The Role of Preference Structure and Moral Hazard in a Multiple Equilibria Model of Financial Crises

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This paper proposes an analysis of financial crises by a multiple equilibria model, based on the assumption of common knowledge. This model modifies and broadens the Corsetti, Guimaraes and Roubini (2003) model based on global games theory. In the first part we assert the implications for the International Monetary Fund (IMF) as an international lender of last resort, utilising existing literature based on multiple equilibria models. In the second part, we extend the analysis and highlight the interesting implications. The model predicts the IMF should not be too conservative in its decisions, while avoiding the excessive liquidity supports, which can lead to moral hazard distortions. [JEL Classifications: F33, F34]

1. - Introduction

In the last decade there have been several currency and financial crises in emerging market economies: from Mexico in 1994 to the Argentine default in 2001. Economic literature has tried, from time to time, to find the main causes for every crisis and their proper remedies. In this connection, a particularly relevant role for economic research has been played by the Asian

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Crisis of 1997. Within this circle of research, the literature recognizes some main causes: as well as solvency problems there are liquidity problems and currency imbalances. Moreover, a determining role is played by agents' expectations in the economy.

In the new literature, emerging after the Asian crisis, some models are based on the analogy between speculative crises and bank runs, as in Diamond and Dybvig (1983) — see for instance Chang and Velasco (2001); Radelet and Sachs (1998); Rodrik and Velasco (1999); Jeanne and Wyplosz (2001). This parallel is due to the fact that debt runs and the banking sector are strictly interlinked. In most cases, banks are vulnerable to such runs because they have short-term (or foreign currency denominated) debts, while the most of their assets are long-term and denominated in domestic currency, with high costs for the liquidation of the investments. Several third-generation models concentrate on the importance of currency and maturity mismatches (Krugman, 1999; Aghion, Bacchetta and Banerjee, 2000; Cespedes, Chang and Velasco, 2000; Jeanne and Zettelmeyer, 2002).

These types of models are mainly based on an information structure that is common knowledge among the agents in the economy. Every agent is aware of the equilibrium strategy undertaken by all the other agents. This structure implies that there's no heterogeneity among the agents' behaviour inside the economy and multiple equilibria. In this view, panic crises can be seen as a switch from one equilibrium to another due to changes in agents' expectations. The multiplicity of equilibria is bounded by the possibility that fundamentals are so strong that they can rule out the possibility of a crisis. It is important to note that this type of financial and currency fragility is not caused by market irrationality. Instead, both the equilibria are consistent with the rational expectations hypothesis.

The main limit of these models is the lack of explanation of

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1 In Sachs J. (1998), foreign creditors consider the stock of international reserves as liquid collateral against their loans to the country.

2 A useful example of a model with self-fulfilling speculative attacks and bank runs can be found in Obstfeld M. (1998).
the factors determining agents' coordination on one equilibrium rather than another. Moreover, these models, having multiple equilibria, lead to the conclusion that the best policies to avoid a crisis should completely eliminate the possibility that the agents coordinate on the crisis equilibrium. A prescription of this kind is the institution of an international lender of last resort (ILOLR) which can prevent bank runs from happening, by a complete bailout of the country. This is the solution proposed by Jeanne (2000), among several others. The current international financial institution which could best play this role is the International Monetary Fund (IMF). The main failure of this prescription is the trade-off between liquidity provision by the ILOLR and moral hazard distortions.

Most recent contributions have tried to address the open issues left by the preceding literature. The foundation of this theoretical development is based on the global games approach, presented by Carlsson and Van Damme (1993). These models allow for the possibility that part of the information about the economic fundamentals, instead of being public as the common knowledge assumption asserts, can be private. In an important contribution, Morris and Shin (1998) have built a theory of speculation, based on global games, in which agents have private and incomplete information about the state of fundamentals and other agents' information and behaviour. These models give rise to different implications of economic policy from the ones deriving from the multiple-equilibria models, having a unique equilibrium.

Hence, agents face strategic uncertainty about the behaviour of the rest of the market, but they are able to build an individual equilibrium strategy, the main result being the achievement of a unique equilibrium.

The new literature suggests that a crisis becomes more likely when the fundamentals are weaker. The recent contribution of

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3 For two different opinions about the viability of the IMF operating as an ILOLR see Jeanne O. - Wyplosz C. (2001) and Fischer S. (1999).

4 Dooley M. - Verma S. (2001) show that IMF lending can create moral hazard distortions and make crises more costly.

5 A critical approach to multiple equilibria models can be found in Morris S. - Shin H. (2000), while a comparison between the two approaches can be found in Corsetti G. - PeSETI P. - Roubini N. (2002).

The model presented in this paper, starting from the existing literature, analyzes some particularly relevant topics of this literature. Given the ever growing role of transparency and the availability of public information, we have adopted the hypothesis of common knowledge among agents, as in the traditional literature of bank runs (Diamond and Dybvig, 1983). Hence we will have multiple equilibria. In order to develop a useful comparison between the results coming from different hypotheses on the information structure, we based our model on the framework of CGR in the developing of our model. It analyzes debt crises in a real economy (without currency). In sections 2-4 we present the model and we will see the interactions between liquidity and solvency problems and IMF interventions. In section 5 we will see the particular role played by the different preference structures of the IMF and fund managers in determining the equilibrium, showing a generalized structure. In section 6, we will consider incentive distortions that can emerge because of moral hazard, obtaining results in line with CGR (though not completely general).

2. - The Model

As previously noted, the core framework of this model is based on that of CGR. Consider a small open economy with a three-period horizon – $t = 0, 1, 2$. In the economy, there is a continuum of domestic agents of mass 1, with every agent of mass zero. Analogously, there is a continuum of international fund managers of mass 1, with zero individual mass.

The starting endowment of the economy is $E$. The domestic agents can borrow foreign short-term debt up to $D$ — with term $t = 1$ — from fund managers. Furthermore the IMF can supply the country with a liquidity support $L$. For simplicity we suppose
that all international lending takes place at the same interest rate $r^*$, which is normalized to zero.

Domestic agents invest in long-term risky projects, which yield a stochastic rate of return $R$ in $t = 2$. The expected return is naturally above the international interest rate: $E_0 R > 1 + r^*$. The early liquidation of the investments (in $t = 1$) is costly, with a cost $k > 0$. Hence, if the project is discounted and liquidated early, it yields $R/(1+k)$. We hypothesize that the capital is infinitely divisible: so the early liquidation of part of the investments in $t = 1$ doesn't affect the rate of return of the other investments.

