Redistribution and Optimal Monetary Policy: Results and Open Questions

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What are the properties of optimal fiscal and monetary policies with heterogeneous agents? This is a pressing question, given the wealth of evidence on heterogeneity in cash holdings and labor income. Yet, until recently it remained largely unexplored. In this paper, I show that with heterogeneity the Friedman rule is optimal only if positive nominal interest rates do not ameliorate constraints on redistribution. With an empirically plausible cross-sectional correlation between money holdings and labor income, the Friedman rule is optimal if the government favors redistribution to the poor. I discuss these findings and propose several directions for future research. [JEL Classification: E52, E61, E63, H21].

Quali sono le proprietà delle politiche monetarie e fiscali ottime quando l'eterogeneità introduce un conflitto tra redistribuzione ed efficienza? Si tratta di una domanda pressante, dato il monte di evidenza sulla disuguaglianza nei redditi e nella ricchezza nella popolazione. Eppure, pochi studi se ne sono sinora occupati. Questo saggio dimostra che la Friedman rule*, che prevede l'azzeramento dei tassi di interesse nominali, è ottima solo quando un aumento dei tassi di interesse non influenza i vincoli sulle politiche redistributive. Con ipotesi realistiche sulla correlazione tra reddito individuale e domanda di moneta, la* Friedman rule *è ottima solo se il governo desidera redistribuire in favore dei poveri.*

1. - Introduction

The optimality of the Friedman rule is one of the main results in the literature on optimal government policies with commitment. Friedman (1969) argument is that any positive value of the

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nominal interest rate effectively amounts to a distorting tax on cash holdings. Thus, equalizing the return on money to that of other assets minimizes distortions and improves efficiency. This logic presumes that the government has access to other less distortionary sources of revenues, but this need not be true in general. Phelps (1973) argues that in a public finance model in which the government sets distortionary taxes on commodities and labor income, it may well be optimal to tax cash holdings if the interest elasticity of money demand is lower than the price elasticity of other commodities in absolute value.

Chari, Christiano and Kehoe (1996) adopt Phelp's public finance approach and only allow for distorting taxes. They show that Friedman rule is optimal for a class of preferences that is broadly used in macroeconomic models. Essentially, the requirement is that income elasticity of money demand is equal or greater than one, and that preferences are weakly separable between consumption and labor. Their argument is an application of the inverse elasticity rule of commodity taxation. If the income elasticity on money demand is greater than one, it is optimal set the tax on cash holdings lower than on other commodities. This is impossible given the zero lower bound on nominal interest rates, which implies that the Friedman rule holds for a large class of preferences.

The optimality of the Friedman rule is robust. For one, the required restrictions on preferences are those that guarantee consistency with balanced growth (Alvarez, Kehoe and Neumeyer, 2004), and when these restrictions are not satisfied, the optimal nominal interest rate is very close to zero (Correia and Teles, 1999). The Friedman rule fails to hold when the tax system is incomplete. For example, a departure from the Friedman rule might be optimal as a way to tax monopoly profits when a corporate income tax is not available for this purpose (Schmidt-Grohe and Uribe, 2004*a*). Similarly, the Friedman rule, which requires that inflation is equal to the negative of the real interest rate, generates relative price distortions in economies with sticky prices and thus may fail to hold (Schmidt-Grohe and Uribe, 2004*b*). However, if the relative price distortions can be removed with a full set of commodity taxes or subsidies, the Friedman rule is reinstated.

What happens when redistributional considerations are introduced? This is a pressing question, given the wealth of evidence on heterogeneity in transaction patterns, asset holdings and labor income. Recent results suggest that the trade-off between redistribution and efficiency generically undermines the optimality of the Friedman rule. In this paper, I study the optimal fiscal and monetary policy with heterogenous agents and show that the Friedman rule is optimal only if positive nominal interest rates do not ameliorate constraints on redistribution *via* labor taxes. In particular, for economies that exhibit empirically plausible cross-sectional correlations between money holdings and labor income, the Friedman rule is optimal only if the government wishes to redistribute in favor of the poor.

