue data from the Blue Ribbon report. Franchise values from Forbes.

only 49% for the bottom 15 clubs; in the Premier League the top ten clubs devoted only 53% of their aggregate income to payroll, compared to 68% for the bottom ten.

The greater inequality of resources in the Premier League is illustrated by the fact that aggregate income of the bottom ten clubs equaled 35% of league income, compared to 43% for the

TABLE 2

PREMIER LEAGUE 1998/1999 SEASON

<i>Team</i>	<i>Rank</i>	<i>Wpc</i>	<i>Attendance m</i>	<i>Payroll \$m</i>	<i>Revenues \$m</i>	<i>Market cap \$m</i>
Manchester United	1	0.75	1.05	59.1	177.5	776
Arsenal	2	0.74	0.72	42.4	77.8	
Chelsea	3	0.72	0.66	48.3	94.5	171
Leeds United	4	0.64	0.68	29.7	59.2	88
West Ham United	5	0.54	0.49	28.3	42.5	
Aston Villa	6	0.53	0.70	26.6	55.8	90
Liverpool	7	0.51	0.82	58.0	72.4	
Derby County	8	0.51	0.55	22.8	35.2	
Middlesbrough	9	0.51	0.65	31.1	44.8	
Leicester City	10	0.49	0.39	25.6	38.1	21
Tottenham Hotspur	11	0.47	0.65	34.7	68.1	102
Sheffield Wednesday	12	0.43	0.51	21.7	30.6	
Newcastle United	13	0.46	0.70	39.2	71.5	152
Everton	14	0.42	0.69	32.4	40.7	
Coventry City	15	0.41	0.39	21.1	30.2	
Wimbledon	16	0.42	0.35	18.4	23.5	
Southampton	17	0.39	0.29	18.2	21.5	18
Charlton Athletic	18	0.37	0.38	13.2	26.0	21
Blackburn Rovers	19	0.37	0.49	35.9	34.0	
Nottingham Forest	20	0.30	0.46	18.9	27.2	27
Total		0.50	23.2	625.4	1071.4	1466
<i>First Division quoted teams</i>						
Sunderland (promoted)	2	0.80	0.74	16.0	38.5	64
Birmingham	4	0.63	0.40	10.0	13.5	27
Bolton	6	0.61	0.35	16.1	20.2	43
Sheffield United	8	0.53	0.31	12.1	10.3	11
West Bromwich Albion	12	0.47	0.28	7.3	10.8	13

Payroll and revenue data from company accounts (reported in Deloitte and Touche Annual Review of Football Finance). Market capitalization at end of the playing season.

bottom 15 in Major League Baseball. However, this difference in inequality is not sufficient to explain the widely divergent pattern of franchise values. The estimated franchise values for the bottom 15 in baseball equaled \$2.7bn in 1999, 41% of the total for the league. Franchise valuations are not available for all English clubs, but by the late 1990s twenty English clubs had obtained a stock

exchange listing and therefore we have data on their market capitalization, and in 1999 five of these teams finished in the top half of the Premier League and five in the bottom half. Those in the top half accounted for 78% (\$1.1bn) of the Premier League market capitalization and those in bottom half accounted for only 22%, a much more uneven distribution of market valuation than that of revenues. This can be accounted for by the fact that teams in the bottom half are much more likely to face the threat of relegation (as Noll, 2002, observes «demotion usually causes teams to be worse off financially») while even if they avoid relegation, they are much more likely to overextend themselves financially in order to avoid the drop.

3. - Effort Contribution in Symmetric Contests with and without Promotion and Relegation

In this section we look at the value of the league and compare the amount of effort that teams will choose to contribute in open and closed leagues. Throughout we will assume that leagues are essentially contests, where teams compete to win a single prize at the end of each season. In a closed league, all teams have a chance of winning the prize in the season. In open leagues, however, only the teams present in the highest ranked division can win the prize in the current season, and that the only incentive in lower divisions is the prospect of promotion to the highest division¹⁴.

To fix ideas we begin by comparing the present value of a team in an open and closed system assuming that each team always faces an equal probability of success in their division. This implies that teams have no choice in the level of effort or investment they supply and that all such contributions are equal and normalized to zero. In the second model, we make effort endogenous, although again all teams are assumed to be symmetric in the sense that

¹⁴ Note that this (European) concept of a division as part of the hierarchical structure is therefore distinct from the American concept of a division. In the European system the concept of inter-divisional play within the League makes no sense, since it violates the hierarchical ordering.

equal spending produces an equal probability of winning the prize and the prize is equally valuable to all teams. Asymmetries are analyzed in Section 4.

3.1 *Model 1: Symmetric Teams with Equal Winning Probabilities (no effort)*

3.1.1 Closed System

Imagine n is the total number of teams. Every period there is a contest and $\delta < 1$ is the discount factor between periods. In every period a team has a probability $1/n$ to win. The value of winning the championship title (the prize) is normalized to 1. The present discounted value of being in a closed league (C) is then simply:

$$V(C) = 1/[n(1-\delta)]$$

Thus the value of the of participating in the closed league is decreasing in the number of teams, since the probability of winning the prize falls, and increasing in the discount factor.

3.1.2 Open System

Imagine the total number of teams is divided among k hierarchical divisions, with $n_1 + n_2 + \dots + n_k = n$. Also, imagine one team is promoted/relegated in every period. Call V_i the NPV of being in division i . Then:

$$\begin{aligned} V_1 &= \frac{1}{n_1} + \delta \left(\frac{n_1 - 1}{n_1} V_1 + \frac{1}{n_1} V_2 \right) \\ V_2 &= 0 + \delta \left(\frac{n_2 - 2}{n_2} V_2 + \frac{1}{n_2} V_1 + \frac{1}{n_2} V_3 \right) \\ &\quad \dots \\ V_k &= 0 + \delta \left(\frac{n_k - 1}{n_k} V_k + \frac{1}{n_k} V_{k-1} \right) \end{aligned}$$

It is immediate to verify that $\sum_{i=1}^k n_i V_i = 1/(1-\delta) = nV(C)$, i.e. the total value of an open league (O) coincides with that of a closed league, as long as the same total number of teams is involved.