In $t = 0$ domestic agents invest their own endowments and the borrowed resources $(E + D)$ in domestic risky projects $I$, and in international liquidity reserves $M$. It is worth noting that, in $t = 0$, there isn't a real decision process. In fact the game is between fund managers and the IMF. $E$, $D$, $M$, $I$ are all given parameters.

In $t = 1$ fund managers decide whether to roll over their loans or withdraw. Denoting with $x$ the fraction of managers who decide to withdraw their loans, $xD$ is the fraction of debt that is not rolled over and must be paid back. CGR assume incomplete information, hence every fund manager has to choose her equilibrium strategy facing strategic uncertainty about the others’ actions. In this environment, the managers’ behaviour will be heterogeneous. Here we assume common knowledge among the agents, leading to no heterogeneity in agents’ behaviour and only two extreme equilibrium conjectures will take place, that is $x = 0$ (everybody rolls over their loans) and $x = 1$ (nobody rolls over their loans). Conditional on $x = 0$ in $t = 1$, GNP in $t = 2$ will be:

$$Y = RI + M - D$$

When $D > RI + M$ there is default and GNP is 0. In case of default, all lenders are paid pro-rata, until exhausting the resources available to the country\(^6\).

Conditional on $x = 1$ in $t = 1$, $D$ is the amount of liquid reserves necessary to the country. To meet short-term obligations $D$, \(^6\)In what follows we take GNP as a measure of national welfare.
domestic agents can use their stock of liquid resources $M$; if this is not enough, they can also liquidate a fraction $z$ of the long term $I$, getting $zRI/(1+k)$, with $z \in [0, 1]$. Moreover the country can receive funds up to $L$ by the IMF. Let $\Lambda = M + L$ denote total liquidity available to the country, made by the predetermined component $M$ and the contingent component $L$. The country will incur some liquidation costs and efficiency loss when $D > \Lambda$; it will default when $D > \Lambda + RI/(1+k)$ because domestic agents are not able to meet their short-term obligations despite complete liquidation of long term investments.

In $t=2$ the country total resources consist of $RI(1-z)$ plus any liquid resources left over from the previous period, i.e. $(\Lambda-D)_+$. Its liabilities consist of any IMF loan $L$. Hence, GNP will be:

$$Y = RI(1-z) + (\Lambda-D) - L_+.$$ 

If $\Lambda$ is not enough to pay back the debt $D$, agents must liquidate part of their investments, up to $zRI/(1+k)$, then:

$$D = \Lambda + \frac{zRI}{1+k}.$$ 

Having $(\Lambda-D)_+ = 0$, solving for $z$ the above formula and substituting in $RI(1-z)$, GNP becomes:

$$Y = RI - (1+k)(D-\Lambda) - L_+.$$ 

Naturally the GNP is zero in the event of default, that happens if $(1+k)(D-\Lambda) + L_+ > RI$.

2.1 Payoffs and Information

Fund managers and the IMF face a structure of payoffs that depends on the decisions taken, as in Rochet and Vives (2005) and in CGR. When the country does not default, rolling over loans to
the country in $t = 1$ yield a benefit that is higher than withdrawing. The difference in utility between rolling over loans and withdrawing is equal to a positive constant $b$. On the other hand, when the country defaults, managers who do not withdraw their loans pay a cost: the difference in utility between rolling over and withdrawing is equal to $-c$ ($<0$).

The IMF is conservative in its decisions, in the sense that it is willing to lend to illiquid countries, trying to reduce early liquidation costs; but it is not willing to lend to insolvent countries, for which its liquidity support would become subsidized loans. For simplicity we assume that the structure of payoffs of the IMF is analogous to that of fund managers. If the country ends up not defaulting, lending $L$ rather than denying it, gives a benefit equal to $B$. If the country defaults, by lending $L$ rather than denying it, the IMF get a negative utility equal to $-C$. Different values of $b$, $c$, $B$, $C$ may represent changes in the strategies of fund managers and the IMF. For the sake of simplicity, we initially consider the case of identical preferences between the IMF and fund managers, that is $B = b$, $C = c$. Note that, the utility for fund managers and the IMF is independent of the extent of default\(^7\).

In accordance with our assumption of common knowledge — and unlike CGR — there will be public information (though incomplete). Hence there is a single public signal about the state of fundamentals $R$, equal for all and known by everybody, so that the mean and the variance of its probability distribution function is common knowledge. Hence, agents do not face strategic uncertainty about their optimal strategy. In $t = 1$, the signal is:

\[(1) \quad S = R + \eta\]

whereas $\eta$ is normally distributed with mean 0 and variance $1/\rho$.

Both international fund managers and the IMF, on the basis of the received signal, decide their strategy — rolling over their

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\(^7\) This hypothesis implies that we abstract from distributional issues of the available resources between the country and the creditors, as well as between the IMF and fund managers, following a debt crisis.
loans rather than withdrawing, and providing the liquidity support $L$ respectively. Fund managers roll over their loans if the public signal is such that their payoff is non-negative; likewise the IMF provides the liquidity if, given the public signal, this action is associated with a non-negative payoff. Given the parameters describing the preference structure — $b$, $c$, $B$, $C$ — we can calculate the threshold values of the signal under which agents switch their behaviour.

Remember that, with the assumption of common knowledge, the equilibrium behaviour of fund managers is symmetric in equilibrium — there is no heterogeneity. In fact, because of receiving the same signal and having the same preference structure, they always undertake the same action in equilibrium. Moreover, every fund manager has an infinitesimal dimension, then he/she cannot influence the rest of the market and the macroeconomic performance of the country in $t = 2$. Hence none of them are interested in deviating from the action undertaken by the rest of the market, which will be optimal in equilibrium.

We have a different assumption about the IMF. It has not infinitesimal dimension (it is modelled as a large player) and by its own actions is able to influence the rest of the market. Hence the IMF does not need to conform itself to the actions undertaken by the rest of the market and can choose its behaviour independently.