I begin by analyzing a simple one period example that clearly illustrates how restrictions on redistribution stemming from the structure of labor taxes or incentive compatibility constraints generate departures from the Friedman rule. I then study an economy where heterogeneity in transaction patterns arises endogenously from differences in labor productivity. Low skill agents hold more cash as a fraction of total purchases, consistent with cross-sectional evidence in the US (Mulligan and Sala-i-Martin, 2000, Erosa and Ventura, 2002) and other countries (Guiso, Haliassos and Jappelli, 2001). The government taxes labor income at a proportional rate and sets the nominal interest rate. The model simplifies to the one considered in Chari, Christiano and Kehoe (1996) once heterogeneity is removed. The Friedman rule turns out to be optimal only if the Pareto weight on low skill agents is high enough. This result is based on a simple logic. The linearity of the labor income tax constrains the government's ability to redistribute and inflation thus assumes an auxiliary role. The labor income tax weighs more heavily on high skill agents, whereas inflation is a greater burden for low skilled agents. Thus, if the government favors high skill agents, a departure from the Friedman rule is optimal. If the government favors low skill agents, it is optimal to subsidize cash holdings. Since the zero lower bound constraint on the nominal interest rate prevents this, the Friedman rule is the best policy if the Pareto weight on low skill agents is high enough.

da Costa and Werning (2007) also study an economy where agents differ in labor productivities and the government sets labor income taxes and the nominal interest rate. They allow labor income taxes to be non-linear and assume that individual labor productivities are private information, following Mirrlees (1971). The government can use individual specific lump sum taxes to achieve any amount of redistribution. However, private information on individual abilities introduces incentive compatibility constraints. In particular, low skill agents' labor supply must be distorted to prevent high skill agents from mimicking those with low skills. The incentive compatibility constraints limits redistribution and the nominal interest rate could play an auxiliary role if it relaxes these constraints. da Costa and Werning assume that money and labor effort are gross complements and show that the Friedman rule is optimal. Under this condition, the demand for money rises with labor effort, for given consumption. Then, a reduction in the nominal interest rate relaxes incentive compatibility constraints, since an increase in the demand for money also increases labor effort. Again, given the zero lower bound on the nominal interest rate, the best policy to ameliorate the incentive problem to follow the Friedman rule.

While these results appear *prima facie* inconsistent, they can be interpreted as an application of the uniform commodity taxation principle. Atkinson and Stiglitz (1976) show that, if the labor income tax schedule is sufficiently unconstrained or the income elasticity does not vary across goods, all commodities should be taxed at the same rate, irrespective of agents' weight in the social welfare function. A proportional labor tax generates a conflict between efficiency and redistribution that induces the government to abandon uniform commodity taxation if redistributional goals are present and income elasticities vary across goods. Similarly, with non-linear labor income taxes and private information on individual labor productivities, differential commodity taxation may be optimal as a screening device. In a monetary economy, uniform commodity taxation translates into optimality of the Friedman rule. Moreover, the zero lower bound constraint the nominal interest rate implies that the Friedman rule will also hold in all those cases that would call for a lower tax rate on cash purchases relative to other consumption goods in a real economy.

These results confirm the connection between optimality of the Friedman rule in monetary economies and optimality of uniform commodity taxation in real economies, established by Charm, Christiano and Kehoe (1996) for representative agent models. They also establish a general principle for heterogeneous agent economies. For models that display an empirically plausible correlation between cash holdings and labor income, the Friedman rule is optimal only if the government wishes to distribute in favor of the poor. I prove this result under proportional labor income taxes in this paper. The analysis in da Costa and Werning (2007) suggests that this principle also holds with non-linear labor taxes. Under utilitarian social welfare, they prove that the Friedman rule is optimal when cash holdings and labor income are gross complements. This restriction on preferences implies a positive correlation between cash holdings and labor income, which is inconsistent with the empirical evidence. If cash holdings and labor income are gross substitutes, a departure from the Friedman rule would relax incentive compatibility constraints and improve efficiency. However, the Friedman rule may still be optimal if the government values distribution towards low productivity agents enough.

The paper proceeds as follows. Section 1 analyzes a simple one period economy and illustrates the conditions for the optimality of the Friedman rule in relation to the constraints on redistribution. Section 2 draws a connection between these findings and the uniform commodity taxation principle, as well as the optimality of the Friedman rule in representative agent economies. It also discusses the empirical evidence on transaction patterns and asset holdings and relates it to the theoretical findings. Section 3 studies optimal policies in a dynamic economy with endogenous heterogeneity in transaction patterns. Section 4 discusses the time consistency of optimal policies with heterogeneous agents. Section 5 discusses the effect of aggregate and idiosyncratic shocks, and Section 6 concludes by pointing to numerous open questions for future work.

