International Monetary Fund Resources and Contagion Mechanisms: A Hypothesis

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This paper examines analytically the possibility that, due to the limitedness of its resources, the International Monetary Fund (IMF) could spread financial crises rather than preventing them, thus developing into a contagion channel. The model we build, based on the most recent global-games literature, allows us to show that this risk is sensible from a theoretical point of view. We conclude that the IMF, when planning its interventions, should take into account this kind of contagion it contributes in creating. Some policy implications are derived. [JEL Classifications: F33, F34]

1. - Introduction

«Back in 1997, then US Treasury Deputy Secretary Lawrence Summers liked to compare modern finance to a jet plane. The technology of modern finance, like a jet plane, lets you get to your destination faster then older transportation technology. But the rare crashes that occur along the way are also more spectacular. [...] This analogy now seems to suffer from one problem. Emerging-market economies crash more frequently than 747s».

In the last ten years, international financial crises followed one another at an impressing rate and showed new characteristics

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[1] Roubini N. - Setser B., *Bailouts or Bail-ins? Responding to Financial Crises in Emerging Economies*, 2004.

that made them particularly harmful. From the 1995 *tequila crisis* to the last Brazilian one in 2002, we can count up to fourteen crisis episodes, to number just the most important cases, characterized by great pressures on countries capital accounts and by impressive capital runs. A fairly widespread theory between economists, attributes the last years crises to debtor countries illiquidity, and claims that financial panic, investors self-fulfilling expectations and contagion from similar-perceived countries, may explain — sometimes even more than emerging markets' own soundness — sudden capital runs. In this context, an *International Lender of Last Resort* (ILLR), acting as a lender of last resort for illiquid (but solvent) countries, can play an important role in reducing the frequency of crises. An ILLR may mitigate investors' coordination problems, thus preventing capital runs and reducing the probability of a crisis for each level of the fundamentals.

Nowadays the institution that comes closest to an ILLR, is the *International Monetary Fund* (IMF). Yet, the magnitude of the most recent financial crises showed that the resources the IMF can rely on to exert this function are limited-limited especially if compared to the liquidity gaps that emerged in the last years. This essay is aimed at examining a specific problem that may stem out of the limitation of the Fund resources. We asked ourselves whether, due to the inadequacy of its funds, the IMF could spread financial crises rather than preventing them: in this sense, it risks developing, from a crisis prevention and management tool, into a channel of contagion. In periods in which crises take big proportions, the limitation of IMF resources may foster investors' fear that it does not have enough funds to bail out all the economies. In this case, the breakout of a crisis in one country can generate panic between creditors, trigger capital runs from other risky economies, and facilitate crisis transmission. In this respect, Claessens *et* al. (2000) write:

«Other reasons [for contagion] could include concern about the supply of funds from international lenders of last resort. In late 1998, for example, the *International Monetary Fund* (IMF) found itself called on to rescue so many countries that economists

wondered whether it would be able to deal with many more liquidity crises. Thus a liquidity crisis in one country could trigger a run on other countries out of fear that the last eligible country would be out of luck».

Traditionally, literature has developed the themes of *International Lending of Last Resort* and contagion separately. In examining the limitation of ILLR resources, economists focused on the necessity of implementing partial bailouts, interventions aimed at persuading creditors to invest in the crisis-hit country (catalytic finance). The first theoretical models developed to examine the effectiveness of catalytic finance, treat the probability of a crisis as exogenous, generate multiple equilibria, and are quite pessimistic toward the effectiveness of this kind of interventions. Goldfajn and Valdes (1999) build an open economy model *à la* Diamond-Dybvig that allows showing that, under certain conditions, a partial bailout may increase the probability of both domestic financial crises and twin crises. Zettelmeyer (1998), by means of creating a self-fulfilling run model derived from Diamond and Dybvig (1983), finds that, with multiple equilibria, a partial intervention raises investors incentives to withdraw their funds and ends up worsening an ongoing crisis.

Conversely, more recent models that treat the probability of a crisis as endogenous, seem to prove the effectiveness of catalytic finance. Morris and Shin (2003) global-game model shows that, if catalytic finance is not perceived as a substitute for government and investors' actions, it is effective in stimulating private investments and reducing the *ex-ante* probability of a crisis. Corsetti, Guimarães and Roubini (2003, 2004) show that an increase in the size of ILLR interventions in the event of a crisis, decreases the threshold values for the fundamentals below which investors choose to withdraw their funds, thus making a crisis less likely *ex-ante*. The authors ascribe this effect to the coordination of investors' expectations that derives from the intervention of the lender. Penalver (2004) finds that, if the ILLR lends liquidity at an interest rate below the market rate, debtor country's government will be encouraged to implement policy measures that reduce the probability of future crises. Investors will be more

eager to invest in the country and will fill its liquidity gap. Finally, Rochet and Vives (2004) show that, under certain conditions, solvent but illiquid debtors are forced to default due to investors' coordination problems. The intervention of an ILLR helps in solving this kind of problem.

Though these works examine an important implication of the limitation of ILLR resources — the necessity of partial bailouts and their effectiveness — they do not relate funds inadequacy to the possible contagion that could originate from it.

On the other hand, literature on contagion highlighted many channels of crisis transmission. Trade links shown by Corsetti, Pesenti, Roubini and Tille (1999), the common lender channel theoretically exposed by Sbracia and Zaghini (2001) and empirically tested by Kaminsky and Reinhart (2000), several types of herd behavior, such as the one made known by Calvo (1999), are just a few of the most important mechanisms. Nonetheless, economists confined themselves to reporting without any formal framework the possibility that an ILLR may become a source of contagion, without developing, as far as we know, any model.

In this essay, we will use for the first time a theoretical model to examine analytically the risk that IMF limited resources may turn it into a contagion channel. To this purpose, we employ a modified version of Corsetti, Guimarães and Roubini (2003, 2004) global-game model[2]. The fundamental hypothesis underlying our analysis is that IMF resources are limited but enough to partially bail out all the economies potentially exposed to the risk of default. The first important result we derived is very similar to the one obtained by CGR: the partial intervention of an ILLR that commits itself to intervening in order to compensate investors' losses in the event of a default, makes creditors less keen on liquidating their investments, and is effective in reducing the *ex-ante* probability of a crisis in emerging-market economies. The changes we make to the original model — in particular its extension to two countries and the fact that IMF intervention is made conditional to a crisis — allows us to gain new insights. If the IMF has limited resources,

[2] Hereafter, we will refer to the original model using the acronym CGR.

the default of a country will reduce the funds available to manage potential crises in other economies. Creditors will perceive their investment in the other emerging markets to be less sheltered, and will become more eager to liquidate them, thus exposing these countries to greater risks. We prove that the limitation of IMF resources creates a positive correlation between the probabilities of default of different economies, and makes crises to transmit more easily between countries, via investors' expectations. The IMF, in other words, may become a contagion channel.

Finally, we analyze the effect of this kind of contagion on the Fund optimal behavior. We find that the IMF should take into account this mechanism in order to effectively plan its interventions in emerging markets. In particular, it is necessary to provide more funds to the countries from which a crisis is more likely to spread. Moreover, in order to ensure the effectiveness of interventions, it is essential that the IMF be able to credibly commit itself to the actual disbursement of the promised funds.

In what follows, we highlight the limitation of IMF funds, analyzing their trend during the crises of the last years (section 2). Section 3 presents the modified version of *CGR* global-game model. In section 4, we draw the conclusions of our analysis.

2. - IMF Resources

At present, the main function of the IMF — which makes it very similar to an ILLR — consists in helping countries experiencing BoP disequilibria, by means of supplying various kinds of loans. Yet, the resources it can rely on to exert this activity, are limited. In appendix A we list the different types of resources used by the IMF to provide its loans, and show that the existence of borrowing arrangements signed with official creditors, is not enough to question the limitedness of its funds. In the following paragraph, we analyze the trend of its resources during the most recent crises[3].

