A Model for Determining Consumption and Social Assistance Demand in Uncertainty Conditions

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This article focuses on the relation between demographic impact and social insurance and consists of two distinct yet closely related sections. The first section introduces a three overlapping generation model which tries to determine the ageing effects on the individual consumption and saving decisions in a partial equilibrium approach. The second section, instead, aims at providing a theory on the need to set up an ad hoc non self-sufficiency fund. It is shown that this mandatory and universal social insurance can guarantee a relatively higher ex-post individual welfare level when the individual fails to get insured and makes his decisions in uncertainty conditions, that is, reacting to risks in an imperfect way through the setting up of a precautionary saving fund. [JEL Classifications: J14, J22, H55]

1. - Foreword

The silent demographic revolution characterising the main industrialised countries is an unavoidable factor which has major economic, social, cultural and psychological implications. The change in the population age pattern is likely to be the major issue over the next few decades, as the growing ageing of the western societies is likely to bring about a flurry of problems that are

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currently underestimated, but that will entirely break out within a few decades.

It should be noted that the current framework of most welfare state systems, particularly as to social security spending, is unable to adequately tackle the problems related to a more and more relevant ageing of the population, because social assistance costs are bound to skyrocket from the next decade on. What is even more worrying is the lack of a debate on ageing related problems, particularly when it comes to the need to ensure the non-self-sufficient elderly an acceptable quality of life.

This embarrassing situation can clearly be seen in Italy, where the social assistance systems show huge limits both in terms of cost-effectiveness and intergenerational equity. The virtually total lack of measures capable of guaranteeing the total assertion of rights of citizenship and safeguarding the elderly's dignity, in particular those who are not self-sufficient, highlights the absolute need to set up a social institution which may properly meet the ever growing demand for social assistance.

In order for the weakest social classes to avoid a sharp welfare loss, it is essential, given the impossibility to increase tax charges, to reallocate public spending “qualitatively”. The latter should combine economic growth and stability goals with social justice objectives. The transfer of resources into a non self-sufficiency-oriented public Fund that can assure an overall cover against such risk would be instrumental in pursuing this target.

This short essay consists of two separate yet interconnected sections. The first section will introduce a three overlapping generation model, which will try to determine the ageing effects on individual saving and consumption decisions in the different stages of the life cycle of a representative individual. From the results, it can be inferred that a fertility rate decline will inevitably lead to a rise in the individual saving rate, so as to compensate the higher costs for social assistance services during the old age, whose equilibrium consumption will probably be relatively lower. The model focuses on the partial equilibrium aspects, in order to understand the direct effects of demographic shocks on intertemporal consumption plans in the different stages of the life cycle.
The second section, by contrast, will provide a theoretical and analytical explanation for the need to set up a government-run social institution, namely, an *ad hoc* non self-sufficiency Fund of a mandatory and universal nature. This would be financed through a mere transfer of resources from overall pension bill, with a view to protecting the elderly from the non self-sufficiency risk. From this analysis it can be implied, through simplified assumptions about preferences and the risk aversion level in the individuals, that such mandatory social insurance would guarantee a level of relatively higher *ex post* individual welfare in the event the single individual fails to get insured and makes his decisions in a state of uncertainty, that is, reacting to the risk in an imperfect way by setting up a precautionary saving fund.

2. - Theoretical Overview

2.1 *The Reference Model*

The initial assumption is a three overlapping generation economy where every economic agent goes through three different life periods: working life, seniority and old age. In particular, the term “elderly” will refer to individuals aged 65 to 75; the term “late elderly” will indicate individuals aged more than 75.

The choice of a model of this kind results from a distinction within the non productive population, which becomes necessary when one wants to concentrate on the non self sufficient elderly’s social assistance. The traditional overlapping generation models usually tend to divide the overall population into young and old people, assuming that the consumption typology of every group relates to the needs that the individuals of that group must meet. Such an analysis cannot cover the actual conduct of the economic agents in the different stages of their life cycle, because the

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1 By non productive population we mean that population who has left the job market upon reaching the retirement age. Such definition excludes there is a young non productive population. In other words, it is assumed that the young population is the population present on the job market.
individuals that make up the elderly’s population may be facing
different states of nature, closely related to the age of the aforesaid
agents, which involve the fulfilment of different needs and, as a
result, the consumption of different goods and services.

This assumption appears to be of particular significance, in
that it is clear that the needs of the elderly differ depending on
whether they are aged 65 to 75 — and in a relatively good psycho-
physical condition — or they are over 75, with a likelihood of a
kind of disability to occur\textsuperscript{2}.

These basic evaluations show that the three overlapping
generation model turns out to be the most useful for a complete
analysis, because if one deems the needs between workers and
elderly different, then a difference should also be made between
the needs of elderly and late elderly.

The choice to divide the overall non productive population
into two different categories is the result of an actual fact: in Italy,
there are about 2 million non self-sufficient people out of a
population of 10 million of over sixty-five, with a ten per cent
incidence for the population aged 65-74, a 20 per cent incidence
in the 75-79 range and an about 50 per cent incidence in the
population aged more than 80\textsuperscript{3}.

Having defined such theoretical assumptions, we can assume
that every young worker offers a performance on the labour
market therefore receiving a money wage of $w_t$, which is shared
among consumption of goods $c_t$, saving $s_t$ and mandatory social
contributions $f_t$. In the second stage of the life cycle, the agent
leaves the labour market having reached retirement age and,

\textsuperscript{2} See for example the Bimonthly newsletter of Agenzia per i servizi sanitari
regionali (Agency for Regional Health Services) AssR (2003), in particular the
reference to the “Study Commission for the prevention and treatment of non self-
sufficiency, especially regarding the elderly”, the conclusions of which were made
public in December 2002.

\textsuperscript{3} See ISTAT (Central Statistics Institute) (2003). Future projections do not seem
to indicate a decline of such percentages: instead, thanks to the progress in
medicine and social assistance technologies according to the most realistic
scenarios, the non self-sufficiency rates are likely to stabilise, when not to decrease.
However, the rise of the elderly population will considerably raise the number of
non self-sufficient people, thus creating a social problem requiring adequate
resources to be solved. See for example CODA MOSCAROLA F. (2003).
according to the previous social assistance contributions paid, receives a pension of $p_{t+1}$. Consequently, his endowment for the period will be equal to the sum of the social assistance contribution paid by the young and his own savings accrued during his working life; such endowment will be solely allocated for health care and basic social assistance $n_{t+1}$ and savings $s_{t+1}$. In the third period, the individual reaches the “late elderly” stage, thus receiving a pension of $p_{t+2}$ also in this stage. As a result, the individual’s endowment in the last stage of his life cycle will be equal to the sum of the saving as previously accrued and the pension transfer. Assuming there is no hereditary motive, such endowment will be deemed as saving $s_{t+1}$, which will be entirely used for the purchase of specific social assistance $a_{t+2}^4$.