3. - Equilibria

Agents follow a trigger strategy. Every fund manager considers two equilibrium conjectures, that are: $x = 0$, everybody rolls over their loans to the country; and $x = 1$, everybody withdraws their loans in $t = 1$. Only these two actions are relevant in equilibrium. In fact, all agents and the signal are identical, so all fund managers will coordinate on the same action.$^8$

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$^8$ There cannot be ranges for which fund managers are indifferent between the two equilibrium conjectures. A complete proof can be found in CORSETTI G. - PESENTI P. - ROUBINI N. (2002).
Conditional on \( x = 0 \), the solvency condition, is: \( RI \geq D - M \). The minimum rate of return under which the country defaults is:

\[
\bar{R}_{x=0} = R_S = \frac{D - M}{I}
\]

It does not depend on the liquidity support \( L \), just because there isn’t any early withdrawal. When \( R < R_S \) the country is not solvent and will always default.

Conditional on \( x = 1 \), the liquidity support \( L \) provided by the IMF influences the minimum rate of return under which there is default. Above all, the dimension of \( L \) will affect the definition of equilibria, in fact \( L \) may be either bigger or smaller than the financing gap of the country \( (D - M) \).

If \( L < D - M \) the country certainly experiences some liquidation costs. The solvency condition is: \( RI \ (1 - z) \geq L \). The threshold rate under which there is default is:

\[
\bar{R}_{x=1} I - (1 + k) [D - M - L] = L
\]

that gives:

\[
\bar{R}_{x=1} L = R_S (1 + k) - k \frac{L}{I}
\]

Instead, if \( L > D - M \) the IMF can solve on its own, by the provision of \( L \), the liquidity problem. The relevant threshold rate becomes:

\[
\bar{R}_{x=1} = R_S
\]

Hence, \( \bar{R}_{x=1} \) depends on the liquidity support \( L \). If \( L \) is bounded, the IMF can lessen the liquidation costs of a run by the fund managers only within a certain range of fundamentals. Instead, if \( L \) is unbounded, there isn’t a liquidity problem anymore and the country will default only if not solvent.

However, as we have seen above, the IMF intervenes only if its payoff is non-negative, otherwise it doesn’t provide any liquidity support \( L \) to the country. If fund managers coordinate on the
action \( x = 1 \) and the IMF does not intervene, the relevant threshold will be:

\[
\bar{R}_{x=1} = \bar{R} = R_s(1 + k)
\]

If threshold values of signals are identical, in equilibrium the IMF provides the liquidity support \( L \) only when fund managers decide to roll over their loans.

Now we are able to determine the threshold values of signals that characterize the equilibrium. The signal about the state of fundamentals, though being common knowledge, is not of complete information. Then, fund managers may undertake their actions with some mismatching in comparison with solvency thresholds. As noted above, the signal is the same for all, and all fund managers undertake the same action in equilibrium.

Considering the two equilibrium conjectures, we know that, conditional on \( x = 0 \), the country defaults if and only if \( R < R_s \).

Then, given the signal \( S \), fund managers withdraw their loans if and only if:

\[
-c \text{Prob}(R < R_s | S) + b \left[1 - \text{Prob}(R < R_s | S)\right] \geq 0
\]

The threshold value of \( S \) under which they decide to withdraw is:

\[
-c \text{Prob}(R < R_s | S^*) + b \left[1 - \text{Prob}(R < R_s | S^*)\right] = 0
\]

that becomes:

\[
\text{Prob}(R < R_s | S^*) = \frac{b}{b + c}
\]

Since \( \text{Prob}(R < R_s | S^*) = N(R_s - S^*) \),

we will have:

\[
R_s - S^* = N^{-1}\left(\frac{b}{b + c}\right)
\]

so:

\[
S^* = R_s - N^{-1}\left(\frac{b}{b + c}\right)
\]

\( S^* \) is the threshold, conditional on \( x = 0 \), under which fund
managers do not roll over their loans to the country — when \( S < S^* \) everybody withdraws.

Conditional on \( x = 1 \), and given a signal \( S \) about the state of fundamentals, the IMF decides to intervene if and only if its payoff is non-negative, in other words, if and only if it expects \( R > \bar{R}_L \). As we have seen above, the liquidity provision \( L \), provided by the IMF, may be either bigger or smaller than the financing gap of the country. Now we consider the case when \( L < D - M \). In this case, the relevant payoff is:

\[
-C \text{Prob}(R < \bar{R}_L | S) + B [1 - \text{Prob}(R < \bar{R}_L | S)] \geq 0
\]

As above, we have the threshold under which the IMF does not provide the liquidity support \( L \) to the country:

\[
S^*_{IMF} = \bar{R}_L - N^{-1}\left(\frac{B}{B + C}\right)
\]

As we can see from the two thresholds determined up to now, they also depend on the preferences of the considered agents. Hence, when the preference structure of fund managers and the IMF is identical, the threshold, conditional on \( x = 1 \), under which everybody switch their behaviour is the same for all.

Therefore, the threshold under which fund managers decide not to roll over their loans is:

\[
S^* = \bar{R}_L - N^{-1}\left(\frac{b}{b + c}\right)
\]

Since \( R_S < \bar{R}_L \), then \( S^* < S^* \). For values of the signal larger than \( S^* \) — and so also larger than \( S^*_{IMF} \) — the IMF provides the country with the liquidity support \( L \) and fund managers decide to roll over their loans to the country. Instead, for values of the signal smaller than \( S^* \) the IMF doesn’t provide the liquidity support \( L \) and fund managers withdraw their loans. Note that, provided the hypothesis of common knowledge, both fund managers and the IMF know to have the same preferences as each other and the same threshold value conditional on \( x = 1 \). In this case \( \bar{R}_L \) is the only relevant threshold value of fundamentals.
PROPOSITION 1: In the model with common knowledge, and with \( L < D - M \), there is a unique equilibrium for values of the signal \( S \leq S^* \) and \( S > S^* \). There are multiple equilibria for values of the signal belonging to the range \( S^* < S \leq S^* \).

Graph 1

Determinaton of Equilibria as in Proposition 1.