[3] In this section, the data we denote as present refer to 31 december 2004.

2.1 *The Last Years Crises*

Have there been periods in which IMF funds clearly seemed to be inadequate to the size and number of necessary interventions? To answer this question, we analyzed the uncommitted usable resource's trend from December 1996 to July 2004. Graph 1 shows their movements in the analyzed period, and relates it to the development of total committed resources, the funds attributed by the IMF to some troubled countries[4]. It comes out from the graphic that the resources available for the provision of new loans underwent a indisputable depletion, between the end of 1997 and the first months of 1999, touching a minimum of $29.8 billion in December 1998. As it is clear from Graph 2 (*Appendix A*), which relates total committed resources to the commitments towards each economy, this plunge was caused by IMF huge interventions that followed the 1997 Asian crisis and the ones that stroke Russia, Brazil and Ukraine the following year[5]. The worrying depletion of the Fund resources caused by these bailouts was made evident also by the activation, in 1998, of the borrowing arrangements available to the IMF, GAB and NAB, to finance respectively the Russian and the Brazilian rescues. Moreover, it was not accidental that during the Eleventh General Review, which took place in 1998 as well, a 45% increase in quotas was decided: clearly, due to the great size of those years' crises, even the IMF directive boards were worried about the inadequacy of its resources. Thus, from the analysis of the available funds during the last years, it comes out that in the period 1997-1999 they were possibly inadequate to manage so many simultaneous crisis episodes.

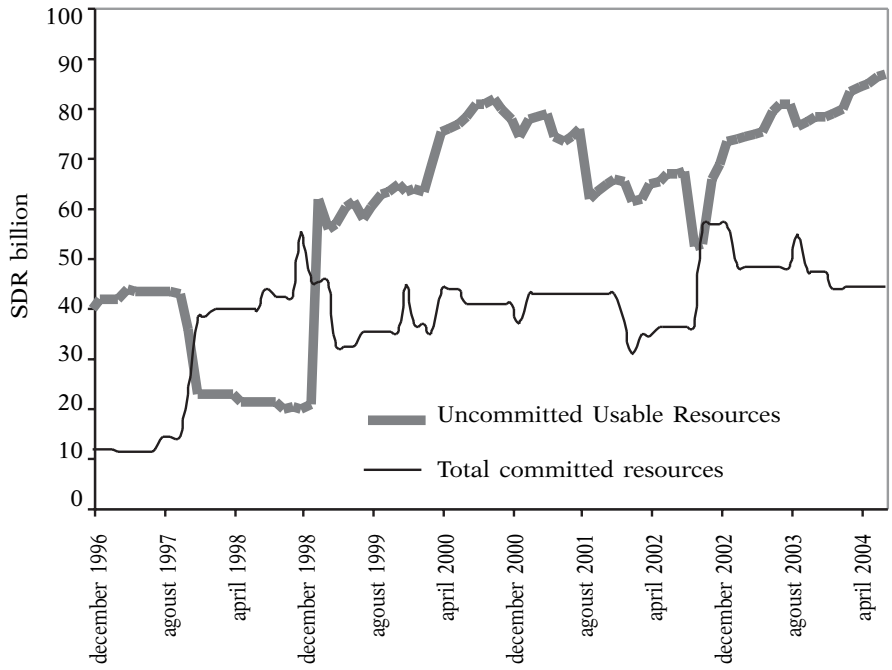
Due to the 1997 Asian crisis, the amount of liquidity the IMF could use to provide new loans, fell sharply. According to some analysts, this initial depletion of resources could have shown investors the limitation of IMF funds. This could have raised

[4] A description of these aggregates, as well as a detailed explanation of the procedures followed for their calculation, is contained in *Appendix A*.

[5] The size of these bailouts was unprecedented in the history of the Fund, so that in order to cope with them, a new type of loan was introduced, the *Supplemental Reserve Facility*, which could overcome the quantitative limits usually applied to rescue packages.

GRAPH 1

IMF RESOURCES AVAILABLE FOR NEW LOANS



foreign creditors' fear that IMF resources would have not been enough to bail out all the economies. Hence, they could have had more incentives to withdraw their capitals from the countries perceived to be the riskiest — Russia and Brazil — thus facilitating the outbreak of a crisis in these two economies. Therefore, the Fund could have eased the contagious spread of the crisis towards Russia and Brazil. In order to prove undoubtedly that this really happened, it would be necessary to run an empiric test, which goes beyond the aim of this essay. Nevertheless, the evidence from the data is consistent with this hypothesis[6].

[6] Notice that the increase in quotas decided during the Eleventh Revision allowed the Fund to cope with less problems with the crises that in the following years hit Argentina, Brazil, and Turkey.

We therefore conclude our analysis of IMF resources stating that, between 1997 and 1999, the limitation of IMF resources is very likely to have enhanced the transmission of crises from Asian economies to Russia and Brazil.

The data we examined suggest that the Fund could have really acted as a channel of contagion.

3. - A Model of Catalytic Finance

In order to examine analytically the risk that the IMF develops into a contagion channel, we used a global-game model that constitutes an adaptation of CGR (2003, 2004).

3.1 *Temporal Structure of the Model*

There are two emerging-market economies, *A* and *B*. In each country, action develops in three periods.

Period 1 (Investment): Each economy has initial endowments *E*, borrows *D* from a continuum of foreign investors and invests (*E*+*D*) in an international liquid asset, *M*, and in risky and illiquid investment projects, *I*, which mature in two period. If they are kept in place for two periods, they yield a stochastic rate of return equal to R_j (where *j* specifies the country to which we refer); if they are liquidated early, after one period, they yield $(R_j / 1 + k)$, with $k > 0$. *E, D, I, M* and *k* are given parameters. The interest rate applied to private credit is normalized to zero for simplicity.

The size of λ_j is announced. The definition of this value will be given in the next paragraph.

Period 2 (Withdrawal): R_j realizes. Foreign investors have the possibility to withdraw their funds in advance. If we denote with *x* the fraction of creditors who choose the withdrawal, *xD* will be the liquidity need of the country. It can be met by international reserves *M* or, if these are not enough, by the (costly and inefficient) liquidation of a fraction *z* of investment projects.

Period 3 (Possible default): Investment projects mature. The

country resources will be equal to the return of completed projects, plus the liquidity left over after paying xD to foreign creditors in the previous period. The country will have to provide $(1-xD)$ to those who left their capital invested for two periods. The difference between returns and disbursements constitute the country net returns:

$$Y_j = \max\left\{R_j(1-z)I + (M-xD)_+ - (1-x)D, 0\right\} = \max\left\{R_j I\left[1-\frac{zk}{1+k}\right] + M - D, 0\right\} \quad (1)$$

where the notation $(M-xD)_+$ indicates the $\max\{M-xD, 0\}$.

If net returns are negative, the country is forced to default; this happens when

$$R_j(1-z)I < (1-x)D \quad (2)$$

In order to expose the two economies to the risk of a crisis in different periods, we built a game which develops over *six* times: at times 1, 2, and 3 respectively investment, withdrawal of funds, and possible default, take place in country *A*; at times 4, 5, and 6 the same sequence of events takes place in *B*.

3.2 *IMF Intervention*

As anticipated in the previous paragraph, at time 1 the IMF commits itself to provide country *A* with a certain amount of funds λ_A in the event that, at time 3, it is forced to default. The disbursement of these funds is very likely to be subject to some degree of conditionality. In the event of a crisis, the hit country should commit itself to implement some policies and internal reforms that can be useful to prevent possible future defaults and to ensure that the country be able to refund the IMF. We assume that A will use these resources to partially compensate its creditors for their losses. So, at $t = 3$, in the event of a default, each creditor of country *A* will receive a reimbursement equal to λ_A (given

parameter). We assume it not to entirely cover investor's loss. Similarly, at time 4 the Fund commits itself to provide each creditor of country B with λ_B at $t = 6$, if a crisis occurs.