First of all, it is necessary to stress the distinction between variable $n$ and variable $a$, because it includes the implications of the model in question. Variable $n$ refers to the combination of health care and basic social assistance the elderly purchases to satisfy his needs. However, the age rise poses a relevant issue, in that it can cause particular states of nature forcing the late elderly to necessarily consume, being of primary importance, general or specific social assistance, defined by variable $a$.

As mentioned earlier, the empirical data indicate a 40%\textsuperscript{5} likelihood for the Italian late elderly population to face a condition of non self-sufficiency; it follows that specific social assistance consumption will concern a large share yet not the totality of the late elderly population.

We may rightly argue that the observations made fail to accurately describe the social reality, the majority of people over 75 being physically self-sufficient. It is however necessary to look at the specific social assistance variable from two different viewpoints closely related to two hypothetical states of nature the

\textsuperscript{4}In other words, I am simply assuming that the pension transfer is considered as a sort of “forced” saving by the elderly, which may enable him to face the risks related to the health state in the last stage of his life cycle. This theory is justifiable when looking at the growing medical care and social assistance consumption costs upon the increase of the individuals’ age because of deterioration of their health state. See for example ASSR (2003).

\textsuperscript{5} See ISTAT (2003).
late elderly may go through. The first viewpoint refers to the possibility of running into some form of non self-sufficiency, which obviously requires a purchase of specific social assistance to compensate for the loss of the main vital functions. The second, on the contrary, pertains to a positive state of nature where the individual is self-sufficient but nevertheless purchases social assistance to protect his own rights and fulfil his interests. This may appear contradictory as the demand for social assistance undoubtedly varies according to the different state of nature. Actually, it is not, simply because a condition of self-sufficiency in the old age does not necessarily coincide with the individual’s happiness. Indeed there are numerous factors to support this assumption and are generally related to the social conditions under which the older individuals live. The concept of specific social assistance we refer to in this essay is a broad one\(^6\). This involves the social assistance needed to conduct vital daily activities in case of loss of the ability to carry on them, but also the support to pursue social opportunities for the full development of the human potential, i.a. access to educational, cultural, spiritual and recreational resources of society, but also the guarantee to live in conditions of dignity and safety, as well as being free from exploitation and mental and physical abuse\(^7\).

If we agree on this theory, then we can understand, as in the model shown, that the late elderly will demand and consume specific social assistance when needed so as to compensate for the loss of his vital functions; but he will also do the same to avoid a condition of social alienation and/or loneliness precluding him the right to live an active life in society.

In addition, we notice that in the same model “individual welfare” is considered as the consumption of an aggregate good during the first period of the life cycle, as the consumption of

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\(^6\) If we agree on the United Nation’s theory on protection for the elderly, the broad concept of social assistance defined herein enables us to state that a person’s self-fulfilment involves the following five aspects: independence, participation, protection, self-fulfilment and dignity.

\(^7\) In fact, “The union recognises and respects the elderly’s right to lead a decent, independent life and to participate in the social and cultural life”, EU Charter of Fundamental Rights, art. 25.
health care and basic social assistance services during the second period, and as the consumption of specific social assistance during the third. We will also assume that the accrued saving will not yield an interest rate\(^8\).

In order to make comparisons between monetary magnitudes, we define:

— \( p_c \): price of consumer goods \( c_t \), for hypothesis equal to 1 and constant in time.

— \( \rho_{t+1} \): price of the good \( n_{t+1} \). It jointly depends on the price for medical care (medicines, drugs, etc.) \( p_u \) and on the price of basic social assistance \( w_{t+1}/(1+r_{t+1}) \) to the extent of \( \rho_{t+1} = \theta[w_{t+1}/(1+r_{t+1})] + (1-\theta) p_u \). It appears clear that if parameter \( \theta = 1 \), then \( \rho_{t+1} \) coincides with the discounted wage expected at time \( t+1 \).

— \( \rho_{t+2} \): price of good \( a_{t+2} \). It is the reduced wage expected at time \( t+2 \).

The foregoing can be expressed with the following formula:

\[
\begin{align*}
(1) & \quad w_t = c_t + s_t + f_t \\
& \quad s_t + \frac{p_{t+1}}{1+r_{t+1}} = \rho_{t+1} n_{t+1} + s_{t+1} \\
& \quad s_{t+1} = \rho_{t+2} a_{t+2}
\end{align*}
\]

We define the individual's wealth in his life cycle as the sum of the work generated income in the first period and the pension of the second period, that is:

\[
(2) \quad \Omega = w_t + \frac{p_{t+1}}{1+r_{t+1}}
\]

The preferences of an agent of the \( t^{th} \) generation are represented by a generic function of intertemporal, additional and in-time separable utility, \( u(c_t)+u(n_{t+1})+u(a_{t+2}) \), where \( u(c_t) \) is the present utility of an individual of \( t^{th} \) generation arising from the consumption of goods \( c_t \), \( u(n_{t+1}) \) is tomorrow's utility resulting

\(^8\) This is a restrictive hypothesis. It simplifies the analysis and it is due to the effects of the partial equilibrium approach used to develop the model.
from the consumption of health care and basic social assistance services \(n_{t+1}\), and \(u(a_{t+2})\) the future utility resulting from the consumption of specific social assistance \(a_{t+2}\).

Later on we will assume that the function of the single period utility is concave, and the marginal utility of consumption is positive and decreasing, that is, \(\partial u(\cdot)/\partial x_i > 0\) and \(\partial^2 u(\cdot)/\partial^2 x_i < 0\) with \(i = t, t+1, t+2\) and \(x = c, n, a\). Therefore, when choosing an individual falling into the \(t^{th}\) generation, the problem will be the following:

\[
\begin{align*}
\max & u(c_t) + u(n_{t+1}) + u(a_{t+2}) \\
\text{s.t.} & \quad \Omega = c_t + \rho_{t+1}n_{t+1} + \rho_{t+2}a_{t+2}.
\end{align*}
\]

We express the impatience in time of the representative individual by introducing an intertemporal preference rate \(q\) and re-writing the consumer’s target function as follows:

\[
\max u(c_t) + \frac{1}{1+q}u(n_{t+1}) + \frac{1}{(1+q)^2}u(a_{t+2})
\]

For the sake of simplicity, we will define:

\[
\frac{1}{1+q} = \beta
\]

to obtain:

\[
\max u(c_t) + \beta u(n_{t+1}) + \beta^2 u(a_{t+2})
\]

Let us assume now that a shock occurs, affecting the fertility rate in period \(t\), causing its immediate reduction. In a closed economy, assuming there is a productive sector where general equilibrium conditions are observed, the effect of the fertility rate decline in period \(t\) causes a rise in the real wage of the subsequent period \(w_{t+1}\) due to a reduction of the job offer and resulting increase in the bargaining power of the workers, but also a reduction of interest rate \(r\); this because the growth of the real wage involves a shifting in the production function to a new optimal combination of productive factors characterised...
by an increased use of physical capital\(^9\). The growth of the real wage of the next generation combined with the reduction of the real interest rate will unavoidably cause an increase of the expected discounted wage, namely the price of good \(n_{t+1}\). We also notice that such a shock does not change the level of the current wage \(w_t\)\(^10\).