Under the assumption of common knowledge, a consequence of these models is that there may be multiple equilibria for some values of fundamentals. If \( S > S^* \), the perception about the state of fundamentals is so good that (irrespective of which equilibrium threshold is selected) a complete withdrawal will never occur \( (x = 0) \). If \( S < S^* \), the state of fundamentals seems so bad that the market will always coordinate on the crisis equilibrium \( (x = 1) \). But, when \( S^* < S < S^* \), the actions of the market and the equilibrium definition will depend on which relevant threshold is chosen by fund managers. If \( S^* \) is chosen, there isn't any withdrawal \( (x = 0) \); and although the IMF does not intervene, there will not be a crisis. But, if \( S^* \) is chosen, there would be the opposite situation. This model leads to conclusions substantially different from those arising in CGR, where the model predicts a unique equilibrium. It is important to note that these differences are exclusively due to different assumptions about the structure of information.
These models, based on multiple equilibria, do not have an endogenous mechanism of the equilibrium threshold selection and what determines the choice of one threshold over the other is not explained by the model. Within this context, the IMF is only able to influence the threshold $R_l$, but not the others and does not have any role in the threshold selection. Crises are always possible. Therefore, if the IMF operates with limited liquidity provisions (as in CGR), it is not able to eliminate the possibility of a liquidity crisis.

The IMF is able to completely eliminate the possibility of a liquidity crisis only by intervening with a liquidity support $L > D - M$, that is acting solely as an ILOLR. In this case, we have a unique relevant solvency threshold, that is $R_s$. Then, there is a unique equilibrium, with a crisis happening if and only if $R < R_s$. Also the only equilibrium conjecture by fund managers is about $R < R_s$. In this case, there can be only a solvency crisis. Also now the signal may not coincide with the exact value of fundamentals, hence, we can express the following proposition:

**PROPOSITION 2:** In the same model, with $L > D - M$, there is a unique equilibrium determined by the threshold $S^*$. Then, this model, founded on the hypothesis of common knowledge, has economic policy implications about the IMF corresponding perfectly to the ones deriving from the multiple equilibria literature — see, for instance, Jeanne and Zettelmeyer (2002); Chang and Velasco (2001). The IMF is able to solve the liquidity problem only if it has enough resources to eliminate the multiplicity of equilibria. Therefore it must act as an international lender of last resort.

### 4. - Preferences and Thresholds Definition

Our hypothesis of common knowledge does not imply that the threshold values of signals must coincide with the threshold values of fundamentals. Thus, the possibility of a crisis depends

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9 Arbitrary probabilities are often associated with the possible thresholds between which the equilibrium is determined. Naturally these probabilities should be known *ex-ante*, and they deeply influence the equilibrium determination.
on the relative positioning of the threshold values of the signals with respect to macroeconomic fundamentals. We have seen that these thresholds depend on the payoff parameters $b$, $c$, $B$, $C$. Therefore the equilibrium depends on the agents’ preferences. In this section we continue to assume identical preferences between fund managers and the IMF.

Being $\eta \sim n(0, 1/\rho)$, $\eta$ is symmetric to the origin, then the cumulative distribution function $N(0) = 1/2$. Hence we have $N^{-1}(1/2) = 0$.

$$\frac{b}{b + c} \leq \frac{1}{2},$$

that is when: $b < c$, $N^{-1}\left(\frac{b}{b + c}\right) < 0$ and the threshold signals are shifted to the right of the threshold rates.

\hspace{1cm}

**GRAPH 2**

**EQUILIBRIA WHEN $b < c$**

<table>
<thead>
<tr>
<th>$R_s$</th>
<th>$\bar{S}$</th>
<th>$\bar{R}_L$</th>
<th>$S^*$</th>
</tr>
</thead>
</table>

Naturally, the actions of the market are always the same regarding the thresholds. If $S \leq \bar{S}$, everybody withdraws their loans and the country defaults. Note that a crisis occurs for all values being within the range $(R_s; \bar{S})$, though the country is still

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$^{10}$While, in the perfect information framework, the equilibrium is independent of them.
solvent. If $S > S^*$ nobody withdraws, the IMF intervenes and the country does not default. When $S^* < S < S^*$, we have, as above, multiple equilibria. If the equilibrium conjecture chosen is $x = 1$, when $S < S^*$ fund managers do not roll over their loans to the country and the IMF decides not to intervene. In this case, the relevant threshold rate of fundamentals under which a default occurs becomes $\bar{R}$, where we know that $\bar{R} > \bar{R}_L$. If $\bar{R}$ is however under the realization of fundamentals, despite the withdrawals and the non-intervention by the IMF, a default does not occur. Note that, although its fundamentals are not weak, the country incurs a speculative attack anyway. This situation can reflect the case of capital outflows due to a lack of confidence by the agents. Otherwise, for every other realized value of fundamentals a default occurs.

In the case of $b > c$, the threshold signals are shifted to the left of the threshold rates of fundamentals.

**Graph 3**

**EQUILIBRIA WHEN $b > c$**

If $S < S^*$ a crisis always occurs. If $S > S^*$, fund managers roll over their loans and the IMF intervenes, supporting the country. A crisis does not occur even if the state of fundamentals is not so favourable. In fact, in the range defined within $S^*$ and $\bar{R}_L$, the realizations of fundamentals are such that if $x = 1$ occurs, the country would not have enough resources to pay back fund
managers and a default would occur, even with an IMF intervention. But the preference structure makes fund managers more willing to roll over their loans and the IMF more willing to intervene. In the multiple equilibria range, where \( S^* < S < S^* \), what occurs depends on the relevant threshold selection. If \( S^* \) is chosen, then we have \( x = 0 \); but in the range between \( S^* \) and \( R_S \) the country is insolvent and a default occurs anyway, despite the roll over of the debt. This situation may be due to excessive agent confidence. With rates greater than \( R_S \) there is no default. We will have a default however, if \( S^* \) is chosen as the relevant threshold.

As in CGR, threshold values of signals are decreasing in \( b \) and increasing in \( c \). Also the relevant IMF threshold is decreasing in \( B \) and increasing in \( C \). Different values of these parameters can lead to very different agent and IMF behaviour. Also intuitively speaking, it is clear that greater values of \( b \) and smaller values of \( c \), which imply a greater utility for the correct roll over of a loan, and a lesser punishment for a wrong roll over, make fund managers more confident (more risk takers) in their loan roll overs to the country. The opposite is true for \( b < c \). Thus this model can be very useful in reflecting effects observed in reality, notably excesses and deficits of confidence.