2. - Redistribution and the Friedman Rule: A Simple Example

To illustrate the forces shaping the optimal setting of monetary and fiscal policies in economies with heterogeneous agents, I begin by describing a simple one period economy. A subset of commodities, cash goods, are purchased with currency, while the others, credit goods, are not subject to this requirement. The distinction between cash and credit goods is built into preferences, as in Lucas and Stokey (1983). The nominal interest rate, R, simply corresponds to the relative price of cash goods. Agents supply labor and differ in labor productivities. Letting $T(l)$ denote the tax on labor income, l, government policy is given by $\{R, T(l)\}$. In the first example, $T(l)$ is linear in l. In the second example, labor taxes are non-linear and individual productivities are private information. In both examples, the optimal nominal interest rate depends on the interaction between the demand for cash goods and labor supply.

2.1 *Linear Income Taxation*

The economy is populated by a continuum of agents who differ in labor productivity, ξ. Half of the agents have productivity, ξ^1, and the other half, ξ^2, with $\xi^2 > \xi^1 > 0$. Their preferences are given by:

$$U(c_1, c_2, l/\xi^i) = u_1, (c_1) + u_2(c_2) - l/\xi^i \tag{1}$$

where c_1 and c_2 denote cash and credit goods respectively. Here $u_i(\cdot)$ for $i = 1, 2$ are strictly increasing and strictly concave functions, defined on the positive reals.

Agents choose $\{c_1^i,\ c_2^i,\ l^i\}_{i=1,2}$ to maximize *(1)* taking as given the nominal interest rate and the tax on labor income, $\{R,\ T\ (l)\}$. Their budget constraint is:

$$Rc_1^i + c_2^i \leq l^i - T(l^i) \tag{2}$$

Following the tradition of the Ramsey taxation literature, I assume that the labor income tax is affine, so that $T\ (l) = -\ \bar{\tau} + \tau l$. Here, $\bar{\tau} > 0$ corresponds to a lump sum transfer and τ is the proportional rate on labor income. The solution to an agent's problem is then characterized by the following three equations:

$$\frac{u_1'(c_1^i)}{u_1'(c_2^i)} = R \tag{3}$$

$$\xi_i u_2'(c_2^i) = \frac{1}{(1-\tau)} \tag{4}$$

$$Rc_1^i + c_2^i = l^i(1-\tau) + \bar{\tau} \tag{5}$$

The resource constraint is:

$$0.5\sum_{i=1,2}(c_1^i + c_2^i - l^i) + g \leq 0 \tag{6}$$

where $g > 0$ is exogenous government consumption. The government budget constraint is:

$$g \leq 0.5\sum_{i=1,2} T\ (l^i) \tag{7}$$

Conditions *(3)-(7)* for $i = 1,\ 2$ characterize an equilibrium for this economy and represent a mapping between government policy, $\{R,\ \tau,\ \bar{\tau}\}$ with $R \geq 1$, and equilibrium allocations. The constraint on R corresponds to the zero lower bound constraint on nominal interest rates in monetary economies. The Ramsey equilibrium is simply the best equilibrium from the standpoint of the social welfare function. Let social welfare be given by:

$$\sum_{i=1,2} \eta^i U(c_1^i, c_2^i, l^i / \xi^i) \tag{8}$$

where $\eta^i \in [0, 1]$ with $\sum_i \eta^i = 1$ correspond to the Pareto weights for the two groups of agents. Formally, the government chooses $\{R, \tau, \bar{\tau}\}$ to maximize *(8)* subject to *(3)-(6)* for $i = 1, 2$.

The most effective way to characterize the Ramsey equilibrium is to pose the government problem in the allocation space. This requires expressing the set of equilibrium conditions *(3)-(6)* in terms of allocations only. This reformulation of the problem is known as the primal approach to Ramsey policies (see Chari and Kehoe, 1999). This approach is particularly interesting for this economy since it clarifies the constraints on redistribution that are implicit in the policy instruments available to the government.