On the other hand, the distribution of the total value changes quite dramatically. For instance if we have 4 hierarchical divisions, with a total of 40 teams and $\delta = 0.8$, then the above system can be solved to obtain: $V_1 = 0.383$, $V_2 = 0.0898$, $V_3 = 0.0213$, $V_4 = 0.00609$, while $V(C) = 0.125$. Equivalently, a team in the top division has the same value “as if” it were in a closed league with approximately only 13 teams (rather than 40). The equivalent number of teams in a closed league increases to 56 for a team in the second division, 235 for a team in the third division and 821 for a team in the fourth division! Obviously, the differences between the V_i 's decreases as the discount factor gets bigger, and it disappears for δ equal to 1.

In this simple benchmark model the only effect of promotion and relegation is to change the distribution of team values.

3.2 Model 2: Symmetric Teams with Endogenous Effort

Endogenizing effort requires us to specify a “contest success function”. We use here the standard logit formulation adopted in much of the literature (see e.g. Nti, 1997). If a team spends x_i , the probability of winning the contest for team i is $s_i = x_i^\gamma / \sum_{j=1}^n x_j^\gamma$. The parameter γ is called “discriminatory power” and it defines the sensitivity of win probabilities to differences in effort. If it is zero, everybody wins with the same probability, no matter how much effort is spent. Conversely, if it tends to infinity, the team that spends more than its rivals wins with probability 1 (the contest in this limiting case is equivalent to an “all-pay” auction). In an open league system, the probability of winning is easily re-interpreted as the probability of being promoted. However, for open systems we also need a rule in order to assign a probability of being relegated as a function of effort/investment relative to the other contestants. For this purpose, we introduce a “contest losing function” that gives the probability of arriving last in a contest:

$$l_i = x_i^{-\gamma} / \sum_{j=1}^n x_j^{-\gamma}$$

Notice that the proposed losing function has a series of desired properties:

— If γ is zero, the probability of arriving first or last is independent from the effort put in the contest, $s_i = l_i = 1/n$ for all i 's;

— If γ tends to infinity, the team that puts the highest effort wins with probability 1 and loses with 0 probability;

— For intermediate values of γ , if all the rivals of team i spend the same amount, while team i outspends (respectively underspends) the individual amount spent by rivals, then $l_i < 1/n$ (respectively $l_i > 1/n$);

— If team i puts zero effort, and all the rivals put some positive effort, then $s_i = 0$, $l_i = 1$ and $l_{-i} = 0$;

— If $n > 2$, then $s_i + l_i < 1$.¹⁵

3.2.1 Closed System

There is no relegation. In a generic period, a team maximizes $\pi_i = s_i - x_i$ with respect to effort. This is a standard model, and it can be verified that, at a symmetric equilibrium, per-period team effort, per-period team profits, and discounted team profits are respectively:

$$(1) \quad \begin{aligned} x &= \frac{\gamma(n-1)}{n^2} \\ \pi &= \frac{\gamma + n(1-\gamma)}{n^2} \\ V(C) &= \frac{\gamma + n(1-\gamma)}{n^2(1-\delta)} \end{aligned}$$

¹⁵ This can be seen by noting that both s_i and l_i decrease with the number of rival teams. Hence, the sum $s_i + l_i$ is bounded above by the value it takes when there are only two teams, in which case $l_i = x_i^\gamma / (x_i^\gamma + x_j^\gamma)$ and $s_i + l_i = 1$.

3.2.2 Open System

The model is as in Section 3.1.2, with the difference that team i in a division j that contains n_j teams now spends effort x_{ij} , in which case its probabilities of winning the league and of being relegated are respectively $s_{ij} = x_{ij}^\gamma / \sum_{n=1}^{n_j} x_{nj}^\gamma$ and $l_{ij} = x_{ij}^{-\gamma} / \sum_{n=1}^{n_j} x_{nj}^{-\gamma}$ the maximization problem is therefore:

$$\begin{aligned}
 \max_{x_{i1}} V_{i1} &= s_{i1} - x_{i1} + \delta[(1 - l_{i1})V_{i1} + l_{i1}V_{i2}] \\
 \max_{x_{i2}} V_{i2} &= 0 - x_{i2} + \delta[(1 - s_{i2} - l_{i2})V_{i2} + s_{i2}V_{i1} + l_{i2}V_{i3}] \\
 &\dots \\
 \max_{x_{ik}} V_{ik} &= 0 - x_{ik} + \delta[(1 - s_{ik})V_{ik} + s_{ik}V_{ik-1}]
 \end{aligned}
 \tag{2}$$

We concentrate for simplicity on a league with only two divisions. It is then possible to obtain the value functions from (2), maximize with respect to effort taking as given the rivals' effort, etc. to find at equilibrium:

$$\begin{aligned}
 x_1 &= \gamma(n_1 - 1) \frac{n_1 n_2^2 - \delta[\gamma n_1(n_2 - 1) + n_1 n_2(n_2 - 1) - 2n_2^2]}{n_1 \{n_1^2 n_2^2 - \delta[n_1(n_2 - 1) - n_2][\gamma(n_1 - n_2) + n_1 n_2]\}} \\
 x_2 &= \gamma(n_2 - 1) \delta \frac{n_1 - \gamma(n_1 - 1)}{n_1^2 n_2^2 - \delta[n_1(n_2 - 1) - n_2][\gamma(n_1 - n_2) + n_1 n_2]} \\
 V_1 &= \frac{[\gamma + n_1(1 - \gamma)][n_2^2 - \delta(\gamma + n_2)(n_2 - 1)]}{(1 - \delta)\{n_1^2 n_2^2 - \delta[n_1(n_2 - 1) - n_2][\gamma(n_1 - n_2) + n_1 n_2]\}} \\
 V_2 &= \frac{\delta[\gamma + n_1(1 - \gamma)][\gamma + n_2(1 - \gamma)]}{(1 - \delta)\{n_1^2 n_2^2 - \delta[n_1(n_2 - 1) - n_2][\gamma(n_1 - n_2) + n_1 n_2]\}}
 \end{aligned}$$

To ensure existence of equilibria in pure strategies it can be shown that it suffices the restriction $0 < \gamma \leq \min [n_1/(n_1 - 1), n_2/(n_2 - 1)]$.