The fundamental hypothesis of our model is the limitation of IMF resources. From this assumption follows that the amount of liquidity the Fund can give to B varies accordingly to what happened in the other economy in the previous periods. If country A has not been forced to default, each creditor of B will receive a reimbursement equal to Λ. If, however, A has incurred a crisis, the Fund will pledge to every creditor of B $\lambda_R = \Lambda - \lambda_A$ (the subscript R stands for residual). Λ and λ_A are given parameters.

3.3 *Investors' Payoffs and Information*

The relevant characteristic of payoffs is that they depend on investors having chosen the behavior that, *ex-post*, reveals to be the "right" one. The difference in utility between debt-rollover and early withdrawal of funds will be a positive constant b, if the country is not forced to default. If, however, a crisis occurs, that same difference will be negative and equal to $-(c-\lambda_j)$, where c is a constant and $\lambda_j < c$. Therefore, the IMF reduces (but not eliminates) the negative payoff each investor will get if, after having chosen debt rollover, the country is hit by a crisis.

A peculiarity of global-game models, which distinguishes them from models with multiple equilibria derived from Diamond and Dybvig (1983), is the *lack of common knowledge*[7]. In our case, in order to decide whether to withdraw their capital from country j, investors get two kinds of information:

— public information on R_j. Investors know that R_j is normally distributed with average $R_{j,M}$ and variance $1/\rho$. They can not observe it directly, but receive about it a

— private information affected by the stochastic noise ε_j, with normal distribution and c.d.f. $G(\varepsilon_j)$: the signal received by

[7] This means that the agents' private information is heterogeneous and affected by some stochastic disturbance: players do not know others' behavior in equilibrium, neither do they have the same perception of the economy.

investors, at time 2 for country *A*, at time 5 for country *B*, is $s_j = R_j + \varepsilon_j$; the precision of private information is α_j.

We assume the distributions of R_j and the c.d.f. of stochastic noises $G(\varepsilon_j)$ to be the same in both the countries. Following CGR, we assume that $\rho/\alpha \to 0$, which allows us to ignore the interaction between the two types of information.

3.4 *Strategies and Threshold Values*

We assume investors to employ a trigger strategy to decide whether to withdraw their funds from country j: they will withdraw their capital if and only if the signal they receive about economy j should follow below (or be equal to) s_j^*, the threshold value:

$$s_j \leq s_j^*$$

The payoff each agent gets in case of debt-rollover, is a positive function of the number of other investors that choose not to withdraw their funds[8]. Therefore, in order to determine the threshold value, every agent will try to estimate the fraction of other creditors that will choose withdrawal. In this respect, the agents' decisions are *strategic complements*, generating coordination problems and raising the probability of a crisis.

Given the fraction of investors who decide to withdraw their capital after just one period, we can derive the threshold value of the fundamental, $\bar{R}_j$, defined as the rate of return below which j will be forced to default[9]:

$$(3) \qquad \bar{R}_j = \frac{D - M}{I} + k\frac{\left[x_j D - M\right]_+}{I} = R_s + k\frac{\left[x_j D - M\right]_+}{I}$$

[8] This happens because, by means of early withdrawing their financings, creditors cause costly liquidation of investment projects, which increase the probability of a default.

[9] See *Appendix B* for the derivation of this threshold value.

3.5 *The Equilibrium in Country* B

In equilibrium in each country the probability of a crisis is *endogenous* and *determined jointly by fundamentals and investors' coordination problems*. Each country is characterized by a Bayes-Nash equilibrium, described by the two threshold values $\bar{R}_j$ and s_j^*. Proceeding backwards, we will now examine the equilibrium in country *B*.

3.5.1 Threshold Values in Equilibrium

The equilibrium in country *B* is characterized by the threshold values s_B^* and $\bar{R}_B$ defined by the following equations

$$(4)\quad s_B^* = R_s\left\{1 + k\frac{G[-G^{-1}(b/b+c-\lambda_B)]\cdot D - M}{D-M}\right\} - G^{-1}\left(\frac{b}{b+c-\lambda_B}\right)$$

$$(5)\quad \bar{R}_B = R_s\left\{1 + k\frac{G[-G^{-1}(b/b+c-\lambda_B)]\cdot D - M}{D-M}\right\}$$

See *Appendix B* for the derivation of these values.

3.5.2 Size of Intervention and Probability of a Crisis

Following CGR we can prove the following

PROPOSITION 1. The *ex-ante* default probability in country *B* is a positive function of the threshold values $\bar{R}_B$ and s_B^*

$$(6)\quad \frac{\partial Pdf_B}{\partial \bar{R}_B} > 0 \quad \text{and} \quad \frac{\partial Pdf_B}{\partial s_B^*} > 0$$

Deriving *(6)* with respect to λ_B we can prove.

PROPOSITION 2. An increase in the amount of liquidity granted to each foreign investor in the event of a default, reduces the

threshold values $\bar{R}_B$ and s_B*; therefore, these are negative functions of the size of ILLR intervention[10].

$$(7) \qquad \frac{\partial \bar{R}_B}{\partial \lambda_B} < 0 \quad \text{and} \quad \frac{\partial s_B{}^*}{\partial \lambda_B} < 0$$

From *(6)* and *(7)* follows

PROPOSITION 3. An increase in the liquidity pledged to every investor in the event of a default, reduces the *ex-ante* probability of a crisis in country *B*.

$$(8) \qquad \frac{\partial Pdf_B}{\partial \lambda_B} < 0$$

The intervention of an ILLR which, following a crisis, provides the debtor country with resources that allow it to partially honor its debts, reduces the *ex-ante* probability of a crisis. This happens because investors know that the loss they will suffer in the event of a default, will be mitigated by the funds supplied by the IMF: the payoff creditors will get if they choose not to withdraw their capital and subsequently the emerging market is forced to default, will be less negative (see par. 3.3). The conditional payoff alteration influences the expected payoff: each creditor will be keener to rollover the debt for each level of the fundamental, as shown in *(7)*. The reduced speculative aggressiveness will lessen the fraction of creditors who choose to early withdraw their capital at time 5. Consequently, the projects' rate of return needed to avoid the default in B, will decrease (see *(7)*). The reduction in both threshold values will cause a fall in the *ex-ante* probability of a crisis (see *(6)*). In this model, the only effect of ILLR intervention on the probability of a crisis is an *indirect* one. Anticipating the IMF aid, in fact, agents alter their own behavior: what reduces the probability of a crisis is the modification in investors' conduct.

This result is very similar to the one achieved by CGR. Yet, in their model the IMF provides the country with resources before a crisis takes place, not following a default. The different timing of intervention lets the IMF reduce crisis probabilities not only

[10] Proofs of PROPOSITIONS 1 and 2 are shown in *Appendix B*.

via investors' expectations, but also, directly, by means of increasing country's liquid resources. Debtor will have more funds than to reimburse creditors who wish to withdraw their capital, and this will reduce the cost of early liquidations. CGR term this as the *direct* effect of IMF intervention on the probability of a crisis. Moreover, in the original model, the IMF is modeled as a player, just like international investors. Realistically, the authors assume that the Fund will decide to finance a country only if it expects its own intervention to avoid a crisis, so as to be able to recover, afterwards, the disbursed funds.