5. - Differences in the Preference Structure

A natural extension of our analysis leads to eliminating the hypothesis of identical preferences between fund managers and the IMF, assuming that they are determined by different parameters. This is straightforward because of the different objectives of fund managers and the IMF. In fact, while fund managers seek to maximize their profits and minimize the risks of losing their capital if a crisis occurs, the IMF seeks to maximize the possibility that a country avoids a crisis or, if a crisis occurs, can survive it. Moreover, it seeks to minimize the probability of losing the liquidity support \( L \). Then, parameters \( b, c, B, C \) depending on different factors, may be distinct.

Given the structure of thresholds of the IMF and fund
managers, conditional on \( x = 1 \), we can see that these thresholds coincide when:

\[
N^{-1}\left( \frac{B}{B + C} \right) = N^{-1}\left( \frac{b}{b + c} \right)
\]

so when \( \frac{B}{C} = \frac{b}{c} \) or, analogously \( \frac{b}{B} = \frac{c}{C} \).

In order for thresholds to coincide it is necessary and sufficient that parameters are equals in relative terms. It follows that:

**Proposition 3:** If preferences are not equal in relative terms between fund managers and the IMF, then the thresholds at which agents switch their behaviour do not coincide.

There could be two scenarios, conditional on \( B/C > b/c \) or \( B/C < b/c \). The proof of Proposition 3 will be analyzed in three distinct lemmas.

**Lemma 1:** If \( B/C > b/c \), then \( S_{IMF}^* < S^* \).

This conclusion derives from the fact that

\[
N^{-1}\left( \frac{B}{B + C} \right) > N^{-1}\left( \frac{b}{b + c} \right)
\]

Being both negative terms inside the thresholds, we have \( S_{IMF}^* < S^* \). If the IMF gains a greater utility following a correct strategy, it is more willing to undertake this strategy.

**Graph 4**

**Determination of Equilibria as in Lemma 1**

| \( S^* \) | \( S_{IMF}^* \) | \( S^* \) |
In this situation, when $S \leq S^*$, $S > S^*$, there is a unique equilibrium, exactly as above. When $S^* < S \leq S^*$ there are multiple equilibria. But this case is different from the preceding one, because thresholds of fund managers and the IMF do not coincide. If $S^*$ is chosen as the relevant threshold, fund managers roll over all their loans to the country and the IMF intervention is superfluous (to analyze the possibility of a crisis refer to the preceding section). Instead, if $S^*$ is chosen as the relevant threshold, since $S \leq S^*$, fund managers decide to withdraw their loans ($x = 1$). But, at this point, a crisis is not certain to occur. In fact, if $S \leq S^*_{IMF}$ the IMF does not intervene and the relevant threshold rate becomes $\bar{R}$. Instead, when $S^*_{IMF} < S \leq S^*$, the IMF decides to intervene anyway — although fund managers withdraw their loans — and the relevant threshold rate of fundamentals is still $\bar{R}_L$. This case is particularly important because there is heterogeneity between the behaviour of fund managers and that of the IMF. Although the withdrawals of fund managers can make a crisis possible, the IMF is able to avoid it with a partial bailout of the country. This structure implies that, even in case of multiple equilibria, the IMF can avoid several liquidity crises with a limited liquidity support.

**Lemma 2:** If $B/C < b/c$, there must be $\bar{S} \leq S^*_{IMF}$ in order to have equilibrium.

In fact, if the preference structure is such that $B/C < b/c$, then we would have a situation where $S^* < S^*_{IMF}$. This situation is a contradiction in terms. In fact, when calculating the threshold $S^*$, it is necessary to consider the probability, conditional on the IMF intervention. But, for every value of the signal $S < S^*_{IMF}$, the IMF does not intervene. Therefore this conditional probability is equal to 0; it is impossible to calculate $S^*$ when the signal is $S < S^*_{IMF}$. Furthermore, for every value of the signal $S > S^*$, fund managers roll over their loans to the country ($x = 0$) and this prevents the existence of threshold $S^*_{IMF}$ that is conditional on the complete withdrawal of loans by fund managers ($x = 1$). Thus, if the two thresholds have values such as $S^* < S^*_{IMF}$, there is a contradiction as regards the meaning of the thresholds. They can never be in that position. When $S^* < S^*_{IMF}$, there isn’t equilibrium.
Therefore, in order to obtain these two thresholds in equilibrium, they always must have the following relation: $S^*_{IMF} \leq S^*$. Hence, this definition includes the first two cases, but not the third because its preference structure contrasts with this conclusion\(^{11}\).

In this third case, when the signal is $S < S^*_{IMF}$, the relevant threshold value for fund managers cannot be $S^*$ but $\overline{S}^*$, calculated based on the equilibrium conjecture $x = 1$ (like the preceding case), but conditional on the non-intervention of the IMF, that is:

\[-c \text{Prob}(R < \overline{R} | \overline{S}^*) + b \left[1 - \text{Prob}(R < \overline{R} | \overline{S}^*)\right] = 0\]

which gives the following result:

\[
\overline{S}^* = \overline{R} - N^{-1}\left(\frac{b}{b + c}\right)
\]

Since $\overline{R}_L < \overline{R}$, we have $S^* < \overline{S}^*$. At this point, there is a problem similar to the preceding one for threshold $\overline{S}^*$ in fact it cannot be greater than $S^*_{IMF}$. This is because, for every value of the signal $S > S^*_{IMF}$, the IMF decides to intervene, then in this interval the threshold $\overline{S}^*$ cannot exist, because for such values of the signal the conditional probability on the IMF non-intervention is equal to 0. In fact, in the first two cases, this probability does not influence the equilibrium determination anyway. Therefore, in this third case, there exists an equilibrium in pure strategies if:

\[
\overline{S}^* \leq S^*_{IMF}
\]

**Lemma 3:** When $B/C < b/c$, $\overline{S}^* \leq S^*_{IMF}$ if equation (11) is satisfied.

Substituting the corresponding formulas to the two thresholds, we have:

\[
\overline{R} - N^{-1}\left(\frac{b}{b + c}\right) \leq \overline{R}_L - N^{-1}\left(\frac{B}{B + C}\right)
\]

\(^{11}\) This result can be achieved by substituting its formula to each threshold value. We can see that in order to have $S^*_{IMF} \leq S^*$, there must be $B/C \geq b/c$.  