The effort in each league is strictly positive unless there is only one team in that league. With the exception of the case $n_1 = 1$ (no effort in the top league since the title is won with probability 1), a team always spends more effort in the top league than in the lower league, *independently* of the number of rivals it faces¹⁶. Also, if teams are distributed symmetrically, the difference between efforts is independent of the discount factor and it amounts to $\gamma (n_1 - 1)/n_1^2 > 0$. Despite spending more on effort, for $\delta < 1$, the value of a team in the top league is always higher than the value of a team in the bottom league. The two values converge as the discount factor gets closer to 1.

3.3.3 Welfare Analysis of Model 2

One question we might want to address is how to distribute a total number n of teams between the two leagues. Welfare analysis of sports leagues is in general problematic. Standard consumer theory suggests that we should concentrate on the utility of fans, but to reach any conclusions this would require us to quantify the utility of competitive balance, own team success and the quality of a tournament. It seems unlikely that policy makers could agree on any unambiguous ranking of outcomes on this basis. In the contest literature, welfare has generally been identified with rent dissipation, meaning the extent to which the profits from participation in the contest are consumed by the effort contributions of the contestants, a measure we will consider. In addition, we will consider the total amount of effort/investment exerted in the contest. However, it is not obvious that aggregate effort (which we might identify with the total quality of the contest) is the right measure of welfare either. The whole point of

¹⁶ The difference is decreasing in δ , hence it takes a minimum for δ approaching 1, in which case it is easily shown to be positive. Recall that we assume no prize at all in the lower league, which is probably not quite accurate. However, it is true to say that, in practice, teams in lower divisions typically spend far less on players than teams in higher divisions, but the gap is not quite so large as in our model.

the competitive balance literature is that it is the higher moments of the effort distribution that count. In some contexts, moreover, it may be that only the effort of the winning contestant really matters¹⁷. However, in a symmetric contest we can at least abstract from the issue of inequality within each division, an issue to which we will return in the next section.

We can illustrate the kinds of trade-off involved in promotion and relegation by looking at total effort and effort levels per team and in each division as the total number of teams increases. Graphs 4 and 5 illustrate the case where the discriminatory power of contest success function is moderate ($\gamma=1$). Along the horizontal axis is shown the total number of teams in the league as a whole (for example, 20 refers to either 20 teams in a closed league or an open league with two hierarchical divisions — labeled Serie A and Serie B¹⁸ — of 10 teams each).

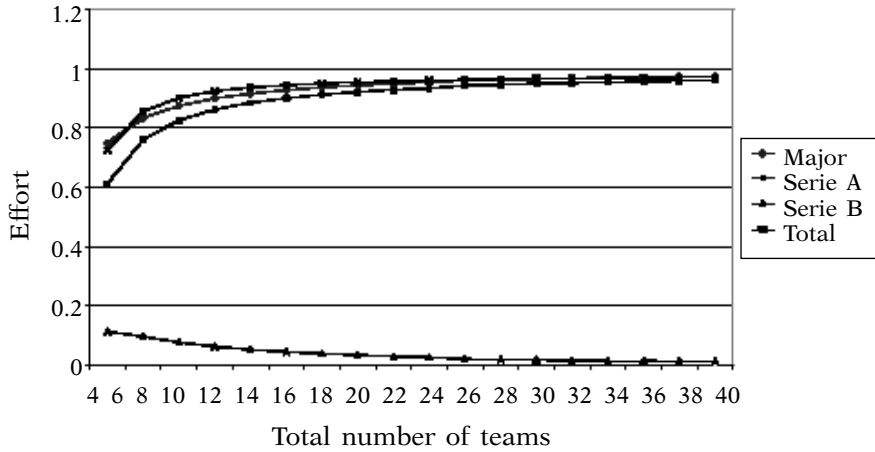
Total effort in the open and closed leagues are almost identical for most league sizes, but inspection shows that this is because teams make almost no effort in Serie B, while the ten teams of Serie A contribute about as much effort as the twenty teams of the closed league. This is quite clear in Graph 5, which shows that the effort per team in Serie A is almost double that of the effort per team in the closed league, while in Serie B effort contributions are negligible unless there are a very small number of teams in the division. Thus in this case the promotion and relegation system seems to produce a contest of relatively high quality among the elite of teams, while the closed system produces lower average quality but spread more evenly among a larger range of teams. Intuitively, relegation creates greater inequality between the teams, so that for a fixed prize, the privileged (Serie A) contribute more effort and the underprivileged (Serie B) do not try very much, but the total effort is roughly the same as in a closed competition.

¹⁷ SZYMANSKI S. (2003) points out that this is more likely to be true in individualistic contests such as foot races, where great weight is attached to record breaking, than in team sports.

¹⁸ These are the names of the top two divisions in Italy. This at least avoids the somewhat confusing English situation where the second ranked division is now called the Football League First Division, from which teams are promoted to the Premier League.