It is easy to understand that the fall of the fertility rate involves, in the model, a reduction of consumption \(c_t\) of the first period so as to enable the consumer to tackle the expected increase in the price of basic social assistance\(^11\). In terms of overall generational welfare, it can be noticed that this is bound to shrink for the first generation (see Graph 1). On the contrary, in terms of the collective welfare level of the period, or in “intragenerational” terms, it can be noticed that this is likely to drop in period \(t\), because on an equal real income basis, young people will reduce the demand for goods to face higher costs of future social assistance, while the effect in period \(t+1\) will be ambiguous. Such ambiguity can be put down to the fact that the new workers will now receive a higher real wage, while the elderly will have to reduce their demand for healthcare and basic social assistance to face the rise in the expected discounted wage of the young caregivers. In other terms, we cannot determine the “welfare effect” in period \(t+1\) due to the presence of a wealth effect caused by the decline of the fertility rate and resulting drop of the job offer\(^12\).

Graph 1 shows the effects of the fertility rate fall on the welfare level of generation 1. As can be observed graphically, the shock will bring about a reduction of the inclination of the budget

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\(^9\) See Hamada K. - Raut L.R. (2003); Barro R. - Sala-I-Martin X. (1995). In the model I assume that wages and interest rate change exogenously. This is due to the partial equilibrium approach that I use.

\(^10\) Please note that I am referring to a closed economy. I will oversee the immigration effects.

\(^11\) This implication can also be found in the Buffer-Stock models. The literature on the subject is rather extensive. See for example Carroll C.D. (1992, 1997, 2004); Deaton A. (1991).

\(^12\) The following paragraph will examine how it is possible to bypass such ambiguities by introducing a particular individual utility function and solving the related constrained maximisation problem.
GRAPH 1

EFFECT OF THE FERTILITY RATE DROP IN $t$ ON THE FIRST GENERATION WELFARE LEVEL*

* It should be noted that the variables indicated after the shock on the fertility rate are marked with an apostrophe.
constraint and, consequently, a reallocation of the resources during the working period so as to decrease the level of the demand for the consumer good as against a precautionary saving aimed at bearing higher costs during seniority.

At the beginning of period \( t+1 \), the individual's pension will decline\(^{13} \) because of the decrease in the number of taxpayers, which will unavoidably end up shifting the constraint leftward, thus leading to a lower consumption of healthcare, social assistance and, as a result, welfare.

Even though the effects on consumption decisions will depend on the income and replacement effects, we can anyway observe that the fall of the fertility rate in \( t \) will cause a drop in the welfare level of generation 1.

### 2.2 Intertemporal Optimization

Previously, it was found that the net effects of the fertility rate drop on consumption decisions depend on the income and replacement effects. As a result, it was not possible to determine the welfare effect in intragenerational terms, meaning among generations in a specific period.

A constrained maximisation exercise is needed to overcome this problem. We can assume that individual preferences may be represented by the intertemporal, additional and separable utility function \( u(x) = x^{1-\sigma} / 1-\sigma \), which is subject to the dynamic budget constraint as previously expressed. In each period the utility function is isoelastic, the intertemporal replacement elasticity being equal to \( \sigma^{-1} \).

The consumer's problem will be the following:

\[
\max \begin{cases}
\frac{c_t^{1-\sigma}}{1-\sigma} + \beta \frac{n_{t+1}^{1-\sigma}}{1-\sigma} + \beta^2 \frac{a_{t+2}^{1-\sigma}}{1-\sigma} \\
\text{s.t. } \Omega = c_t + \rho_{t+1} n_{t+1} + \rho_{t+2} a_{t+2}
\end{cases}
\]

\(^{13} \)I am assuming here a definite sharing system.
In order to obtain the first order conditions, we define the Lagrange function as follows:

\[
L = c_t^{1-\sigma} + \beta n_{t+1}^{1-\sigma} + \beta^2 a_{t+2}^{1-\sigma} - \lambda \left[ c_t + \rho_{t+1} n_{t+1} + \rho_{t+2} a_{t+2} - \Omega \right]
\]

By calculating the first partial derivatives, it will follow that:

\[
\begin{align*}
\frac{\partial L}{\partial c_t} = 0 & \quad \Rightarrow c_t^{-\sigma} = \lambda \\
\frac{\partial L}{\partial n_{t+1}} = 0 & \quad \Rightarrow \beta n_{t+1}^{-\sigma} = \lambda \rho_{t+1} \\
\frac{\partial L}{\partial a_{t+2}} = 0 & \quad \Rightarrow \beta^2 a_{t+2}^{-\sigma} = \lambda \rho_{t+2} \\
\frac{\partial L}{\partial \lambda} = 0 & \quad \Rightarrow c_t + \rho_{t+1} n_{t+1} + \rho_{t+2} a_{t+2} = \Omega
\end{align*}
\]

Through some algebraic calculations we will obtain the values of each variable expressed in the same terms as the others, namely:

\[
c_t = \left( \frac{\rho_{t+1}}{\beta} \right)^{\frac{1}{\sigma}} n_{t+1} = \left( \frac{\rho_{t+2}}{\beta^2} \right)^{\frac{1}{\sigma}} a_{t+2}
\]

\[
n_{t+1} = \left( \frac{\beta}{\rho_{t+1}} \right)^{\frac{1}{\sigma}} c_t = \left( \frac{\rho_{t+2}}{\beta \rho_{t+1}} \right)^{\frac{1}{\sigma}} a_{t+2}
\]

\[
a_{t+2} = \left( \frac{\beta^2}{\rho_{t+2}} \right)^{\frac{1}{\sigma}} c_t = \left( \frac{\beta \rho_{t+1}}{\rho_{t+2}} \right)^{\frac{1}{\sigma}} n_{t+1}
\]

By replacing such equations within the constraint we will obtain the optimal values of the different variables:

\[
c_t^* = \frac{\Omega}{1 + \beta \left( \frac{\beta}{\rho_{t+1}} \right)^{\frac{1-\sigma}{\sigma}} + \beta \left( \frac{\beta}{\rho_{t+2}} \right)^{\frac{2-\sigma}{\sigma}}}
\]
As can be noticed, the effect of $\rho_{t+1}$ and $\rho_{t+2}$, due to a decline of the fertility rate, on the saving rate in period $t$ will depend on the value of parameter $\sigma$.