3.6 *Contagion*

Proceeding backwards, we will now examine what determines the size of IMF intervention in aid of *B*. Due to the assumptions we made in 3.2, the reimbursement promised to each creditor of *B* will be equal to

$$\lambda_B = \begin{cases} \lambda_R & \text{with probability } Pdf_A \\ \Lambda = \lambda_A + \lambda_R & \text{with probability } (1 - Pdf_A) \end{cases}$$

The average of λ_B will be

$$\bar{\lambda}_B = \lambda_R \cdot Pdf_A + (\lambda_A + \lambda_R) \cdot (1 - Pdf_A)$$

$$= \lambda_A + \lambda_R - \lambda_A \cdot Pdf_A \qquad (9)$$

From which it follows that

$$\frac{\partial \lambda_B}{\partial Pdf_A} < 0 \qquad (10)$$

From *(8)* and *(10)* follows

PROPOSITION 4. The limitation of IMF resources creates a positive correlation between the probabilities of a crisis in the two economies.

(11)
$$\frac{\partial Pdf_B}{\partial \underline{\lambda}_B} \cdot \frac{\partial \lambda_B}{\partial \underline{P}df_A}\bigg|_{\Lambda} > 0$$

The ILLR has a limited amount of liquidity. Hence, if one country is forced to default and receives IMF financings, the other one will have a smaller quantity of resources in the event of a crisis. Yet, as we showed in the previous section, in this model the *ex-ante* probability of a crisis is influenced by the size of IMF intervention. As a result, the limitation of Fund resources creates a form of contagion between the two economies: if a crisis strikes in *A*, the probability of a default in *B* will rise. Therefore, the limitation of IMF resources may turn it into a contagion channel.

3.7 *The Equilibrium in Country* A

Due to the hypotheses we made on the parameters in the two economies, the values of s_A^* and $\bar{R}_A$ that characterize *A*'s equilibrium are identical to the thresholds we derived for country *B* in 3.5 (see equations *(4)* and *(5)*). Following the same reasoning as in paragraph 3.5.2, we can prove that the effects of IMF intervention are the same in both economies, in particular

(12)
$$\frac{\partial Pdf_A}{\partial \lambda_A} < 0$$ [11]

3.8 *The Optimal IMF Action in the Presence of Contagion*

3.8.1 The Allocation of Funds

We will now introduce a new period in the game, anterior to the others, time 0. We assume that at time 0 the IMF knows the *ex-ante* probabilities of a crisis in both the countries and has

[11] Clearly, proofs are the same as in paragraph 3.5.2.

limited resources. It must decide how to commit its funds between the two economies in order to minimize the total *ex-ante* probabilities of a crisis. The IMF knows that in each economy an increase in the reimbursement pledged to every creditor, reduces the probability of a crisis (see *(8)* and *(12)*).

Moreover, given Λ,

$$\lambda_R = \Lambda - \lambda_A \tag{13}$$

If we assume that the burden of a crisis is the same in both the countries (which lets us normalize to 1 the cost of a default in both the economies), the problem faced by the IMF will consist in choosing the value of λ_A which

$$\min_{\{\lambda_A\}} : Pdf_{TOT} = Pdf_A(\underset{-}{\lambda_A}) + \overline{\overline{Pdf}}_B(\underset{-}{\lambda_B})$$

where: $\overline{\overline{Pdf}}_B = (1 - Pdf_A) \cdot Pdf_B(\Lambda) + Pdf_A \cdot Pdf_B(\Lambda - \lambda_A)$

In order to minimize total *ex-ante* crisis probabilities, IMF must take into account the effect on *B*'s probabilities of default, of a variation in the funds promised to *A*'s creditors.

By means of deriving $\overline{\overline{Pdf}}_B$ with respect to λ_A we get:

$$\frac{\partial \overline{\overline{Pdf}}_B}{\partial \lambda_A} = -\frac{\partial Pdf_A}{\partial \lambda_A} \cdot Pdf_B(\Lambda) + \frac{\partial Pdf_A}{\partial \lambda_A} \cdot Pdf_B(\Lambda - \lambda_A) + \left.\frac{\partial Pdf_B}{\partial \lambda_A}\right|_{\Lambda - \lambda_A} \cdot Pdf_A =$$

$$= \frac{\partial Pdf_A}{\partial \lambda_{-A}} \cdot \underbrace{[Pdf_B(\Lambda - \lambda_A) - Pdf_B(\Lambda)]}_{+} + \left.\frac{\partial Pdf_B}{\partial \lambda_{A_+}}\right|_{\Lambda - \lambda_A} \cdot Pdf_A \;\underset{-}{}\; 0 \Leftrightarrow$$

$$\Leftrightarrow -\frac{\partial Pdf_A}{\partial \lambda_A} \cdot \frac{\lambda_A}{Pdf_A} \leqslant \left.\frac{\partial Pdf_B}{\partial \lambda_A}\right|_{\Lambda - \lambda_A} \cdot \frac{\lambda_A}{Pdf_B(\Lambda - \lambda_A) - Pdf_B(\Lambda)} \tag{14}$$

An increase in λ_A has a twofold effect on $\overline{\overline{Pdf}}_B$. On the one

hand it implies a reduction in the resources the IMF chooses to commit directly to B ($(\Lambda - \lambda_A)\downarrow$); on the other, it causes a fall in the probability of a crisis in country A and, consequently, raises the chances that each creditor of B is provided with a reimbursement equal to Λ ($(1 - Pdf_A)\uparrow$). The sign of $\partial\overline{\overline{Pdf}}_B/\partial\lambda_A$ depends on which of these two effects is greater. Notice that, if *ex-ante* crisis probabilities in B are very small when its creditors are promised the whole volume of IMF funds, *(14)* can be approximated with

$$(15) \qquad -\frac{\partial Pdf_A}{\partial\lambda_A}\cdot\frac{\lambda_A}{Pdf_A} \gtrless \left.\frac{\partial Pdf_B}{\partial\lambda_A}\right|_{\Lambda-\lambda_A}\cdot\frac{\lambda_A}{Pdf_B(\Lambda-\lambda_A)}$$

The left hand side of *(15)* represents the elasticity of the probability of a crisis in country A with respect to the size of IMF intervention. The right hand side shows the reactivity of country B's crisis probabilities with respect to the volume of funds promised to the other country's creditors, evaluated in $\Lambda - \lambda_A$. If the LHS of *(15)* is bigger than the RHS, $\overline{\overline{Pdf}}_B$ will increase when λ_A rises, and *vice versa*. In order to clarify the discussion we make in the next paragraph, we define country A highly contagious if *(15)* holds with a major sign, moderately contagious otherwise.

The problem faced by the IMF has two different solutions, depending on $\partial\overline{\overline{Pdf}}_B/\partial\lambda_A$ to be bigger or smaller than zero.

CASE I

$$(16) \qquad -\frac{\partial Pdf_A}{\partial\lambda_A}\cdot\frac{\lambda_A}{Pdf_A} > \left.\frac{\partial Pdf_B}{\partial\lambda_A}\right|_{\Lambda-\lambda_A}\cdot\frac{\lambda_A}{Pdf_B(\Lambda-\lambda_A)} \Leftrightarrow \frac{\partial\overline{\overline{Pdf}}_B}{\partial\lambda_A} < 0$$

The strong contagion between the countries causes an increase in the resources destined to A to determine a reduction in the probability of a crisis in B.

ILLR's problem will be

$$\min_{\{\lambda_A\}} : Pdf_{TOT} = Pdf_A(\underset{-}{\lambda_A}) + \overline{\overline{Pdf}}_B(\underset{-}{\lambda_A})$$

Which delivers

$$\lambda_A^* = \max\{\lambda_A\} = \Lambda \tag{17}$$

If the probability of a crisis in *A* is very sensitive to changes in the reimbursement pledged to its creditors, the Fund will find it optimal to allocate the totality of its resources to this country.