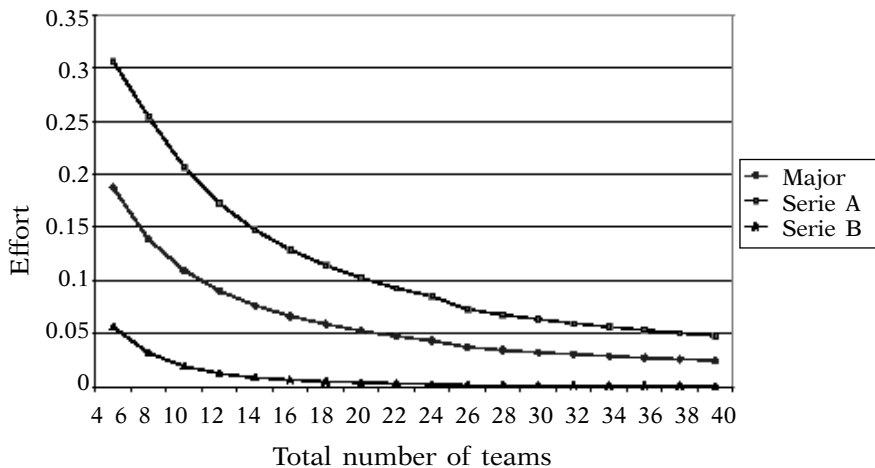
GRAPH 4

TOTAL EFFORT IN A MAJOR LEAGUE AND A TWO DIVISION HIERARCHY (GAMMA=1)



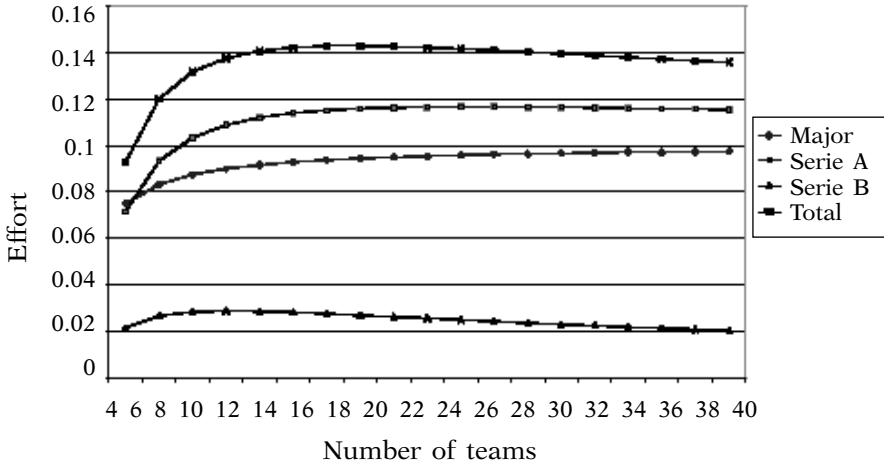
GRAPH 5

EFFORT PER TEAM (GAMMA = 1)



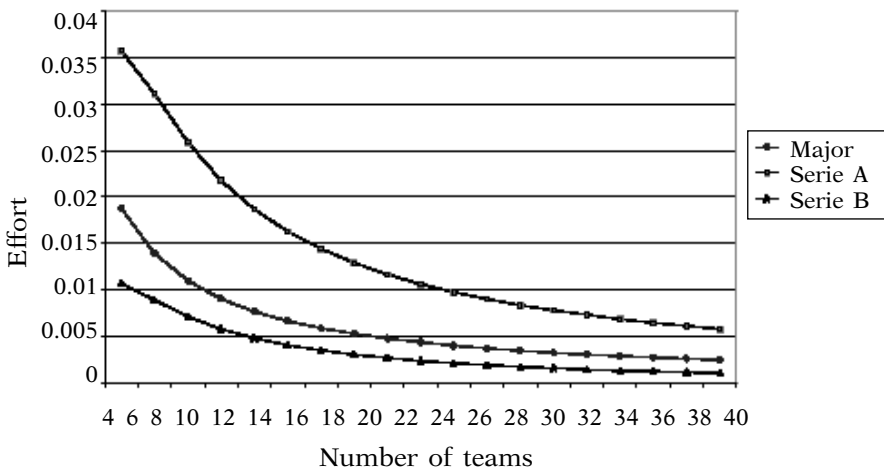
GRAPH 6

TOTAL EFFORT IN MAJOR LEAGUE AND TWO HIERARCHICAL DIVISIONS (GAMMA = 0.1)



GRAPH 7

EFFORT PER TEAM (GAMMA = 0.1)



Graphs 6 and 7 illustrate the case where the discriminatory power of the contest is low ($\gamma = 0.1$). First note that this discourages effort (and rent dissipation) since the marginal returns to effort are low. Graph 6 shows that total effort is now significantly higher in an open league system and Graph 7 makes it clear that this is because effort per team in the closed league is not much higher in the closed league than in Serie B. Because the contest is not very discriminating, there is little incentive to make an effort to win, which is the only instrument providing incentives in the closed league. In the open league, however, teams in Serie A are also competing to avoid the drop, and this extra incentive keeps effort levels per team much higher than in the closed league.

Clearly, the objective of the teams (to minimize rent dissipation) conflicts with the social objective of maximizing rent dissipation. Thus, if the league members jointly determine league policy,¹⁹ they will opt for closed leagues when the discriminatory power γ of the contest is low (recall that γ defines the sensitivity of win probabilities to differences in effort) and open leagues when the discriminatory power is high.