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that is: \[
\bar{R} - \bar{R}_L \leq N^{-1}\left(\frac{b}{b+c}\right) - N^{-1}\left(\frac{B}{B+C}\right).
\]

where both sides of the inequality are positive members. Given the fact that \(\bar{R}_L\) depends on the dimension of the liquidity support \(L\), this variable has a critical role. By using (3) and (5) and substituting them into the left hand side member, we obtain the following rationality constraint:

\[
(11) \quad k \frac{L}{I} \leq N^{-1}\left(\frac{b}{b+c}\right) - N^{-1}\left(\frac{B}{B+C}\right).
\]

Hence, this constraint depends on \(L\) (as mentioned above), but also on \(k\) and \(I^{12}\). This condition must be satisfied to have equilibrium with \(B/C < b/c\). We can see how different values of the cost of liquidation \(k\) impose different strategies on the IMF. If \(k \to \infty\), we can see from (11) that \(L\) is necessarily equal to 0 — because it cannot be equal to negative values. This achievement of the model may seem counterintuitive, but remember that this is the case in which \(L < D - M\). In such a case, with infinite liquidation costs, the country always defaults and for this reason the IMF does not provide any liquidity support. Note that the default is caused only by the fact that the liquidity support \(L\) is not sufficient to cover the liquidity gap \((L < D - M)\). On the other hand, if \(k \to 0\), \(L\) can assume the maximum possible value provided it remains smaller than \(D - M\).

As said before, \(\tilde{S}^*\) must not be greater than \(S_{IMF}^*\). When \(S \leq \tilde{S}^*\), fund managers do not roll over their loans to the country, the IMF does not intervene and a crisis occurs. On the other hand, if \(\tilde{S}^* < S \leq S_{IMF}^*\), in spite of the IMF non-intervention, fund managers, aware of it, decide to roll over their loans to the country anyway. Naturally if \(S > S_{IMF}^*\) the IMF intervenes and all fund managers roll over their loans. Note that when \(\bar{R}_L < \bar{R}\), there is \(S^* < \tilde{S}^*\). In this case, the range of multiple equilibria is larger than in the previous

\[ \text{\footnotesize {12} Being } L < D - M \text{ we can also establish:} \]

\[
k \frac{D-M}{I} \leq N^{-1}\left(\frac{b}{b+c}\right) - N^{-1}\left(\frac{B}{B+C}\right).
\]

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case and then the probabilities of a crisis are consequently higher. This situation is only caused by the preference structure which characterizes a very conservative IMF, one that is very unwilling to intervene.

When \( L > D - M \), even if preferences are not equal in relative terms, the conclusions of the model do not change. In fact, if the IMF is able to fill the whole liquidity gap of the country by itself, there aren’t any liquidation costs left and only solvency crises occur. The threshold rate of fundamentals under which a default occurs is always \( R_s \), as in Proposition 2. The role of a liquidity provision of such dimension is above all that it can eliminate the multiplicity of equilibria.

The conditional threshold on \( x = 0 \) is exogenous, in the sense that it depends only on \( D, M, I \), which are predetermined parameters. This threshold cannot be influenced by the IMF and fund managers’ decisions. Thus, in this case, preferences do not play any role.

A liquidity crisis can be completely avoided only with the IMF acting as an ILOLR, but our framework shows that, even with a limited liquidity support, the IMF can still play a decisive role. When the IMF provides a liquidity support, is able to prevent some crises that would otherwise occur. Moreover, this framework proves that if the IMF follows a very conservative strategy, it makes a crisis more likely, even when the fundamentals of the economy are not weak. Thus, the IMF should not adopt this conservative strategy.

6. - Liquidity and Moral Hazard

One of the main criticisms of the models à la Diamond-Dybvig, based on the hypothesis of common knowledge, regards their inability to capture moral hazard distortions, due to excessive liquidity provision by the IMF. The prescription of an international lender of last resort, that emerges from these types of models, should be deeply reviewed with the presence of moral hazard. For this reason, a major innovation of models based on global games
is the fact that they are able to capture moral hazard distortions. It is possible to find a general treatment of the incentives faced by the government both in CGR and in Morris and Shin (2003)\textsuperscript{13}.

In this section we propose an analysis of the incentives faced by the government in the multiple equilibria framework, borrowing the incentives framework of CGR. We will see if this framework is able to represent moral hazard distortions due to liquidity provisions by the IMF, when it does not fill the whole liquidity gap with its intervention. An international lender of last resort will create, without any doubt, moral hazard distortions\textsuperscript{14}.

Some adjustments to the basic framework of the model are necessary. Assume that the rate of return $R$ is distributed as $R \approx N(R_j, 1/\rho)$. The mean $R_j$ (with $j = N, A$) depends on the effort of the government, if it undertakes costly reforms to improve the economic efficiency of the country (action $A$) or not (action $N$). The probability distribution of $R$ — but not its mean, which is not observable — is common knowledge. For simplicity, we assume perfect information\textsuperscript{15}. Therefore there are two equilibria. One equilibrium is conditional on the conjecture $x = 0$. With complete information, the only relevant threshold is $R_S$. The threshold that determines the other equilibrium is conditional on $x = 1$ by fund managers and on the IMF intervention; it is $\bar{R}_L$. It is necessary to introduce an exogenous mechanism for the selection of the equilibrium threshold, because there are multiple equilibria for a certain range. Thus, we assign a probability $\xi$ to the selection of threshold $R_S$ and, with complementarity, a probability $(1 - \xi)$ to the selection of threshold $\bar{R}_L$. The welfare function of the government is:

$$W = U - \Psi = E_0Y - \Psi$$

\textsuperscript{13}An analysis of the same moral hazard distortions, but with quite different models, can be found in Jeanne O. - Zettelmeyer J. (2004), and Cordella T. - Levy-Yeyati E. (2004).

\textsuperscript{14}For an analysis of moral hazard distortions with an international lender of last resort see Dooley M. - Verma S. (2001).

\textsuperscript{15}In order to have perfect information it suffices to make $p \to 0$. 