CASE II

$$-\frac{\partial Pdf_A}{\partial \lambda_A} \cdot \frac{\lambda_A}{Pdf_A} < \left.\frac{\partial Pdf_B}{\partial \lambda_A}\right|_{\Lambda-\lambda_A} \cdot \frac{\lambda_A}{Pdf_B(\Lambda - \lambda_A)} \Leftrightarrow \frac{\partial \overline{\overline{Pdf}}_B}{\partial \lambda_A} > 0 \tag{18}$$

In this case, on average, crisis probabilities in *B* increase when the funds promised to *A*'s creditors rise. Yet, as we show in *Appendix B*, due to contagion, the IMF will find it optimal to commit more than half its resources to economy *A*. Indeed, by means of reducing crisis probabilities in this economy, the Fund decreases also, indirectly, the probability of a default in the infected country.

In this case the solution to the IMF problem will be

$$\lambda_A^* > 1/2 \cdot \Lambda \tag{19}$$

From *(17)* and *(19)* follows

PROPOSITION 8. In order to allocate its resources between the economies so as to minimize the total probabilities of a crisis, the IMF will decide to give a larger share to the economy from which contagion spreads.

$$\Lambda \geq \lambda_A^* > 1/2 \cdot \Lambda$$

Thus, the Fund should take into account this form of contagion it contributes in creating, when planning emerging markets' bailouts. In particular, the IMF should pre-commit more resources to countries that are more likely to be hit first by a crisis. These commitments should also be larger, the more

contagious is the considered country, as we show in the next paragraph. We must note that the differences in the treatment reserved to each economy do not stem from "political" distortions. They are indeed the result of a process of minimization of the total probabilities of a crisis, and thus constitute an optimal solution.

3.8.2 Contagiousness of Countries

Given the conclusions of our model, form a normative point of view, it is extremely important that the IMF be able to identify the economies from which contagion is more likely to spread. First, notice that, given the structure of the model, country *A* — the first to be exposed to the risk of a crisis — the one from which contagion arises. Hence, in general we can claim that the time order has certain relevance. It could be useful if in every particular circumstance the Fund could identify the countries that are more likely to be hit first by a crisis[12]. In this respect, the IMF could use its role of *monitorer* to look at several factors, such as external debt burden, political stability and production structure[13]. A constant monitoring of the condition of the economies and of the economic situation, could therefore allow the Fund to identify in every circumstance the countries that are more vulnerable to the risk of a crisis.

Moreover, based on the conclusions we derived in the previous paragraph, we can distinguish several degrees of contagiousness of the country from which the crisis spreads. If *(16)* holds, we can define economy *A* highly contagious, moderately contagious if *(18)* holds. Notice that, from the point of view of the country from which the crisis originates, for every level of the parameters

[12] Obviously, the model we are examining does not allow us to study analytically this problem, because, by construction, the time order of crises is exogenous.

[13] Usually the emerging-market economies that are very dependant on the export of one single commodity and do not diversify their production tend to be strongly exposed to external shocks in goods prices and exchange rates, and should therefore be considered quite risky.

relative to the infected country, the higher the reactivity to the IMF intervention of *A*'s crisis probabilities, the more contagious this economy is. If *A*'s elasticity is bigger than the reactivity of economy *B* evaluated in λ_R, crises probabilities in the two countries will move in the same direction when the size of the resources devoted to *A*'s creditors varies. *Vice versa*, if *A* is moderately contagious, they will move in opposite directions when λ_A varies. Hence, there are other elements worth to be taken into account by the IMF, namely economies' fragility and the responsiveness of their crisis probabilities to bailouts and external interventions. Also in this case, a deep analysis of the previous crisis episodes and their resolution, plus the structure and internal characteristics of each country, may help in this identification.

3.8.3 Time Inconsistency of the Interventions

The problem of optimal allocation of funds aimed at the minimization of total crisis probabilities, which we examined in paragraph 3.8.1, can become more complex and tricky, if the IMF cannot credibly commit itself to the actual disbursement of the resources promised to creditors.

To illustrate this point, we assume that the Fund at time 0 solves the problem described in paragraph 3.8.1. Furthermore, we hypothize that in period 3, after having observed whether country A was hit by a crisis, the IMF could change its decisions about the funds to be paid out to *A*'s creditors. Of course, the Fund's goal is again the minimization of the total crisis probabilities in the two economies. When revising the allocation of resources, the IMF can consider as given the probabilities of a crisis in economy *A*[14]. Naturally, in this case the Fund will choose to change its previous decisions and will devolve the whole amount of its resources to economy *B*. Hence, *A*'s creditors will *not* receive the

[14] Notice that the results highlighted in this section remain valid even in the event that the IMF can change its choices at time 2, after *A*'s creditors decided whether to withdraw their funds. Also in this case the Fund can consider the probabilities of a crisis in *A*, invariant with respect to λ_A.

funds they were promised at time 0. Therefore, there exists a problem of time inconsistency of the resources promised by the Fund to the two economies. Notice that this result does not stem from any kind of opportunistic behavior of the IMF, but only from an optimization process aimed at ensuring the efficient allocation of resources.

Obviously, time inconsistency constitutes an obstacle to the effectiveness of IMF interventions. If *A*'s creditors know that the Fund has an *ex-post* incentive to change its decisions, they will consider uncertain the volume of resources they were promised. Accordingly to they own confidence in the credibility of IMF promises, the effectiveness of the intervention on *A*'s crisis probabilities may dramatically decrease and even plunge to zero. Thus, in order to achieve the maximum effectiveness of the intervention is essential that the Fund credibly commit itself to the provision of the resources promised to each country.

3.8.4 An Example of Two-way Contagion

The model we developed in the previous paragraph can be extended to describe a situation in which the *ex-ante* crisis probabilities of the two countries influence each other. The aim of this section is to show that when dealing with this kind of contagion, the allocation of the Fund's resources grows more complex, because contagion may originate from both economies.

Assume that *A*'s creditors expect to receive some additional kind of help with respect to the official commitment the Fund makes in $t = 1$, if the IMF has enough resources left after having possibly bailed out the other country. Referring to the timing described in paragraph 3.1, this is equivalent to assuming that *A*'s creditors, in $t = 1$, believe that, if country *B* is *not* hit by a crisis, the IMF will use the remaining resources at time 6, $\Lambda - \lambda_A$, to additionally refund them of the loss suffered in the event of a default. We specify that, given the current mechanisms of intervention of the IMF, it is extremely unlikely that the Fund itself could promise this kind of help. Indeed, once a certain amount

of resources has been allocated to an economy, it is cancelled from the uncommitted usable resources, and cannot be assigned to any other country. Yet, it is sensible to think that if country A is particularly important from a political or strategic point of view, its creditor will expect a privileged treatment. Thus, they may think they will be refunded of the loss suffered in a crisis more than the Fund officially announced at time 1, if the IMF has enough resources to do it[15].

From the analysis of crisis probabilities carried out in *Appendix B*, we get

$$\frac{\partial Pdf_B}{\partial \lambda_B} \cdot \frac{\partial \lambda_B}{\partial Pdf_A}\bigg|_{\Lambda} > 0 \quad \text{and} \quad \frac{\partial Pdf_A}{\partial \bar{\bar{\lambda}}_A} \cdot \frac{\partial \bar{\bar{\lambda}}_A}{\partial Pdf_B}\bigg|_{\Lambda} > 0$$

were $\bar{\bar{\lambda}}_A = Pdf_B \cdot \lambda_A + (1 - Pdf_B) \cdot \Lambda$ is the amount of resources that *A*'s creditors expect to receive from the Fund.[16]. Thus, the probability of a crisis in *B* are influenced from country *A*'s ones and *vice versa*.