4. - Asymmetric Teams with Endogenous Effort

In the previous section we focused on the incentive to supply effort, in this section we focus on the incentive to share revenues. The justification for revenue sharing in sports leagues is competitive balance — by creating a more balanced contest the league will become more attractive to fans and will generate larger league-wide profits (and welfare). This analysis presupposes asymmetry in revenue generation between the teams — i.e. for a given win percentage some teams will generate a larger income, either because the club draws on a larger fan base or is more intensely supported than other teams. However, revenue sharing

¹⁹ In the next section we examine redistribution policies in an asymmetric model, where we assume league policies require the consent of the strongest teams.

requires agreement among the teams. In particular, teams that enjoy a larger income absent revenue sharing must consent to a redistribution scheme that will see their income fall relative to weaker rivals. Another effect of revenue sharing that has been widely commented on in the literature is its tendency to blunt incentives (see e.g. Fort and Quirk, 1995), a factor which can make revenue sharing attractive for the strong teams. In our analysis we look for conditions where the larger revenue clubs would willingly share income²⁰.

Suppose there existed four feasible locations (e.g. cities) for a sports team, based on the drawing power of those teams. We assume that each location can support only one team, while two locations possess a greater drawing power than the others. In such a universe a number of league configurations are possible. We suppose either that all teams compete in the same championship each year (the closed system) or that there are two divisions of two teams each with promotion and relegation of one team from each division each season (the open system).

For tractability we assume that in the “no sharing” case this means that the two weak teams have a zero probability of winning any match against a strong team. Moreover, we assume that whenever “sharing” occurs the teams competing with each other have an equal probability of winning in that particular contest. Our notion of sharing implies a significant restriction of the strategy space of the teams (in the Appendix we develop a model where more of the teams’ choices are endogenized). The weaker teams will always want to share. We focus on the potential benefit for the strong teams from sharing under closed and open systems.

²⁰ We suppose that smaller revenue generating teams could never compel the larger revenue generating teams to share, otherwise they would simply quit the league and start a rival competition. In England, this is approximately what happened in the early 1990s - the top 5 teams led a breakaway from the Football League (a venerable institution which had recently celebrated its centenary) on the grounds that it no longer wanted broadcasting income, largely derived from their own matches, to be shared with the other 87 members of the League. Having failed to negotiate a significant increase in the share of the top teams, they persuaded 15 other teams to secede with them to form the FA Premier League.

4.1 Closed League with Four Teams, no Sharing

A single prize is awarded to the winner of each contest. Since weak teams never win in a contest against strong teams, they can never win at all and so never contribute effort. Thus the four team case is indistinguishable from a symmetric two team league. Normalizing the value of the prize to unity, the effort levels and payoffs to the strong teams (superscript S) in a closed league (C) with no sharing (N) will be:

$$(3) \quad e^s = \frac{\gamma}{4}, V^s(CN) = \frac{2 - \gamma}{4(1 - \delta)}$$

4.2 Closed League with Four Teams, Sharing

Now all four teams have an equal probability of winning every season regardless of the effort they supply and so no team supplies any effort. We assume that because the contest is now perfectly balanced, this also enhances its attractiveness and therefore the value of the prize, which is now assumed to be $z > 1$. Thus the payoff to each player in a closed league (C) with sharing (S) is:

$$(4) \quad V(CS) = \frac{z}{4(1 - \delta)}$$

Clearly the total value of the league is increased by sharing, but in the absence of side payments the strong teams will only consent to sharing as long as (4) is greater than (3), which happens when $z \geq z^C = 2 - \gamma$. This is true by assumption for $\gamma \geq 1$, and may be true even for smaller values of γ . Note that, as is true in many contest models of this type²¹, a necessary condition for the existence of pure strategy equilibrium is $\gamma \leq 2$. What this section has shown is that the incentive to share revenues depends on the potential gain from a balanced competition (z) and the

²¹ BAYE M. *et AL.* (1994) developed the analysis of mixed strategies when $\gamma > 2$.

discriminatory power of the contest. The latter matters because it affects effort in the no-sharing case. In a highly discriminating contest, strong teams will choose to expend high effort and thus dissipate rents, making the sharing alternative attractive.

4.3 *Open League with Two Two-Team Divisions, no Sharing*

We assume that the worst performing team is relegated from each division in each season and replaced with the best performing team from the lower division. Again, with no sharing, the weak teams can never win against the strong teams, can never win the prize and so contribute no effort. The two strong teams never meet in the lower division, but meet every other season in the top division, and are then assumed to compete for the prize.

To calculate the optimal amount of effort, we need to identify the value of each possible state for a strong team:

$$\max_e V_1^{SS} = p - e + \delta [pV_1^{SW} + (1-p)V_2^{SW}]$$

$$V_1^{SW} = 1 + \delta V_1^{SS}$$

$$V_2^{SW} = \delta V_1^{SS}$$

$$p = e^\gamma / (e^\gamma + e_{-i}^\gamma)$$

where V_1^{SS} is the present value of a strong team currently located in the top division with another strong team, V_1^{SW} is the present value of a strong team currently located in the top division with a weak team and V_2^{SW} is the present value of a strong team currently located in the second division with a weak team. Solving for V_1^{SS} we find:

$$(5) \quad V_1^{SS} = \frac{(1+\delta)p - e}{1 - \delta^2}$$

Maximizing (5) with respect to e we find the equilibrium effort level when team the two strong teams are in the top division, $e^S = \gamma(1 + \delta)/4$. The present value of the payoffs in the three states are:

$$(6) \quad V_1^{SS} = \frac{2 - \gamma}{4(1 - \delta)}, \quad V_1^{SW} = \frac{4 - \delta(2 + \gamma)}{4(1 - \delta)}, \quad V_2^{SW} = \frac{\delta(2 - \gamma)}{4(1 - \delta)}$$

Given that a strong revenue generating team is always promoted when in the second division and is relegated with probability 1/2 when in the top division, in the steady state each of these teams obtains V_1^{SS} with probability 1/2, and V_1^{SW} or V_2^{SW} with probability 1/4. Thus the steady state payoff to strong team in an open league with no sharing is:

$$V^S(ON) = \frac{4 - \gamma(1 + \delta)}{8(1 - \delta)}$$

The difference between $V^S(ON)$ and the closed league payoff with no sharing ($V^S(CN)$ from (3)) is $\gamma(1 - \delta)$, which is always positive. Intuitively, the benefit to the strong teams of an open league arising from the fact that they get to win easily 25% of the time is exactly offset by the cost arising from not being in the top division 25% of the time. However, in a closed league the strong teams meet every season and so have to contribute effort every season if they want to win, whereas in our stylized model a strong team meets only weak opposition 50% of the time and on the occasions have to make no effort at all. More generally, we might expect that as long as the strong teams need to try less hard to win when they play weak teams, then they will prefer the open league system. The greater the discriminatory power of the contest, the more effort is required when strong teams compete, and the greater the benefit to the strong teams of the open system (without sharing).