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Where \( U \) is the utility of the domestic representative agent and \( \Psi \) is the welfare cost of undertaking action \( A \). This cost falls on the government only, and is motivated by exogenous considerations — i.e. electoral costs. Hence \( W \) does not coincide with \( U \) that is measured by GNP\(^\text{16} \). Let \( \Delta R = R_A - R_N \). The utility of the government is different whether the costly action \( A \) is undertaken or no action is undertaken. Also the dimension of \( \Delta R \) influences the government decisions. If fundamentals are inside the range of multiple equilibria, the welfare function and, above all, the utility difference between action \( A \) and action \( N \) will crucially depend on \( \xi \). Naturally, as in CGR, the dimension of threshold \( RL \) also influences the optimal strategy of the government. Since \( RL \) depends on the liquidity provision \( L \) by the IMF, examining the effects of \( L \) on \( \Delta W \) can show possible moral hazard distortions due to an excessive liquidity provision \( L \). It is necessary to consider the position assumed by \( R_N \) and \( RA \) compared to thresholds \( RS \) and \( RL \) in order to analyze the effects \( \xi \) and \( L \) variations on \( \Delta W \).

As seen before, \( \Delta W \) is equal to the difference between \( W(A) \) and \( W(N) \). The welfare function can assume different values depending on the relevant threshold value of fundamentals. When the relevant threshold is \( RS \), GNP is \([RI + M - D]\); instead, considering as the relevant one, GNP is given by \([RI + M - D - k(D - M - L)]\).

There are some limiting cases. In fact, if \( R_N, RA \leq RS \) the equilibrium is unique, fund managers always withdraw their loans in \( t = 1 \) and the IMF does not intervene. The country is not solvent even if it undertakes action \( A \), and a default always occurs. Different values of \( L \) and \( \xi \) cannot influence government actions.

If \( R_N, RA > RL \), we still have a unique equilibrium, where fund managers roll over and the IMF intervention is unimportant. Accordingly, function \( \Delta W \) does not depend on \( L \) and \( \xi \). Then:

\[
\Delta W = [RI + M - D][F(R|RA) - F(R|R_N)] - \Psi
\]

\(^\text{16} \)In this version of the model, because of the information structure, there is no need to integrate the welfare function of the government as in CGR.
The government decides to undertake the costly action $A$ if $\Delta W$ is positive, otherwise it does not exert any effort. $\Delta W$ is positively influenced by the difference $R_A - R_N$ and negatively influenced by costs of reform $\Psi$. $\xi$ does not play any role, because the equilibrium is unique. The liquidity provision $L$ is not influential, but only when $\bar{R}_L < R_N$, otherwise we fall inside the multiple equilibria range.

There is another limiting case when $R_N < R_S$ and $R_A > \bar{R}_L$. If the government does not undertake the action $A$, it defaults. If the government undertakes the action $A$, there will be a unique equilibrium with $x = 0$. We have:

$$W(A) = \Delta W = [RI + M - D] F(R|R_A) - \Psi$$

The government always has a positive incentive to undertake the action $A$, excluding the limiting case of $W(A) \leq 0$. As above, $\Delta W$ is not influenced by variables $\xi$ and $L$.

Scenarios encompassing multiple equilibria are more interesting. The first one is defined by $R_N < R_S$ and $R_S < R_A < \bar{R}_L$. Here, if the government does not undertake the action $A$, it will default for sure ($W(N) = 0$). The only possibility to avoid a crisis is undertaking action $A$. But, since $R_A$ falls inside the range characterized by multiple equilibria, a crisis may always occur, depending on the threshold selection. We have:

$$W(A) = \Delta W = [\xi(RI + M - D) + + (1 - \xi)(RI + M - D - k(D - M - L))] F(R|R_A) - \Psi$$

In this case $\Delta W$ depends on both $\xi$ and $L$. Deriving the above formula, we can understand the role played by these two variables on the welfare of the government:

$$\frac{\partial \Delta W}{\partial \xi} = k(D - M - L)F(R|R_A) \geq 0$$

$$\frac{\partial \Delta W}{\partial L} = kL(1 - \xi)F(R|R_A) \geq 0$$
The net increase of welfare obtained undertaking action $A$ compared to action $N$, is increasing in the two variables $\xi$ and $L$. In fact, being $R_A$ within the range of multiple equilibria, a crisis is always possible and it would occur when threshold $\bar{R}_L$ is chosen. An increase in $\xi$ means an increase in the probability that $R_S$ is chosen as the relevant threshold, then it means a reduction of the probability of a crisis. An increase in $L$ decreases $\bar{R}_L$, and therefore increases the probability that the country could avoid a crisis in $t = 1$. In this case, the liquidity provision $L$ does not decrease incentives for the government to undertake the costly action $A$. This is also caused by the fact that $R_N < R_S$, and then by the fact that the country, without action $A$, would certainly incur a crisis.

In this case, it is possible to speak of strategic complementarity between the actions by the IMF and the government. In fact the IMF is interested in intervening only if the country does not default in $t = 1$, and such possibility increases undertaking action $A$ and with the size of $L$ and $\xi$. Whereas the country, which is discouraged by bleak prospects of success, is interested in undertaking action $A$ only for sufficient liquidity provisions $L$ to considerably increase the probability of a good economic outcome. These implications are perfectly in tune with the implications found in CGR and with the theory of catalytic finance of Morris and Shin (2003).

The second case, regarding multiple equilibria, is characterized by: $R_S < R_N < \bar{R}_L$ and $R_A > \bar{R}_L$. Here the country has the possibility not to incur a default, even without undertaking action $A$. But this possibility is conditional on the choice of the relevant threshold within the range of multiple equilibria. Here, the country is able to avoid anyway a possible crisis undertaking action $A$. The two welfare functions are:

\[
W(N) = [\xi(RI + M - D) + (1 - \xi)(RI + M - D - k(D - M - L))] F(R|\bar{R}_N)
\]

\[
W(A) = [RI + M - D] F(R|\bar{R}_A) - \Psi
\]

The utility difference between the two welfare functions is:
\[ \Delta W = [RI + M - D] F(R|R_A) - [\xi (RI + M - D) + (1 - \xi) (RI + M - D - k(D - M - L))] F(R|R_N) - \Psi \]

As above, we can calculate the partial derivatives to see how changes in \( \xi \) e \( L \) can influence \( \Delta W \). We have:

\[
\frac{\partial \Delta W}{\partial \xi} = -k(D - M - L)F(R \mid R_N) \leq 0
\]

\[
\frac{\partial \Delta W}{\partial L} = -kL(1 - \xi)F(R \mid R_N) \leq 0
\]

The derivatives are both negative in this case. When \( \xi \) or \( L \) increase, the utility gained by undertaking the costly action \( A \) decreases. In fact, because of \( R_N \) falling within the range of multiple equilibria, an increase in \( \xi \) increases the probability that \( R_S \) is chosen as the relevant threshold and then that a crisis does not occur even without action \( A \). At the same time, an increase in \( L \), making \( \tilde{R}_L \) decrease, with the same \( \xi \), decreases the probability of a crisis. Hence, an increase in \( L \) decreases the incentives for the government to undertake the action \( A \), and decreases expected GNP. This result, again in tune with CGR, is particularly important because it proves how a multiple equilibria model can reflect the possible moral hazard distortions.