Previously, it was observed that the fall of the fertility rate would give rise to a reduction in the market interest rate. Given that a rise in the interest rate produces a negative income effect and a positive replacement effect on the saving function, the net effect will depend on which of the two will prevail\(^{14}\).

In this specific case, if $\sigma > 1$ then the income effect will

\[ n_{t+1} = \frac{\Omega}{\rho_{t+1} + \left( \frac{\rho_{t+1}}{\beta} \right)^{\frac{1}{\sigma}} + \left( \frac{\beta}{\rho_{t+1}} \right)^{\frac{1}{\sigma}} (\rho_{t+2})^{\frac{1}{\sigma}-1}} \]

\[ a_{t+2} = \frac{\Omega}{\rho_{t+2} + \left( \frac{\rho_{t+2}}{\beta^2} \right)^{\frac{1}{\sigma}} + \left( \frac{\beta^2}{\rho_{t+2}} \right)^{\frac{1}{\sigma}} (\rho_{t+1})^{\frac{1}{\sigma}-1}} \]

\[ s_t = \rho_{t+1} n_{t+1} + \rho_{t+2} a_{t+2} - \frac{p_{t+1}}{1 + r_{t+1}} \]

it will follow that:

\[ s_t^* = \frac{\Omega}{1 + \left( \frac{\rho_{t+1}}{1/\sigma} \right)^{\frac{1}{\sigma}-1} \left[ \left( \frac{1}{\beta} \right)^{\frac{1}{\sigma}} + \left( \frac{1}{\beta} \right)^{\frac{1}{\sigma}} (\rho_{t+2})^{\frac{1}{\sigma}-1} \right]} + \frac{\Omega}{1 + \left( \frac{\rho_{t+2}}{1/\sigma} \right)^{\frac{1}{\sigma}-1} \left[ \left( \frac{1}{\beta} \right)^{\frac{2}{\sigma}} + \left( \frac{1}{\beta} \right)^{\frac{1}{\sigma}} (\rho_{t+1})^{\frac{1}{\sigma}-1} \right]} - \frac{p_{t+1}}{1 + r_{t+1}} \]

As can be noticed, the effect of $\rho_{t+1}$ and $\rho_{t+2}$, due to a decline of the fertility rate, on the saving rate in period $t$ will depend on the value of parameter $\sigma$.

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\[^{14}\text{See Hamada K. and Raut L.R. (2003).}\]
dominate the replacement effect, thus the net effect of a reduction in the saving interest rate will be positive\textsuperscript{15}. Empirical data show that the US estimates, since the Second World War, have indicated a $1 < \sigma < 4$\textsuperscript{16} value. If we agree on such data, we can state that a decline in the fertility rate will increase the saving rate\textsuperscript{17}.

Now let us focus on another key implication of the model: what are the consequences on the demand for goods and services in the periods following that relating to the shock on the fertility rate, assuming that the shock is of an “exogenous nature”\textsuperscript{18}? The new workers’ wage increase in $t+1$ will involve on the one hand, a rise in the costs borne by the elderly for the purchase of basic social assistance and care and, on the other hand, an increase in the vital endowment of the young people of the new generation.

If we assume a lack of additional shocks on the fertility rate, then we will certainly obtain that $c_{t+1} < c_{t+1}^0$ with $c_{t+1}^0$ referring to young people’s consumption following the shock. In order to determine the “welfare effect” in period $t+1$, we should compare the variation extent of optimal demand for healthcare and basic social assistance on the part of the first generation elderly with that of optimal consumption of the aggregate good of the second generation young people regardless of whether the shock occurs or not. By observing the optimal solutions and assuming a $\sigma > 1$ value, the response will depend on the net variation between first generation elderly’s consumption and that of the young people of the next generation.

The theoretical discussion defined $\rho_{t+1} = \theta \left[ w_{t+1}/(1+r_{t+1}) \right] + (1-\theta) p_u$, where if $\theta = 1$, $\rho_{t+1} = w_{t+1}/(1+r_{t+1})$. Considering that the

\textsuperscript{15} Conversely, if $\sigma < 1$ the substitution effect will dominate the income effect.


\textsuperscript{17} As can be observed from the equation $s^*$ the saving rate in the working period will depend on prices $\rho_{t+1}$ and $\rho_{t+2}$ that is, on the workers’ expected reduced wages in the two following periods. Considering that a fertility rate drop implies an increase of the workers’ wage in the subsequent period and that $\sigma > 1$, then we may state that the shock will cause a rise in the saving rate in the first period of the cycle.

\textsuperscript{18} In order to analyse the direct implications of the model, I assumed that the shock on the fertility rate will be exogenous and with a long-term impact. On the contrary, an extended literature has identified the important links between fertility and economic variables, highlighting their endogenous nature, such as Easterlin R.A. (1966); Leibenstein H. (1974). For a model characterised by an endogenous fertility, see Barro R. - Sala-i-Martin X. (1995), Chap. IX.
fertility rate fall in $t$ will cause a wage increase in $t+1$ and a reduction of the market interest rate, then the resulting increase in the price of basic social assistance utilised by the elderly will be more than proportional compared to the increase in the workers’ wage. In unitary terms and assuming that the shock in $t+1$ is of an exogenous nature, that means that the net variation will show a minus sign. In fact, if we look at the structure of the utility function by which we represented the preferences of the representative individual, the optimal solutions provide us with this indication. In other terms, the “welfare effect” in period $t+1$ will be negative. In addition, when considering that the demand for specific social assistance on the part of the first generation late elderly is determined by the choice made in the previous period, that is by the demand for basic social assistance, then it will shrink compared to the non-shock case.

We can now assume that in every stage following the first period, additional shocks constantly occur. In this case, the negative impact on the welfare level is likely to be increasingly sharp and will end up significantly affecting the welfare of the elderly and late elderly populations, that will have to bear growing costs to assure an increasingly expensive social assistance$^{19}$. In fact, we can observe that a shock on the fertility rate in $t+1$ has effects on population living in $t+2$ (see Table 1). The effects of such shock is on third generation young’s consumption $c_{t+2}$ through the growth of their purchasing power, on second generation elderly consumption $n_{t+2}$ as a consequence of a precautionary saving due to increasing of specific social insurance, and on first generation late elderly consumption $a_{t+2}$, because of the reduction in their purchasing power. The overall welfare effect is negative, considering the hypothesis on utility function and intertemporal replacement elasticity.