The first constraint can be derived by expressing *(5)* in terms of allocations only, using *(3)-(4)* to substitute for $\bar{\tau}$, τ and R, and combining the result with *(7)*. This gives rise to the following constraint:

$$0.5\sum_{i=1,2}\left[u_1'(c_1^i)c_1^i + u_2'(c_2^i)c_2^i - l^i / \xi^i\right] \geq 0 \tag{9}$$

This condition, known as implementability constraint, does not capture all the constraints on Ramsey allocations. There are two wedges in this economy, the cash-credit wedge, which corresponds to *(3)*, and the consumption-labor wedge, captured by *(4)*. Since all agents face the same nominal interest rate and the proportional tax rate on labor income is not agent specific, these wedges must be equalized across agents. In addition, in any equilibrium $R \geq 1$. The resulting constraints on the optimal allocations are:

$$\frac{u_1'(c_1^1)}{u_2'(c_2^1)} = \frac{u_1'(c_1^2)}{u_2'(c_2^2)} \geq 1 \tag{10}$$

$$\xi^1 u_2'(c_2^1) = \xi^2 u_2'(c_2^2) \tag{11}$$

The Ramsey allocation problem corresponds to the choice of $\{c_1^i, c_2^i, l^i\}_{i=1,2}$ to maximize *(8)* subject to *(9)-(11)* and *(6)*.

Constraints *(10)-(11)* clearly identify the limitat to redistribution in this economy. The linear labor income tax induces *(11)*, which implies that relative consumption levels in the population are solely driven by relative productivities and cannot be influenced by the government. Still, differences in labor productivities may generate differences in consumption pattern. Since value of R above 1 effectively correspond to a subsidy to credit good consumption, they could in part offset the restriction imposed by *(11)*.

Constraints *(10)-(11)* will typically be binding and they shape the properties of Ramsey policies. To examine the effect of the constraints on redistribution imposed by the linearity of labor income taxes and *(10)*, it is useful to first solve a version of the Ramsey allocation problem where constraints *(10)-(11)* are dropped. Attaching multipliers $\tilde{\mu} \geq 0$ and $\tilde{\lambda} \geq 0$ to *(9)* and *(6)*, respectively, the first order necessary conditions for this relaxed problem are:

$$\eta^i u_j'(c_j^i) + \mu\left[u_j'(c_j^i) + u_j''(c_j^i)c_j^i\right] = \lambda \tag{12}$$

$$-\frac{(\eta^i + \mu)}{\xi^i} + \eta = 0 \tag{13}$$

for i, j = 1, 2, where $\mu = 0.5\,\tilde{\mu}$ and $\lambda = 0.5\tilde{\lambda}$.

Equation *(12)* immediately implies:

$$\frac{u_1'(c_1^i)}{u_2'(c_2^i)} = 1$$

for i = 1, 2. Let assume for simplicity: $u_j(c) = c^{1-\sigma}/(1-\sigma)$ for $\sigma > 0$. Combining *(12)* for j = 2 and *(13)* obtains:

$$\xi^i\left(\eta^i + \mu(1-\sigma)\right)(c_2^i)^{-\sigma} = \eta^i + \mu$$

for i = 1, 2. This condition clearly violates *(11)* unless $\eta^1 = \eta^2$. Thus, generically, if the government has redistributional goals, that is $\eta^1 \neq \eta^2$, *(11)* will be binding.

Let's now impose *(11)* on the Ramsey allocation problem, while still ignoring *(10)*. Denote with χ the Lagrangian multiplier attached to *(11)*. The first order necessary condition for c_2^i incorporating *(11)* is:

$$\left[\eta^i + \mu(1-\sigma)\right](c_2^i)^{-\sigma} + \chi(-1)^i \sigma (c_2^i)^{-\sigma-1} = \lambda \tag{14}$$

for $i = 1, 2$, with complementary slackness condition:

$$\chi\left[\xi^2 u_2'(c_2^2) - \xi^1 u_2'(c_2^1)\right] = 0$$

Combining *(14)* and *(13)* obtains:

$$\xi^i (c_2^i)^{-\sigma} = (\eta^i + \mu)\left[\eta^i + \mu(1-\sigma) + \frac{\chi(-1)^i \sigma}{c_2^i}\right]^{-1}$$

for $i = 1, 2$. By *(11)*:

$$\chi = \frac{\eta^1 - \eta^2}{(\eta^2 + \mu)} c_2^2 \left(1 + \left(\frac{\xi^1}{\xi^2}\right)^{-1/\sigma}\right)^{-1} \tag{15}$$