In *Appendix B* we solve the problem of the IMF. From it we get two relevant results. First, the probabilities of a crisis in *A* do not necessarily decrease when the amount of resources officially committed to this country by the Fund increases. This happens because an increase in λ_A causes the resources formally promised to A to raise, but also an increase in *B*'s crisis probabilities. The total effect on $\bar{\bar{\lambda}}_A$, and hence on Pdf_A, is uncertain. As we show in the appendix, if

$$\frac{\partial Pdf_B}{\partial \lambda_A} \cdot \frac{\lambda_A}{Pdf_B} > \frac{\lambda_A}{(\Lambda - \lambda_A)} \tag{20}$$

when λ_A increases, $\bar{\bar{\lambda}}_A$ diminishes, rising *ex-ante* *A*'s crisis

[15] Notice that, in order to generate two-way contagion, it is not necessary that the Fund actually pay out these expected extra resources. Yet, the IMF may really find it optimal to provide this additional help if it can, in order, for instance, to restore investors' confidence in a particularly important country, so as to prevent possible future crises.

[16] We are assuming that *A*'s creditors do not discount the liquidity received at time 3 with respect to the one gotten at time 6.

probabilities. The right hand side of equation *(20)* represents the share of resources officially promised to *A* on the additional liquidity that those who invested in this country expect to receive at time 6 if *B* is not forced to default. Hence, if the probabilities of a crisis in *B* increase a lot when λ_A increases, or if the volume of extra funds expected by *A*'s creditors is big, $\partial Pdf_A / \partial \lambda_A > 0$. In this case, we can consider country *B* highly contagious.

The second remarkable result concerns the optimal allocation of the Fund's resources. If $\partial Pdf_A / \partial \lambda_A < 0$, the optimal choice is identical to the one described in 3.8.1. If instead $\partial Pdf_A / \partial \lambda_A > 0$, due to the high contagiousness of country *B*, the IMF will decide to provide it with its own whole resources. Therefore, it is essential that, in situations that can give rise to two-way contagion — the one we described in this paragraph is just an example — the Fund carefully evaluate the potential contagiousness of all the economies so as to plan its own interventions.

4. - Concluding Observations

The financial crises that hit many emerging markets in the last years, forced politicians and economists to cope with the need of reforming the international financial architecture. Though analysts do not agree on the solution to be adopted, many suggest that the IMF should play an important role in preventing crises, particularly acting as an International Lender of Last Resort. Yet, in the last ten years the magnitude of financial crises showed the Fund resources to be limited if compared with the amounts of liquidity necessary to manage these episodes. In this essay, we used for the first time a theoretical model to examine the possibility that the IMF, due to the limitation of its resources, may turn into a contagion channel. As showed in section 2, we should not underestimate this risk, given the resources the IMF can use to provide loans, nor can we rule out such a situation to have taken place in fact in the late Nineties.

In order to analyze this threat and its implications for the conduct of the Fund, we used a modified version of CGR global-

game model (2003, 2004). Compared with the original model, ours is extended to cover two countries, and shows significant changes in the timing structure. As in the original model, we obtain that a partial IMF intervention is effective in reducing the *ex-ante* probability of a crisis. In particular, we proved that an increase in the reimbursement pledged to each creditor in the event of a crisis reduces the probability of a default.

The changes made to the original model let us achieve an important result that was not possible to derive from CGR: IMF limited resources create a positive correlation between the probabilities of a crisis in the two economies. Therefore, our work shows that the Fund, due to the limitation of its means, may develop into a contagion channel.

Moreover, the IMF should take into account this form of contagion in allocating its resources between the economies. In particular, we proved that it should allocate more resources to the country from which a crisis is more likely to spread in order to minimize the total probabilities of a default.

It is clear that the model presented here can be further developed and may be used as a starting point for empirical analyses aiming at evaluating the relevance that in reality had this kind of contagion. To this purpose, an interesting research field could be the one of the late-Nineties financial crises. In particular, it could be important to evaluate to what extent investors' fear about the scarcity of IMF resources following the Asian crisis, could have contributed to trigger the subsequent Brazilian and Russian defaults. The results of this analysis could determine the actual relevance for the diffusion of the crises of the limitation of IMF resources, a contagion channel that we proved to be plausible from a theoretical point of view.

APPENDIX A

1. - The Fund's Resources: Quotas and Borrowing Arrangements

The largest share of IMF resources is made up of the quotas that member countries pay when joining the organization. The last general modification was decided during the Eleventh review, in 1998, and consisted in a 45% increase in quotas, which now amount to $331 billion. Yet, the aggregate describing the volume of liquidity deriving from quotas, which is effectively available to supply new loans are the *uncommitted usable resources*[17]. *Uncommitted usable resources* are fairly smaller than the total quotas paid by members: $143 billion against quotas' $331 bil.

In order to increase its liquidity resources, the Fund signed two borrowing arrangements with official creditors, the *General Arrangements to Borrow* (GAB) and the *New Arrangements to Borrow* (NAB).

To the purpose of the current discussion, it is important to notice that GAB was activated for the last time in 1998 to finance the Russian bailout, while NAB, designed in 1995 to address the huge financing needs requested by the new crises, such as the Mexican one, was activated only one time, in 1998, to cope with Brazil's Balance of Payments disequilibria.

The funds borrowed through GAB and NAB constitute a liability for the IMF, and the interest rate applied to them is above market-average. So in reality these borrowing arrangements tend not to be activated, with the little exception of extremely distressed periods. Moreover, the Fund may have to cope with reputational problems that, in fact, prevent or strongly discourage GAB and NAB activation. For instance, the more the IMF is exposed with

[17] See next paragraph for a detailed description of this and other aggregates representing the resources of the Fund.

its loans, the less global taxpayers — which ultimately constitute its financers — will be keen on accepting the disbursement of new funds. Consequently, borrowing arrangements will be seldom activated. Therefore, in reality, their existence is not sufficient to question the essential limitation of IMF resources.

2. - IMF Resources: The Aggregates

Uncommitted usable resources: they include the national currencies of member countries with a sound BoP balance, the SDR[18] held by the IMF, and, if the GAB or NAB have been activated, the funds available through these arrangements. They do not include *undrawn balances under arrangements*, the funds already committed but not disbursed, yet. In order to gauge the capacity to provide new loans, the Fund account sheets use also another aggregate named *one-year forward commitment capacity*, equal to the *uncommitted usable resources* plus the reimbursements the IMF expects to get in a one-year time. Given that the calculus of this second quantity is based upon forecasting, not upon established data, we preferred to use the *uncommitted usable resources*.

As concerns this aggregate, there are not monthly data available for the periods January-March and May-September 1997, and January-March and May-September 1998. The values relating to this periods have been approximated with the medium of the extremes of the intervals.

Total committed resources: they include the funds committed (*amount agreed*) to the most important countries that were hit by a crisis between 1997 and 2004, countries that were chosen on the basis of the size of their rescue packages and of the magnitude of their crises. Therefore, the aggregate refers only to some economies[19] and constitutes merely a rough measure of the

[18] SDR is an international asset used almost exclusively in the transactions between the IMF and member countries. It is exchangeable for the currencies of countries with a sound BoP situation.

[19] In particular, to Argentina, Brazil, Korea, Indonesia, Philippines, Russia, Thailand, Turkey, Ukraine and Uruguay.