In conclusion, we can state that in an economy characterised by lack of immigration, the fertility rate decline will inevitably

$^{19}$ In this limiting case, the young populations’ choices would change. In fact, they will tend to decrease their consumption to implement a precautionary saving so as to assure resources for low-income periods with growing expenses.
lead to a saving rate increase, so as to bear, during the old age, the higher costs expected for social assistance services, whose equilibrium consumptions will tend to be relatively lower\textsuperscript{20}. (Please refer to the Appendix for the demonstration pertaining to the trend of equilibrium social assistance consumption within a framework of exogenous shock on the fertility rate)

It is necessary underline that such results are due to the effects of partial equilibrium approach of the model. In the future I will examine these issues in a general equilibrium approach, in order to inspect if the results and the conclusions differ.

From these conclusions it is interesting to look whether there is room for public measures which can lead individuals not to change their consumption plans, that is, to avoid reacting to uncertain future health conditions in an improper way by setting up a precautionary saving fund. An answer to this question will be the target of the next paragraph, which will also try to show that an \textit{ad hoc} Fund against non self-sufficiency universal and with mandatory participation — can curb the economic and social costs related to the growing ageing of the population.

\textsuperscript{20} Several works within the extended literature on intertemporal choices estimate dynamic consumption models, pinpointing existing links between the income and consumption trend in the first part of the life cycle, but also showing that the empirical data indicate accrued wealth as the retirement age approaches. See for example ATTANASIO O.P. - BANKS J. - MEGHIR C. - WEBER G. (1999); GOURINCHAS P.O - PARKER J.A. (2002); CAGETTI M. (2003); KENNICKELL A. - LUSARDI A.M. (2004).
3. - Insurance Against the non Self-sufficiency Risk

This section will try to examine, through analytical means, that a mere transfer of resources from the overall pension bill to a social institution aimed at covering the non self-sufficiency risk in the elderly population can curb costs related to the population ageing in terms of individual and collective welfare. The aim will be to show that the setting up of an ad hoc fund against this risk is of vital importance, especially in Italy, a country where households play a crucial role in supporting the disabled elderly population.

First of all, it should be noted that the insurance against the non self-sufficiency risk can be taken out both through the private insurance market and public institutions. Public or private insurance can be seen as a contract linked to occurrence of an uncertain event, and be such as to reduce the random fluctuations of the individuals’ marginal utility21. In particular, public insurance can be seen as an institution that, with a view to protecting individuals against the consequences of negative events, lock up their strategies, for example forcing them to save22.

The risks of a target-oriented conduct and private market-related failures, the affirmation of redistribution ideals and the protection of the poor’s needs are some of the reasons leading to consider setting up a Government-run Universal Fund against non self-sufficiency, capable of offering all social assistance services without altering the individuals’ financial conditions. The target to aim at is the so-called «German model» in which risk socialization is practically total23. In this scheme, in fact, the individual financial

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23 For a discussion on the different non self-sufficiency social assistance models adopted in Europe, see the bimonthly newsletter of Agenzia per i servizi sanitari regionali, ASSR (2003). In the so-called “German model”, the financing to the public social assistance fund is guaranteed through a contribution rate on the taxable income of each individual, which is the same for all the social assistance beneficiaries and throughout the national territory. The extent of the contribution, determined by the national law, is expected to account for 1.7%, and is charged, in equal parts, to employers, workers or former workers (definite sharing system) with a ceiling of about monthly 57 Euro. 90% of the population obtain compulsory
condition is not a selection criterion for social assistance access, in that income redistribution is guaranteed both *ex ante*, in favour of the individuals exposed to a higher risk, and *ex post*, to the benefit of the needy. Presently, such risk is very partially socialized in Italy, because non self-sufficient people are called on to share public social assistance costs; also, they have to privately buy a large number of social assistance services and they have to resort to the help of relatives and friends to meet their needs\textsuperscript{24}.

Let us create a reference model, assuming there is an insurance market where the following conditions are met:

— The risks faced by the different individuals are independent one another; in other terms, the likelihood that an unfavourable state of nature affecting an individual is independent of the likelihood affecting another individual; the law of large numbers applies accordingly.

— There is such absence of informative asymmetry as to prevents the insurer from having the same information as the insured;

Let us retrace the traditional method of textbooks on insurance in perfect competition with zero insurance cover costs. We define by $\pi$ the likelihood for the damage to occur and by $(1-\pi)$ the likelihood for the damage not to occur. The damage ($d$) the individual may suffer refers to the loss of some physical or mental ability involving a non self-sufficiency condition. For the purpose of the analysis, we assume that the non self-sufficiency condition involves the same damage for all individuals in terms of wealth levels\textsuperscript{25}. We also define by $\delta k$ the insurance premium and by $k$ the allowance paid to the insured by the insurance company in case of a damage.

\textsuperscript{24} See BELTRAMETTI L. (2003).

\textsuperscript{25} This theory, albeit restrictive, significantly simplifies the analysis and allows us to draw definite conclusions.
The problem of the insurance company, which in theory is neutral to risks, will be that of maximising the profit, that is:

\[
\text{(17)} \quad \max \pi(\delta k) + (1 - \pi) \delta k
\]

Knowing that the maximum profit of a firm in a competitive market is zero, we can reformulate the target function as follows

\[
\text{(18)} \quad \pi(\delta k) + (1 - \pi) \delta k = 0
\]

By algebraically solving the target function of the insurance company, we obtain the relation between premium and allowance in perfect competition conditions:

\[
\text{(19)} \quad \delta k = \pi k
\]

In other terms, it can be noticed that a competitive market offers a fair premium and a fair allowance\(^{26}\). In this way, the individual will be indifferent to the income levels related to the two different states of nature.

The problem of the individual is instead that of maximising the utility resulting from the two possible states of nature; if we define by \(z_1\) the endowment in the unfavourable state of nature, that is that of the damage occurrence, and by \(z_2\), the endowment in the state of nature in which the individual suffers no damage, the target function of a generic individual will be expressed as follows:

\[
\text{(20)} \quad \max \pi u(z_1) + (1 - \pi) u(z_2)
\]

\[
\text{(21)} \quad \max \pi u(z - d - \delta k + k) + (1 - \pi) u(z - \delta k)
\]

Knowing that the premium is fair \(\delta k = \pi k\), partially diverting from the allowance, it will follow that:

\[
\text{(22)} \quad u'(z_1) = u'(z_2)
\]

\(^{26}\)This means that the non self-sufficiency risk cover is effected at a fair rate, that is, the expected insurance value will be equal to its cost.
The equation shows that the equality of marginal utilities must apply. In other terms, the marginal utility of an additional Euro in case of an unfavourable state of nature shall be equal to the marginal utility of an additional Euro in case of non-occurrence. In order for the marginal utility to be equal \( z_1 = z_2 \).