The setting for this result is somewhat extreme, given the small number of teams involved and the assumed gap in capabilities. However, our result would generalize to the case

where the league divisions are larger and the gaps in abilities are smaller as long as the weak drawing teams will contribute less effort. In such cases relegation is always a cost to the strong team that is relegated but a benefit to the remaining strong teams amongst whom competition is relaxed²². However, as we show in the Appendix there can exist equilibria in an open system where the weaker teams contribute more effort than the strong teams (in order not to be relegated), and under these conditions the strong teams may prefer a closed league.

4.4 *Open League with Two Two-Team Divisions, Sharing in the Top Division*

We begin by assuming that sharing only occurs in the top division (competitive balance concerns are likely to be greatest in the top division, and gaining the consent of the strong clubs will be easier to obtain than when there is equal sharing across all divisions). With sharing the present value of a strong team is reduced compared to the closed league case, since relegated teams miss an opportunity to win the prize. However, the strong teams are always certain to be promoted in the season following relegation. Thus:

$$V_1^{SS} = V_1^{SW} = V_1^S = [z + \delta(V_1^S + V_2^{SW})]/2, \quad V_2^{SW} = \delta V_1^S$$

These lead to:

$$V_1^S = \frac{z}{(2 + \delta)(1 - \delta)}, \quad V_2^{SW} = \frac{\delta z}{(2 + \delta)(1 - \delta)}$$

At the steady state the strong team is at the top with probability 2/3 and at the bottom with probability 1/3.²³ Hence

²² While the very best teams will not be relegated very frequently in a league made up of a larger number of teams, it must be expected to happen at least occasionally.

²³ To see this note that $p_1^S + p_2^{SW} = 1$, while the transition probabilities in the steady state must satisfy $p_1^S = p_1^S/2 + p_2^{SW}$, and $p_2^{SW} = p_1^S/2$.

the payoff to a strong team in an open system (O) with top division sharing ($S1$) is:

$$(7) \quad V^S(OS1) = \frac{z}{3(1-\delta)}$$

Note that this payoff is larger than in a closed division and equal sharing since the strong teams have the advantage of always being immediately promoted whenever they are relegated, and hence win the prize more frequently (i.e. one third of the time rather than one quarter of the time).

If we now compare the strong team's payoff to sharing in the open system ($OS1$) to no sharing in the open system (ON), the necessary condition for sharing to be preferred is found by comparing (7) with (6), leading to:

$$z \geq z^o = 3/2 - 3\gamma(1+\delta)/8$$

Recall that the equivalent condition for sharing to be preferred in a closed league was $z \geq z^C = 2 - \gamma$. For high discount rates the critical value for z will be smaller in the open league, implying a greater willingness to share on the part of the strong teams. However, for lower discount rates it can happen that the critical value to make sharing attractive in the closed league would not be large enough to make sharing attractive in the open league. For example, when $\gamma = 1$, sharing will be preferred in a closed league for all $z > 1$, while if $\delta = 0.25$ sharing is only preferred if $z > 21/16$.

Equivalently, the comparison between the threshold values of z tells us that revenue sharing is a more stringent condition in an open league ($z^O > z^C$, i.e. revenue sharing is less likely to happen) if $\gamma > 4/5(5-3\delta)$, that is to say if the contest is sufficiently discriminatory.

This result is intuitive as the discriminatory power affects profits only without revenue sharing (under our assumptions firms do not react to z with any form of sharing, both in open and in

closed systems). Without sharing, the higher the discriminatory power the lower equilibrium profits. Under *CN* a firm is competing 100% of its time, while under *ON* effort is exerted only 50% of the time. It is thus clear that a higher value of γ , while making sharing less appealing both in open and in closed systems, reduces profits by more in the latter than in the former.

Our assumption, however, is very restrictive. In the Appendix we show that, once we endogenize effort choices in an open league with revenue sharing, the threshold that makes revenue sharing attractive in an open league (z^O) increases, so that the range of values of γ for which revenue sharing is attractive in a closed league but not in an open league expands. If weak teams can end up contributing more effort than strong teams (this can happen because the penalty of relegation is higher: weak teams tend to find it harder to get promoted again) then one of the main attractions of the open system to the strong teams (namely, weaker opponents) has vanished and they resist sharing.

4.5 *Open League with Two Two-Team Divisions, Sharing in Both Divisions*

Each team has an equal probability of winning in each division and no team contributes effort. Thus for any team:

$$V_1 = [z + \delta(V_1 + V_2)]/2, \quad V_2 = \delta(V_1 + V_2)/2$$

so that:

$$V_1 = \frac{(2 - \delta)z}{4(1 - \delta)}, \quad V_2 = \frac{\delta z}{4(1 - \delta)}$$

and the average payoff in an open system with sharing in both divisions (*S2*) is:

$$(8) \quad V^S(OS2) = \frac{z}{4(1 - \delta)}$$

This is the same as the payoff to sharing in a closed division, since teams compete for the top prize half as frequently but with twice the probability of winning. As might be expected, this is lower than the payoff to the strong teams when there is no lower division sharing, so that strong teams will be less willing to share if sharing is applied to both divisions. Comparing (8) with (6), in an open league sharing in both divisions is preferred to no sharing when $z > 2 - \gamma(1 + \delta)/2$. Recalling that the equivalent condition for a closed league is $z > 2 - \gamma$, it is clear that the critical value of z is always lower in a closed league. In other words, strong teams will be less willing to agree to full sharing in an open league than in a closed league.