The last case is characterized by \( R_S < R_N, R_A < \tilde{R}_L \). Here, both welfare functions depend on \( \xi \) and \( L \). The welfare functions and their difference are:

\[
W(N) = [\xi(RI + M - D) + (1 - \xi) (RI + M - D - k(D - M - L))] F(R|R_N)
\]

\[
W(A) = [\xi(RI + M - D) + (1 - \xi) (RI + M - D - k(D - M - L))] F(R|R_A) - \Psi
\]

\[
\Delta W = [\xi(RI + M - D) + (1 - \xi) (RI + M - D - k(D - M - L))] [F(R|R_A) - F(R|R_N)] - \Psi
\]

Partial derivatives are:
is increasing in $\xi$ and $L$; and the size of this increasing ratio depends on the term $[F(R|R_A) - F(R|R_N)]$. If this term is large, an increase in $\xi$ will cause a greater increase in $\Delta W$. The same reasoning applies to $L$.

However, an increase in $\xi$ means an increase in the probability of choosing $R_S$ as the relevant threshold, and then a decrease in the probability of default choosing either $R_N$ or $R_A$. Examining the derivative with respect to $\xi$, the welfare function is increasing in $\xi$, hence an increase in $\xi$ should increase the incentive to undertake the costly action $A$. On the other hand, an increase in $\xi$ decreases the probability of a crisis even if action $A$ is not undertaken, but our framework is not able to capture this effect.

The reasoning about $L$ is similar to the preceding one. In fact an increase in $L$, should lead to an increase in welfare and should increase incentives to undertake action $A$. In fact a reduction of $\bar{R}_L$ decreases the possibility of a crisis. But the possibility of a crisis is reduced for both $R_A$ and $R_N$. Also here, our framework does not capture the diminishing effect of an increased liquidity support $L$ on the incentive for the government effort. In this case, moral hazard distortions should emerge, but they are absent in our framework.

In conclusion, the case in which $R_S < R_N < \bar{R}_L$ and $R_A > \bar{R}_L$ can correctly represent such moral hazard distortions and represents a major censure to the institution of an ILOLR. While the case in which $R_N < R_S$ and $R_S < R_A < \bar{R}_L$ properly describes the phenomenon of strategic complementarity and catalytic finance. Note that these implications do not conflict with those stated in the previous section. But this framework does not provide a correct representation of incentives within the range of multiple equilibria. This lack derives from the existence of multiple equilibria — and then the partition of possible outcomes.
of fundamentals — and also from the information structure which, being complete, completely eliminates uncertainty. Thus, there are the well-known problems of lack of heterogeneity in agents’ behaviour, exogenous mechanisms (not explained inside the model) of threshold selection, etc.

Holmstrom and Tirole (1998) show that in a domestic context the government must increase the liquidity supply in the case of a higher liquidity shock and decrease this supply otherwise. The international implications for the IMF that arise from moral hazard analysis, are substantially aligned with the domestic observations of Holmstrom and Tirole (1998). Hence, these implications have a broader range, underlining the importance of the IMF as the international liquidity regulator.

However it is very important that the obtained results are substantially (though not completely) in accord with the results obtained both in CGR and in Morris and Shin (2003) under the hypothesis of global games. This demonstrates the validity of the theoretical framework as well as the flexibility of the information structure, which is able to express an accurate representation of incentives under different hypotheses.

7. - Conclusions

In this paper we have shown a model of financial crises. We have exploited the basic framework of CGR (based on the global games assumption) to develop a multiple equilibria model based on the hypothesis of common knowledge. Being a real model, we have left out the aspects regarding currency fragilities.

In the first part we have reached conclusions in agreement with the preceding literature about crises models with multiple equilibria. The emerging of a currency crisis can be eliminated only if the IMF act as an international lender of last resort, providing a liquidity support sufficient to eliminate the multiplicity of equilibria.

But, differing from the preceding literature, the probability of a crisis is also correlated to the strength of fundamentals. Thus,
the IMF can considerably reduce the probability of a crisis without providing a liquidity support able to eliminate the multiplicity of equilibria.

In the second part, we have been able to observed how the preference structure influences our analysis. A generalized preference structure makes the determination of equilibria more realistic, creating distinct equilibrium positions for the IMF and fund managers. A greater or lesser willingness, of the IMF, to provide a liquidity support can modify the probability of a crisis, with all the other parameters remaining unvaried. A more willing IMF is able to avoid a crisis with a partial bailout, even if fund managers withdraw their loans. On the other hand, a very conservative IMF can make a crisis more likely. An interesting continuation of this research would be extending this generalized preference structure to models based on global games.

Since the preceding literature about models à la Diamond-Dybvig does not take into account moral hazard distortions that could be caused by an international lender of last resort, in the last section we have made up for this lack. Apart from one particular case, we obtained very similar results as the ones in CGR and in Morris and Shin (2003). In opposition to the main predictions arising from multiple equilibria models, the possibility of self-fulfilling crises does not necessarily justify the solution of a complete bailout in the event of a crisis. In fact an international lender of last resort certainly creates moral hazard distortions. Moreover, the argument that liquidity provisions always induce debtor moral hazard is not correct. In fact there could be either moral hazard distortions or strategic complementarity between the country and the IMF. The latter gives rise to an important role for catalytic finance.

Hence, this paper’s main conclusion is that the IMF should provide sufficiently limited liquidity supports to avoid moral hazard distortions, yet also be more willing to intervene in order to prevent pure liquidity crises.
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