By replacing this condition in the first derivative and solving algebraically, it will follow that:

\[
(23) \quad k = d
\]

The result so obtained indicates that in a competitive market where the insurance company offers a fair premium, the best solution for the individual is to pay such a premium as to match allowance and damage\(^27\).

Going back to the pattern of the previous sections, we assume that the representative individual is aware that there will be a certain likelihood \( \pi \) of being non self-sufficient in the last period of his life cycle. For the sake of clarity, we also assume that the problem of non self-sufficiency risk protection is only handled by the individual in the old age\(^28\).

From Italy-related data, it can be inferred that the likelihood of facing some form of non self-sufficiency is 20% for the population aged more than sixty-five\(^29\). We can also assume that the individual does not save during his working life and that therefore his endowment in his old age is equal to the pension transfer standardised at 1 Euro. If we establish that non self-sufficiency involves a 50% damage in terms of overall endowment to the individual, we wonder which premium a risk-averse individual will be willing to pay to be insured against such a negative state of nature.

\(^{27}\) It is easy to understand that in this specific case, the risk-averse individual will always and entirely get insured.

\(^{28}\) Recent literature on wealth accumulation in the life cycle highlights that empirical data indicate precautionary saving forms in the older age which can be attributed to demographic or income shocks, which is at odds with the behaviour of consumer’s conduct in the life cycle model. See for example, ATTANASIO P.O. - BANKS J. - MEGHIR C. - WEBER G. (1995); CAGETTI M. (2003).

\(^{29}\) The likelihood, as observed earlier, will tend to increase with the growing age of the individuals.
If the market is in perfect competition conditions, the allowance paid to the insured by the insurance company will be equal to the damage \((k=d)\); the individual, therefore, will be insured by 0.5 Euro paying a premium of \(\delta k\). The premium and allowance being fair, it will follow that \(\delta k = \pi k\) hence \(\delta = \pi\). Assuming that \(\pi=20\%\), then the individual will pay a premium of \(0.2\times0.5\) Euro = 0.1 Euro. In this case, regardless of whether the favourable state of nature occurs or not, the individual will consume exactly the same quantity.

Another way to be insured against the risk could be to resort to the annuity market, because it has a huge impact on the consumption model in the life cycle. Yaari (1965) was the first to move in this direction analysing, through the use of an overlapping generation model, the impact of uncertainty on life duration in the consumption pattern. He concluded that the presence of a fair annuity market could lead to the same consumption mechanisms of the certain scenario. More recently, literature extends Yaari’s\(^{30}\) work to multiple directions. Latest research on the so-called annuity “puzzle” leads to conclude, however, that these are not fair in terms of present value due to the presence of adverse selection and of administrative costs the insurance company must bear\(^{31}\).

A different way to protect individuals from the risk could be that of taking out policies which may share it among all individuals\(^{32}\). With about 2 out of 10 million elderly being non self-sufficient, the overall monetary loss will total 1 million Euro. In other terms, each elderly will be faced with an expected loss of 0.1 Euro. When

\(^{30}\) For a systematic overview of specific literature reference should be made to i.a. LEVHARI D. - MIRMAN L. (1977); DAVIES J. (1981); BUTLER M. (2001).

\(^{31}\) See MITCHELL O. - POTERBA J.M. - WARSHAWSKY M.J. (1997). This brilliant work shows that annuities are not fair in terms of current value because as for individuals with an average life expectancy, they insure solely 75 to 84 cents per dollar of premium paid.

\(^{32}\) The risk sharing issue within the framework of demographic ageing is analyzed by KRUSE A. (2002). This work compares the intergenerational redistribution effects in the different pension schemes caused by demographic and economic changes. It also identifies a potential trade-off between financial stability of the pension system and intergenerational redistribution. BOHN H. (1999) demonstrates that, with a fertility rate being affected by an idiosyncratic shock, a social protection system providing defined benefits is ex ante more efficient than a defined contribution system or a private insurance system.
assuming that each individual may wish to diversify the risk, transferring a part of it to the others, it will follow that 10 million elderly will be mutually insured, by joining a Fund aimed at covering the overall damage caused by non self-sufficiency. The overall damage amounting to 1 million Euro, each elderly shall pay a premium of 0.1 Euro. This means that if 2 out of 10 elderly suffer a 0.5 Euro damage, everyone shall pay a 0.05 Euro to each non self-sufficient elderly, that is an overall premium of 0.1 Euro. A way to implement this risk sharing could be to set up a non self-sufficiency fund, run by the government and subject to mandatory participation.

As found in these two simple examples, the overall social assistance cost remains unchanged independently of risk sharing or a collective cover. In this case, there are no reasons to support the need for a mandatory collective approach other than an individual one. It should, however, be noted that the voluntary individual approach is only feasible if the theories expressed earlier are observed, meaning only if the markets are complete and, as a result, able to perfectly neutralize the individual uncertainty.

The main problem arising within the framework of complete markets is to ascertain if and how complete they are. Individual risks can undoubtedly be reduced by signing insurance or financial agreements, but it is also true that most individual risks can hardly be insured officially due to adverse selection or moral hazard problems. In the presence of realistic theories on conduct and opportunities, the private market for the cover of certain risks does not exist, is inefficient, or inaccessible to most of the population. In such cases, public insurance can replace them through social protection schemes, thus leading to an improvement of general welfare.

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33 See JAPPHELLI T. - PISTAFERRI L. (2000). We can note that the larger a community, the easier the task of insuring individual risks; conversely, the larger a community, the weaker the solidarity constraints and, therefore, the willingness to help the needy. BORCH K.H. (1990); ECKHOUTD L. - GOLLIER C. (1992) also point out that the complete insurance that can eliminate the unpleasant consequences of any kind of risk is not related to moral hazard and anti-selection problems hindering the private offer of different kinds of insurance.

In the complete market model, the elderly save only if they expect reduced endowment in the next future; however, they are totally indifferent to the endowment-related risk. On the contrary, if markets are not complete, the individual will react to the risk only in an imperfect way, through the setting up of a precautionary saving fund. The incentive to distribute resources among periods is accompanied, in this case, by that of limiting the negative effects of uncertainty through saving\textsuperscript{35}.

Therefore, let us try to understand the impact of uncertainty on individual intertemporal choices when markets are incomplete. The main difference from the complete market case is that now the intertemporal dynamic budget constraint only applies after random variables have produced an effect, that is, \textit{ex post}.