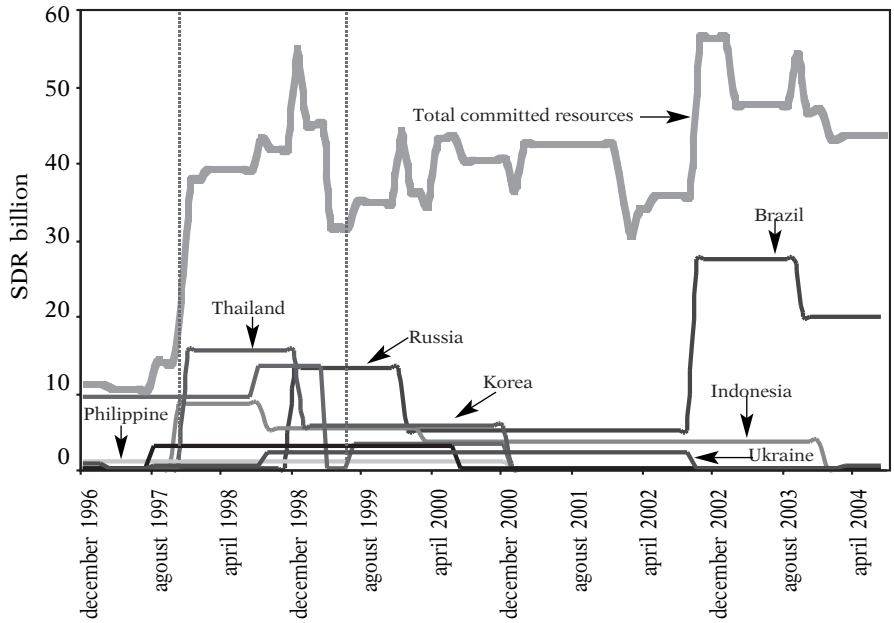
resources the IMF totally provided to its members during this period.

Please note that the *amount agreed* refers to the resources the Fund consents to commit to crisis-hit countries within specific credit agreements. For several reasons, it does not always match the amount of liquidity that is actually disbursed.

The two series of data are on a monthly base; the source of the both of them is the IMF web site. For a more in-depth description of these and other aggregates portraying the Fund resources' situation, see the accounting documents named *Financial Statement* and *IMF's Financial Resources and Liquidity Position*, available at *http://www.imf.org/external/fin.htm*.

GRAPH 2

COMMITTED RESOURCES: THE 1997-1999 CRISES



APPENDIX B

1. - Threshold Value for the Fundamental

For each country j, we define $\bar{R}_j$, the value of the rate of return such that, given x_j (the fraction of investors who decide to withdraw their capital after just one period) if $R_j > \bar{R}_j$, the country is not forced to default. From *(2)* we know that $\bar{R}_j$ must be such that

$$\bar{R}_j(1-z)I = \bar{R}_j I - (1+k)[x_j D - M]_+ = (1-x_j)D$$

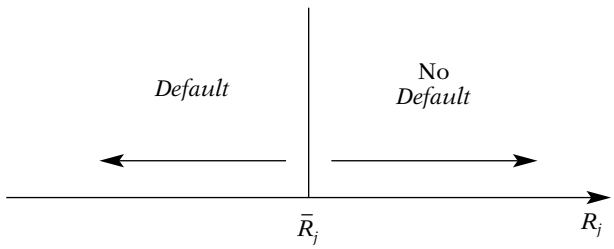
The rate of return below which j will be forced to default is

$$\bar{R}_j = \frac{D-M}{I} + k\frac{\left[x_j D - M\right]_+}{I} = R_s + k\frac{\left[x_j D - M\right]_+}{I}$$

where R_s is the value of R_j below which j incurs a crisis even if no investor withdraws his funds[20]. Therefore, given x_j, the distribution of R_j is partitioned into two areas, as Graph 3. Thus, the fundamental must be bad enough for the investors to trigger a crisis.

GRAPH 3

THE DISTRIBUTION OF $\bar{R}_j$ GIVEN x_j



[20] Through withdrawal, creditors cause costly liquidations of investment projects, which increase default probabilities.

2. - The Equilibrium in Country B: Derivation of the Threshold Values

I. - *Threshold Value for the Fundamental*

In equilibrium each creditor will try to estimate the fraction of other investors whom will receive a signal $s_B \leq s_B^*$ and will consequently decide to withdraw their capitals. Through the accumulation function of ε_B, agents will estimate the probability that, if $R_B = \bar{R}_B$, investors will receive a signal smaller than s_B^*. The fraction of investors who will choose to withdraw the financings at time 5, when $R_B = \bar{R}_B$, will be

$$x_B = \text{prob}(s_B \leq s_B^* | \bar{R}_B) \equiv G(s_B^* - \bar{R}_B) \tag{21}$$

Substituting *(21)* in *(3)* we get the first equilibrium condition

$$\bar{R}_B = R_s \left\{ 1 + k \frac{[G(s_B^* - \bar{R}_B) \cdot D - M]}{D - M} \right\} \tag{22}$$

II. - *Threshold Value for the Private Signal*

Given the conditional payoffs b and c, and given λ_B, the expected payoff of each investor is

$$w = b[1 - G(\bar{R}_B - s_B)] - (c - \lambda_B) \cdot G(\bar{R}_B - s_B) \tag{23}$$

where $G(\bar{R}_B - s_B)$ is the probability, estimated through the c.d.f. of ε_B, that, when an investor gets a signal equal to s_B, R_B is inferior to $\bar{R}_B$. Each creditor finds it optimal not to withdraw his capital up to the point in which his expected payoff is equal to zero. Equating *(23)* to zero we obtain the second equilibrium condition

$$G(\bar{R}_B - s_B^*) = \frac{b}{b + c - \lambda_B} \tag{24}$$

III. - *Equilibrium*

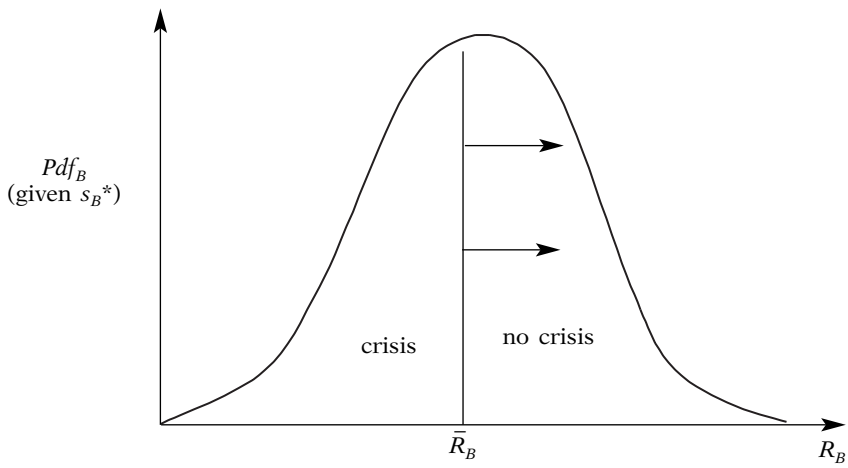
Solving the system formed by *(22)* and *(23)* we get the equations *(4)* and *(5)* presented in the text, that is to say the threshold values that characterize the equilibrium in country *B*.

3. - Proposition 1: Proof

We know that the investment projects' rate of return is normally distributed with mean R_M and variance $1/\rho$. For the graphic proof of Proposition 1, see Graphs 4 and 5, in which is shown the effect on the area of probability of a crisis, respectively of an increase in $\bar{R}_B$ given s_B^* and of a rise in s_B^*, taken $\bar{R}_B$ as a constant.