Thus in this section we have shown that under some circumstances sharing can be more attractive in an open league, but only when that sharing is limited to participants in the top division. When there is sharing across all divisions, revenue sharing is always less attractive in an open league than in a closed league.

5. - Conclusions

One of the most striking but least analyzed differences between the (closed) American and (open) European models of professional sport is the system of promotion and relegation. This paper applies contest theory to the analysis of open and closed league systems.

We find that a promotion and relegation system typically enhances the incentive to contribute effort and hence to dissipate rents, which may be considered an enhancement of social welfare. On the other hand, promotion and relegation is also likely to inhibit incentives to share resources. Redistribution in team sports is frequently considered beneficial not only for the owners, since the incentive to compete is weakened, but also for consumers, who are said to prefer more balanced contests. To the extent that this is true, promotion and relegation may reduce social welfare.

Whatever the welfare implications, we argue that the effects

of promotion and relegation on incentives are broadly consistent with what we observe on either side of Atlantic. In Europe, where the system is applied, teams compete intensively — to the point of bankruptcy in fact — and seem unwilling to share resources. In the US, where the system does not apply, economic competition is less intense and teams do share resources. While this paper cannot be said to be the final word, we believe it points to a potentially fruitful avenue for both empirical and theoretical research.

APPENDIX

Consider an open league with four teams (two weak and two strong) and two divisions. In this annex, we allow the weak teams to be able to beat the strong teams in the top division, but only if there is revenue sharing. We suppose that in the lower division, where there is no sharing, the strong team always wins against a weak team but has to supply some minimum amount of effort to achieve this result.

(a) No Sharing

As weak teams never win against a strong team, we can concentrate on the effort choice for the strong team. This is derived from the following optimization problem:

$$\begin{aligned} \max_e V_1^{SS} &= p - e + \delta [pV_1^{SW} + (1-p)V_2^{SW}] \\ V_1^{SW} &= 1 - e_H + \delta V_1^{SS} \\ V_2^{SW} &= -e_L + \delta V_1^{SS} \\ p &= e^\gamma / (e^\gamma + e_{-i}^\gamma) \end{aligned}$$

This is basically the same problem as in Section 4.3, with the only difference that the strong team has to supply some (exogenous) minimum level of effort to win against a weak team both in the top division (e_H) and in the low division (e_L). The only endogenous effort is the one supplied against an equally strong rival. At a symmetric equilibrium ($e_{-i} = e$) one gets:

$$e = \gamma \frac{1 + \delta(1 - e_H + e_L)}{4}$$

$$V^S(ON) = \frac{2V_1^{SS} + V_1^{SW} + V_2^{SW}}{4} = \frac{4 - \gamma - \delta\gamma(1 - e_H + e_L) - 2(e_H + e_L)}{8(1 - \delta)}$$

which is a simple generalization of (6).

(b) *Sharing in the Top Division*

The problem is considerably extended here compared to Section 4.4. We now denote by z the gross value of the prize. Team i of type $h = \{S, W\}$ competing against a team of type $k = \{S, W\}$ in division $d = \{1, 2\}$ puts effort e_{id}^{hk} and wins that division with probability $p_{id}^{hk} = (e_{id}^{hk})^\gamma / [(e_{id}^{hk})^\gamma + (e_{-id}^{hk})^\gamma]$. The maximization problem for the strong team is:

$$\max_{e_{11}^{SS}} V_1^{SS} = zp_{11}^{SS} - e_{11}^{SS} + \delta \left[p_{11}^{SS} V_1^{SW} + (1 - p_{11}^{SS}) V_2^{SW} \right]$$

$$(A1) \quad \max_{e_{11}^{SW}} V_1^{SW} = zp_{11}^{SW} - e_{11}^{SW} + \delta \left[p_{11}^{SW} V_1^{SS} + (1 - p_{11}^{SW}) V_2^{SW} \right]$$

$$V_2^{SW} = -\tilde{e}_L + \delta \left[p_{21}^{SW} V_1^{SS} + (1 - p_{21}^{SW}) V_1^{SW} \right]$$

We are still assuming that, if relegated, a strong team wins with probability one since it plays against a weak team, but has to put in some effort \tilde{e}_L to do so. On the other hand, the endogenous efforts are those exerted in the top division against an equally strong or a weaker team. Although there is sharing, and teams in the top division are all competing for the gross prize z , the efforts are not symmetric since firms are taking into account the probability of getting to other states that do not give the same payoffs to strong and weak teams. Hence we have to characterize also the effort supplied by the weak teams:

$$\max_{e_{11}^{WS}} V_1^{WS} = z(1 - p_{11}^{SW}) - e_{11}^{WS} + \delta \left[(1 - p_{11}^{SW}) V_1^{WS} + p_{11}^{SW} V_2^{WW} \right]$$

$$(A2) \quad V_2^{WS} = \delta \left[p_{21}^{SW} V_2^{WW} + (1 - p_{21}^{SW}) V_2^{SW} \right]$$

$$V_2^{WW} = -\tilde{e}_W + \delta (V_1^{WS} + V_2^{WS}) / 2$$

Hence we are endogenizing only the effort that a weak team puts to win the title, while it puts zero effort when relegated and competing without sharing against a strong team. To simplify calculations, we also assume that, when in the lower division, weak teams facing each other put an effort \tilde{e}_W and win with probability 1/2.