Moving back to the previous model and assuming, in simple terms, that the interest rate is null, we can define the target function of the average consumer as follows:

\begin{equation}
E(u) = u(c_t) + \beta u(n_{t+1}) + \beta^2 E[u(\tilde{a}_{t+2})]
\end{equation}

The problem of the individual’s choice, therefore, will be the following:

\begin{equation}
\begin{aligned}
\max_{s.v.} & \quad E(u) \\
\text{s.t.} & \quad w_t = \bar{w} \\
& \quad p_{t+1} = \bar{p}_{t+1} \\
& \quad \tilde{a}_{t+2} = [(w_t - c_t) + (p_{t+1} - n_{t+1})] + \tilde{d}
\end{aligned}
\end{equation}

where \(\tilde{d}\) is the monetary damage caused by the loss of self-sufficiency that, in theory, will occur with a \(\pi\) likelihood. We can notice that the social assistance consumption in period \(t+2\) is uncertain and dependent on the extent of the monetary damage. The individual may therefore be faced with an unfavourable state of nature, causing a minor social assistance consumption \((\tilde{a}_{t+2}^-)\), or in a favourable state of nature characterised by sound physical and mental state \((\tilde{a}_{t+2}^+\)). Bearing in mind these observations, the target function of the consumer can be rewritten as follows:

\textsuperscript{35}Ibidem.
(26) \[ E(u) = u(c_t) + \beta u(n_{t+1}) + \beta^2 [\pi u(a_{t+2}^-) + (1 - \pi) u(a_{t+2}^+)] \]

As can be noticed, the target function is a multi-period equation which can be converted into a sequence of two-period equations assuming that the consumer is short-sighted. This theory may appear restrictive; yet, when accepting the empirical consequences of the uncertain life model as for the vital wealth profile, we may likewise argue that the non self-sufficiency rate being low among young people, then the non self-sufficiency risk uncertainty will solely affect the conduct and intertemporary decisions of the elderly\(^{36}\). In the early period \(t+1\) the individual is in fact old and, having examined the results of the income and interest rate which was assumed to be null, will have a new expectation influencing his choice of health care and basic social assistance in \(t+1\) and social assistance consumption in \(t+2\).

The problem with converting a multi-period equation into a sequence of two-period equation can be solved through the dynamic programming system\(^{37}\), which consists in defining the optimal value function of the problem, i.e.

(27) \[ u_t(w_t) = \max_{c_t, \ldots, c_T} \sum_{n=0}^{T-t} \beta^n u(c_{t+n}) \]

where \(E_t\) indicates the expectation subject to the available information in period \(t\). The above equation accounts for the maximum value of the expected utility discounted at time \(t\), which the consumer obtains from the optimal consumption sequence corresponding to an initial wealth endowment. By defining a value function, the multi-period problem comes down to a sequence of two-period recursive problems which can be expressed through the Bellman equation\(^{38}\), namely,

\(^{36}\) Hansen G.D. - İmrohoroğlu S. (2005) find that, in absence of an annuity market and with a social system insuring against uncertainty, the saving in the life cycle model regards the old age. In other words, the peak of the consumption in the “hump shaped” model is only reached in the old age.


\(^{38}\) See Jappelli T. - Pistaferrri L. (2000) for the solution to the multi-period problem in a sequence of two period problems.
This equation indicates that with the information available in period $t$, the consumer formulates expectations on future income and interest rates and chooses a consumption as if such expectations should be fulfilled. In the following period, the consumer, after observing the income performance, will formulate a new expectation leading him to choose the kind of consumption in that specific period.

In our case, the problem of the consumer in period $t$ will be:

$$
\max_{c_t, n_{t+1}} u_t(w_t) = \max u(c_t) + \beta E_t[u_{t+1}(n_{t+1})] = \max u(c_t) + \\
+ \beta E_t[u_{t+1}(w_t + p_{t+1} - c_t - s_{t+1})]
$$

In our case, the problem of the consumer in period $t$ will be:

$$
\max u_t(c_t) + \beta u_{t+1}(n_{t+1})
$$

The target function obtained indicates that in the first period, the individual will choose, given his specific preferences, such an optimal combination of goods, health care and basic social assistance as to maximize his intertemporary utility. The consumer being, as assumed, “short-sighted”, he will not take into account the uncertainty on his future health state. In other words, in the early period $t+1$, the quantity of $n_{t+1}$ will no longer be determined because the elderly shall reconsider his future choices while bearing in mind the uncertainty on his future health conditions.

The problem of the consumer in period $t+1$ will be accordingly:

$$
\begin{align*}
\max_{s.t.} & \quad E(u) = u_{t+1}(n_{t+1}) + \beta E[u_{t+2}(\tilde{a}_{t+2})] \\
& \quad p_{t+1} = \tilde{p}_{t+1} \\
& \quad \tilde{a}_{t+2} = (p_{t+1} - n_{t+1}) + d \\
& \quad \tilde{a}_{t+2} = s_{t+1} + d
\end{align*}
$$

where $d$ is the damage, which will be negative with a $\pi$ likelihood and null with a likelihood of $(1-\pi)$.

The following first order conditions can be obtained from the system:
First of all, it should be noticed that $n_{t+1}$ changes upon change of the likelihood of being non self-sufficient. Should the consumer be risk-averse, it will follow that:

\[ \frac{\partial E[u]}{\partial n_{t+1}} = u'(n_{t+1}) + \beta \left[ \pi \frac{\partial u[a_{t+2}]}{\partial n_{t+1}} + (1-\pi) \frac{\partial u[a_{t+2}^+]}{\partial n_{t+1}} \right] = 0 \]

\[ u'(n_{t+1}) = \beta [\pi u'(a_{t+2}^-) + (1-\pi)u'(a_{t+2}^+)] \]

which gives rise to a precautionary saving $\frac{\partial n_{t+1}}{\partial \pi} < 0$.

However, we must understand that the optimal value of $n_{t+1}$ will depend on the convexity on the marginal utility function, thus from the sign of the third derivative of the utility function. The Jensen inequality applied to convex functions will lead to $uE(x_{t+1}) < Eu(x_{t+1})$, that is, the marginal utility of expected consumption is lower than the expected value of marginal utility. In this case, to observe the first order condition, the consumer will aim at reducing his consumption in the first period to reduce the expected marginal utility in the second period, thus increasing that of the previous period. Actually, the utility function convexity requires $u'' > 0$, hence $s_{t+1}^* > 0$\(^{39}\).

For the purpose of the analysis, an isoelastic utility function $u(x) = x^{1-\sigma} / (1-\sigma)$ will therefore be used, with $\sigma > 0$, $u' = x^{-\sigma}$, $u'' = -\sigma x^{-\sigma-1}$, $u''' = -\sigma(1+\sigma)x^{-\sigma-2}$ and where the degree of absolute risk aversion $-u''(x) / u'(x)$ is on the decrease. In short, we will use a particular case of the isoelastic function for the analytical handling: the logarithmic function $u(x) = \ln(x)$.