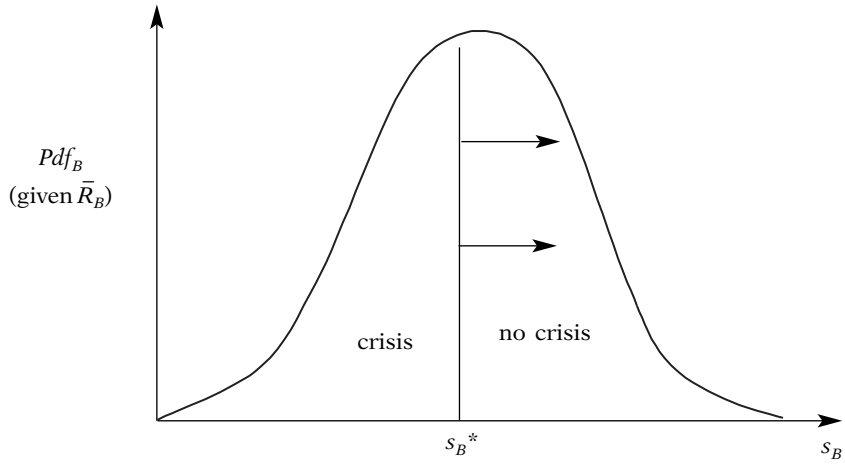
GRAPH 4

PROBABILITY OF DEFAULT AS A FUNCTION OF R_B



GRAPH 5

PROBABILITY OF DEFAULT AS A FUNCTION OF s_B



4. - Proposition 2: Proof

In order to proof Proposition 2 we must derive *(4)* and *(5)* with respect to λ_B

$$\frac{\partial \bar{R}_B}{\partial \lambda_B} = \left(\frac{R_S \cdot k \cdot D}{D - M}\right) \cdot g\left[-G^{-1}\left(\frac{b}{b + c - \lambda_B}\right)\right] \cdot (-1) \cdot g^{-1}\left(\frac{b}{b + c - \lambda_B}\right) \cdot$$

$$\cdot \frac{b}{(b + c - \lambda_B)^2} < 0$$

$$\frac{\partial s_B{}^*}{\partial \lambda_B} = \left(\frac{R_S \cdot k \cdot D}{D - M}\right) \cdot g\left[-G^{-1}\left(\frac{b}{b + c - \lambda B}\right)\right] \cdot (-1) \cdot g^{-1}\left(\frac{b}{b + c - \lambda_B}\right) \cdot$$

$$\frac{b}{(b + c - \lambda_B)^2} - g^{-1}\left(\frac{b}{b + c - \lambda_B}\right) \cdot \frac{b}{b + c - \lambda_B} < 0$$

5. - Proposition 8: Proof

We want to show that if

$$\frac{\partial \overline{\overline{Pdf}}_B}{\partial \lambda_A} > 0$$

the solution to the IMF problem is $\lambda_A^* > 1/2 \cdot \Lambda$.

The f.o.c. derived from the Fund minimization problem is:

$$\frac{\partial Pdf_A}{\partial \lambda_A}[1 - Pdf_B(\Lambda) + Pdf_B(\Lambda - \lambda_A)] - \left.\frac{\partial Pdf_B}{\partial \lambda_A}\right|_{\Lambda - \lambda_A} \cdot Pdf_A = 0$$

Assume that $\lambda_A^* = \Lambda/2$.

Evaluated in this value, the f.o.c. becomes:

$$\frac{\partial Pdf_A}{\partial \lambda_A}(\Lambda/2) \cdot [1 - Pdf_B(\Lambda) + Pdf_B(\Lambda/2)] - \frac{\partial Pdf_B}{\partial \lambda_A}(\Lambda/2) \cdot Pdf_A(\Lambda/2) \Leftrightarrow$$

$$\frac{\partial Pdf_A}{\partial \lambda_A} \cdot [1 - Pdf_B(\Lambda) + Pdf_B(\Lambda/2) - Pdf_A(\Lambda/2)] \Leftrightarrow$$

$$\underbrace{\frac{\partial Pdf_A}{\partial \lambda_A}(\Lambda/2)}_{-} \cdot \underbrace{\left[1 - Pdf_B(\Lambda)\right]}_{+} < 0$$

The second line stem from the fact that the crisis probabilities of the two countries, due to the assumptions we made, are equal *ex-ante*. Hence,

$$\left|\frac{\partial Pdf_A}{\partial \lambda_A}(\Lambda/2)\right| = \left|\frac{\partial Pdf_B}{\partial \lambda_A}(\Lambda/2)\right|$$

To derive the third line, it is sufficient to notice, following an analogue reasoning that $Pdf_A(\Lambda/2) = Pdf_B(\Lambda/2)$.

Hence, in $\lambda_A^* = \Lambda/2$, the derivative of the objective function with respect to λ_A is less than zero. Taking into account that the IMF is solving a minimization problem, we can conclude that for the first order derivative to be equal to zero, it is necessary that $\lambda_A^* > 1/2 \cdot \Lambda$.

6. - Two-way Contagion

Following the same reasoning carried on in paragraph 3.5.2, noticing that now *A*'s crisis probabilities depend on the amount of liquidity that its creditors expect to receive, and assuming they are not aware of the contagion generated by *A*, we get $Pdf_B(\underline{\lambda}_B)$ and $Pdf_A(\underline{\bar{\lambda}}_A)$, where $\bar{\bar{\lambda}}_A = (1-Pdf_B) \cdot \Lambda + Pdf_B \cdot \lambda_A$. From this last expression, we may derive $\partial\bar{\bar{\lambda}}_A / \partial Pdf_B < 0$. The funds received on average by *B*'s creditors are $\bar{\bar{\lambda}}_B = (1-Pdf_A) \cdot \Lambda + Pdf_A \cdot \lambda_R$, from which we get $\partial\lambda_B / \partial Pdf_A < 0$. Hence,

$$\frac{\partial Pdf_B}{\partial \lambda_B} \cdot \left.\frac{\partial \lambda_B}{\partial Pdf_A}\right|_{\Lambda} > 0 \quad \text{and} \quad \frac{\partial Pdf_A}{\partial \bar{\bar{\lambda}}_A} \cdot \left.\frac{\partial \bar{\bar{\lambda}}_A}{\partial Pdf_B}\right|_{\Lambda} > 0$$

The IMF solves the following problem:

$$\min_{\{\lambda_A\}} \cdot Pdf_{TOT} = Pdf_A(\underline{\bar{\bar{\lambda}}}_A) + \overline{\overline{Pdf}}_B(\underline{\lambda}_B)$$

where: $\overline{\overline{Pdf}}_B = (1 - Pdf_A) \cdot Pdf_B(\Lambda) + Pdf_A \cdot Pdf_B(\Lambda - \lambda_A)$

Let's analyze the effect of a variation in λ_A on the amount of funds that *A*'s creditors expect to receive.

$$\frac{\partial \bar{\bar{\lambda}}_A}{\partial \lambda_A} \gtreqless 0 \Leftrightarrow -\Lambda \cdot \frac{\partial Pdf_B}{\partial \lambda_A} + \frac{\partial Pdf_B}{\partial \lambda_A} \cdot \lambda_A + Pdf_B \gtreqless 0 \Leftrightarrow$$

$$\frac{\partial Pdf_B}{\partial \lambda_A} \cdot \frac{\lambda_A}{Pdf_B} \lessgtr \frac{\lambda_A}{(\Lambda - \lambda_A)}$$

If $$\frac{\partial \bar{\bar{\lambda}}_A}{\partial \lambda_A} > 0$$

then $$\frac{\partial Pdf_A}{\partial \lambda_A} < 0$$

In this case, the solution to the minimization problem is identical to that derived in 3.8.1: $\Lambda > \lambda_A^* > 1/2 \cdot \Lambda$.

If, instead: $$\frac{\partial \bar{\bar{\lambda}}_A}{\partial \lambda_A} < 0$$

then $$\frac{\partial Pdf_A}{\partial \lambda_A} > 0$$

From this follows that

$$\frac{\partial \overline{\overline{Pdf}}_B}{\partial \lambda_A} > 0$$ [21]

Since the *ex-ante* crisis probabilities in both the economies are decreasing functions of λ_A, the solution to the IMF optimization problem is $\lambda_A^* = 0$.

[21] See paragraph 3.8.1 for this derivation.

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