We have solved the problems represented by (A1) and (A2) with respect to the three endogenous efforts. The expressions are rather cumbersome and therefore we show only some numerical examples below. Once the efforts are known, the expected value of a strong team can also be calculated and compared to the expected value it would get without any sharing. A strong team can be found in one of the three following states: *a)* against a strong team in the top division with absolute probability p^{1S} , *b)* against a weak team in the top division with absolute probability p^{1W} , *c)* against a weak team in the bottom division with absolute probability p^{2W} . Taking into account the transition probabilities between states, the absolute probabilities in a steady state must satisfy:

$$p^{1S} = p_{11}^{SW} (p^{1W} + p^{2W})$$

$$p^{2W} = (1 - p_{11}^{SW}) p^{1W} + p^{1S} / 2$$

$$p^{1S} + p^{1W} + p^{2W} = 1$$

These can be solved to get:

$$(A3) \quad p^{1W} = p^{2W} = (1 - p^{1S}) / 2$$

$$p^{1S} = p_{11}^{SW} / (1 + p_{11}^{SW}) = (e_{11}^{SW})^\gamma / [2(e_{11}^{SW})^\gamma + (e_{11}^{WS})^\gamma]$$

It can be checked that these values satisfy $p^{1W} \equiv (1 - p_{11}^{SW}) p^{2W} + p^{1S} / 2$.

Finally, having obtained the equilibrium efforts and thus the probability of each state, the expected value of a strong team is:

$$V^S(OS1) = p^{1S}V_1^{SS} + (1-p^{1S})(V_1^{SW} + V_2^{SW})/2$$

Graph A1 plots the solution for the following parameterization: $\gamma = 1$, $\delta = 0.8$, exogenous efforts e_H , e_L , e_W all set to 0. The left panel reports the expected value for the strong team against the value of z : the dotted line corresponds to revenue sharing (OS1), while the continuous line refers to the no sharing case (ON). Unless z is very high, a strong team will never want to adopt revenue sharing. In the right panel we compare efforts. Revenue sharing corresponds again to the dotted lines: in particular the highest one is the effort spent by the weak team, the middle and bottom lines plot the effort spent by the strong team against a weak and a strong team respectively. We can thus tentatively conclude that, for low values of z , revenue sharing does not occur despite the effort spent is still quite limited compared to no sharing: without sharing the strong team benefits from zero effort to win against a weak team at the top (this happens with probability 25%). Now, despite the “collusive” effect of lower efforts with sharing and low values of z , when it competes against a weak team at the top it has to put effort. This effect of costly effort prevails and makes sharing dominated by no sharing.²⁴ Only when z is sufficiently high, the higher expected gross value of the prize compensates for the higher effort and revenue sharing would be preferred by the strong team.

Another interesting observation from the diagrams is that, under revenue sharing, the highest level of effort is actually put by the weak team (although in the only state where it puts an endogenous effort). The threat of relegation to the bottom, where the weak team is not competitive when it meets a strong team, means that it will fight very hard to remain at the top when it happens to be there.

Recall that the equivalent condition for sharing to be preferred

²⁴ To be more precise, one has to take into account also the probabilities of realization of each of the three possible states, which are not shown in the Graphs. Since efforts under revenue sharing are not “too” different from each other, it turns out that they are in the range of 1/3 - see eq. (A3).

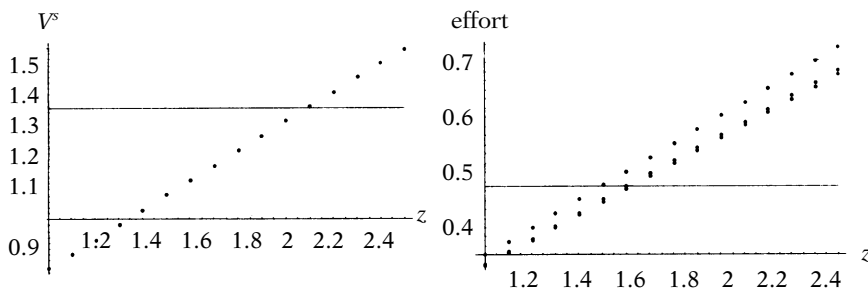
in a closed league was $z > 2 - \gamma$. Given the parameterization in the figure ($\gamma = 1$), this means that, in a closed league, revenue sharing would always happen ($z^C = 1$). On the contrary, in an open system, z has to be sufficiently high ($z^O \approx 2.15$). Hence, once we endogenize the effort of the weaker teams it becomes *more likely* that the strong teams will *reject revenue sharing in an open system* when they would accept it in a closed system.

By looking at other parameterizations, we can describe the following tendencies:

a) when the comparison is *between* an open league and a closed league, the range of γ 's that make sharing less likely in an open league is now considerably expanded. For instance, under the parameterization of Graph A1, the threshold value of z is always more stringent in an open league ($z^O > z^C$) for $\gamma > 0.3$; on the other hand in the main text the limiting condition reduced to $\gamma > 4/(5-3\delta) \approx 1.54$ in this case;

b) similar results are obtained by putting reasonable values for the various exogenous efforts; for instance if \bar{e}_W is positive (i.e., the effort put by the weak teams when they are both in the bottom division), then the likelihood of revenue sharing in an open league decreases even further. This is the intuition: the weak team — when at the top — competes even harder to avoid relegation that becomes a worse state, as a consequence the strong team has to face a tougher rival when it shares at the top.

GRAPH A1
EXPECTED VALUE OF A STRONG TEAM (LEFT PANEL) AND EFFORT
(RIGHT PANEL) IN AN OPEN LEAGUE WITH AND WITHOUT
SHARING IN THE TOP DIVISION. PARAMETERS: $\gamma = 1$, $\delta = 0.8$



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