We assume that the pension value is given and is equal to 1 Euro and that, to put it briefly, the market interest rate is null. Previously we found that the likelihood of an elderly to face some

\(^{39}\) We must however note that in order for that to be true the utility function must not be quadratic, because in this case the individual is risk-averse ($u'' < 0$) but, at the same time, he does not react to the risk through a precautionary saving ($u''' = 0$).
form of non self-sufficiency is equal to \( \pi = 0.2 \). Also, we assume that the non self-sufficiency damage involves a 50% loss of the endowment. Given the utility function of a logarithmic nature, we can determine the consumer’s optimal choices in conditions of uncertainty, or when the individual personally faces the non self-sufficiency risk without insurance due to incomplete private insurance market.

To make it simple, we fix the service prices equal to unit \( \rho_{t+2} = \rho_{t+1} = 1 \) and assume that in the first period there is no saving, so that the endowment will be equal to the pension transfer in the early period. The choice problem will be the following:

\[
\begin{array}{l}
\max_{u} \quad E[u] = \ln(n_{t+1}) + \beta E[\ln(\bar{a}_{t+2})] \\
s.t. \quad a_{t+2}^- = (100 - n_{t+1}) - 50 \\
\quad a_{t+2}^+ = 100 - n_{t+1}
\end{array}
\]

hence:

\[
E[u] = \ln(n_{t+1}) + \beta[0.2\ln(50 - n_{t+1}) + 0.8\ln(100 - n_{t+1})]
\]

We expressed impatience to time through a psychological rate of intertemporal preference whose value increases as future consumption decreases. Considering that the individual is aware of the risk he may face in the third period of his life cycle, we assume that the value of such rate is relatively low, in that it will attach importance to specific social assistance consumption during the old age. Therefore, setting \( q = 0.2 \), our target function will be:

\[
E[u] = \ln(n_{t+1}) + 0.83[0.2\ln(50 - n_{t+1}) + 0.8\ln(100 - n_{t+1})]
\]

By partially diverting from \( n_{t+1} \) and algebraically solving an equation of second degree, we will obtain the economically consistent optimal value of \( n_{t+1} \)

\[
n_{t+1}^* = 0.39
\]

The values of in the negative and positive state of nature will be respectively:
(38) \[ a_{t+2} = 0.11 \]
\[ a_{t+1} = 0.61 \]

What can be inferred from such results? They indicate that the individual will react to the risk only in an imperfect way, through precautionary saving. In other words, an incentive is created to reduce the negative effects of uncertainty through saving.

In order to understand these observations, it is necessary to compare the results obtained with those that would be obtained in case of complete markets with a competitive insurance offering a fair premium, or the results which could be obtained with a government-run institutional fund allowing a risk diversification through a mandatory participation by a transfer of resources from the overall pension bill to the fund.

As mentioned earlier, the individual, with both insurance covers, will pay a premium of 0.1 Euro and regardless of the damage occurrence, he will receive the same resources, amounting to 0.9 Euro. In both situations, the individual will have to face a state of certainty. If we take into account the previous discussions, we find that the setting up of a collective fund against the non self-sufficiency risk can eliminate the uncertainty related to such risk.

If we keep the prices \( \rho_{t+2} = \rho_{t+1} = 1 \) constant and assume there is no saving in the first period, the choice problem, given the utility function \( u(x) = \ln(x) \), will be:

(39) \[
\begin{align*}
\max & \quad \ln(n_{t+1}) + \beta \ln(a_{t+2}) \\
\text{s.v.} & \quad 0.9 \text{ euro} = n_{t+1} + a_{t+2}
\end{align*}
\]

Hence the Lagrange formula

(40) \[ L = \ln(n_{t+1}) + \beta \ln(a_{t+2}) - \lambda [n_{t+1} + a_{t+2} - 0.9 \text{ Euro}] \]

By partially diverting, it will follow that

(41) \[ n_{t+1} = \frac{a_{t+2}}{\beta} \quad a_{t+2} = \beta n_{t+1} \]
Assuming that in this case too the subjective impatience rate is \( q = 0.2 \), the following optimal solutions will be obtained:

\[
(42) \quad n_{t+1}^* = 0.49 \quad a_{t+2}^* = 0.40
\]

As seen in these solutions, the uncertainty and incomplete market case involves an excessive distribution of resources in the favourable state of nature. This can be put down to the fact that the individual cannot affect the level of saving in the two different states of nature. To protect himself from the risk of suffering the non self-sufficiency damage, he is forced to save even when not necessary, for instance when the damage fails to occur. The fund scheme, by contrast, allows the selection and transfer of the saving in the two states of nature, while the precautionary saving enables solely a consumption stabilization between the different periods.

It would also be more reasonable to assume that the presence of the fund induces individuals to a less cautious, more impatient conduct towards consumption of the first period considered, because the guarantee of public protection helps raise the consumption level. In other words, the impatience rate could increase, highlighting the effects previously discussed.

What does all this mean in terms of welfare? If we talk in \textit{ex ante} terms, the welfare level turns out to be the same. However, in \textit{ex post} terms, this is not true, because in the non insurance case there is an excessive distribution of resources in the favourable state of nature, as against a precautionary saving in the first period. When examining the assumptions related to the form of the utility function and the attitude of the individual toward the risk, if the individual could be fairly insured, he would be willing to waive some units of good \( a_{t+2} \) to obtain some additional units of good \( n_{t+1} \).

Given the logarithmic form of the utility function, when calculating the level that would be reached in the fund and non fund cases, a substantial difference can be inferred: in numerical terms, considering the previously assumed data and the optimal solutions obtained, the individual utility will suffer a rise of
approx. 17.5% when shifting from an individualistic system to a system with full risk socialisation\textsuperscript{40}.

Even though the exercise is based on simplified assumptions on the individual's preferences, risk aversion and aggregation of individual preferences, the results indicate that the \textit{ex post} creation of a non self-sufficiency fund could raise individual and collective welfare.

Although it is hard to establish the sources generating the economic resources to ensure the most suitable performances for every citizen, a fundamental issue arises: the absolute need to set up an \textit{ad hoc} non self-sufficiency fund as an independent social assistance program, with the task of running a public insurance against such risk.

\textsuperscript{40} Likewise it can be found that in \textsc{Fornero E. - Mandrone E.} (2000), when realistic assumptions affect conducts and opportunities, the private market for the cover of certain risks does not exist or is inefficient. In such case, it is replaced by public insurance with social protection schemes leading to an improvement in general social assistance.
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