Combining *(14)* and *(12)* for $i = 1, 2$:

$$\frac{(c_1^i)^{-\sigma}}{(c_2^i)^{-\sigma}} = 1 + (-1)^i \sigma \frac{\chi}{c_2^i}\left[\eta^i + \mu(1-\sigma)\right]^{-1} \tag{16}$$

The inability of the government to redistribute via labor income taxes generates a motive for distorting the cash-credit good wedge. Let's consider the case $\eta^1 > \eta^2$, when by *(15)*, $\chi > 0$. The government favors redistribution to type 1 agents and can induce such a redistribution, even with a linear labor tax rate, by subsidizing cash good consumption for type 1 agents and taxing it for type 2 agents. Similarly, if $\eta^2 > \eta^1$ and $\chi < 0$, the government can redistribute to type 2 agents by subsidizing their cash good consumption and taxing it for type 1 agents. However, the government cannot set agent specific subsidies to cash good consumption and, in fact, cannot subsidize cash good consumption at all. By *(10)*, the cash-credit wedge must be equalized across agents and must be greater than 1. This restriction is clearly violated in *(16)*.

Even if condition *(16)* does not hold when constraint *(10)* is formally incorporated into the analysis, it clearly identifies when it will be optimal to depart from the Friedman rule. If $\eta^2 > \eta^1$, that is when the government favors type 2 agents, a departure from the Friedman rule relaxes the constraint on redistribution implicit in the proportional labor income tax. Instead, when the government favors redistribution towards type 1 agents, a subsidy to cash goods could play this role. Since such a subsidy violates the zero lower bound on the net nominal interest rate, the Friedman rule will be optimal for $\eta^1 > \eta^2$.

2.2 *Non-Linear Income Taxes and Private Information*

This example makes two changes relative to the previous set up. First, individual productivities are assumed to be private information. Second, the government selects a labor income tax schedule $T(l)$ that is allowed to be arbitrarily non-linear. As in Mirrlees (1971), private information implies that the optimal allocation must satisfy incentive compatibility constraints to induce agents to truthfully reveal their type. This requirement shapes the properties of the optimal tax schedule and may influence the optimal value of the nominal interest rate.

I will revert to the general specification of preferences, assuming that $U(c_1, c_2, l/\xi)$ is increasing in the first two arguments, decreasing in the third and strictly concave. In addition, the single crossing condition will be imposed so that the indifference curves between credit good consumption and labor effort l/ξ become flatter as productivity increases[1]. Under

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[1] See SALANIÉ B. (2000), for a discussion.

the single crossing condition, the incentive compatibility constraint is binding only for the high productivity type and is given by:

$$U(c_1^2, c_2^2, l^2/\xi^2) \geq U(c_1^1, c_2^1, l^1/\xi^2) \tag{17}$$

The zero lower bound constraint for the general utility specification corresponds to:

$$\frac{U_1(c_1^1, c_2^2, l^1/\xi^1)}{U_2(c_1^1, c_2^2, l^1/\xi^1)} = \frac{U_1(c_1^2, c_2^2, l^2/\xi^2)}{U_2(c_1^2, c_2^2, l^2/\xi^2)} \geq 1 \tag{18}$$

The taxation principle holds in this environment (see Guesnerie, 1998), so the government problem can be formulated in the allocation space directly. The government chooses $\{c_1^i, c_2^i, l^i\}_{i=1,2}$ to maximize *(8)* subject to *(17)*, *(18)* and *(6)*.

Let's first consider the government problem when *(18)* is dropped. Denoting with μ and $\tilde{\lambda}$ the multipliers on *(17)* and on the resource constraint, respectively, the first order necessary conditions for this problem are:

$$\eta^1 U_j(c_1^1, c_2^1, l^1/\xi^1) - \mu U_j(c_1^1, c_2^1, l^1/\xi^2) = \lambda, \text{ for } j = 1,2 \tag{19}$$

$$\eta^1 U_3(c_1^1, c_2^1, l^1/\xi^1)/\xi^1 - \mu U_3(c_1^1, c_2^1, l^1/\xi^2)/\xi^2 = -\lambda \tag{20}$$

$$(\eta^2 + \mu) U_j(c_1^2, c_2^2, l^2/\xi^2) = \lambda, \text{ for } j = 1,2 \tag{21}$$

$$(\eta^2 + \mu) U_3(c_1^2, c_2^2, l^2/\xi^2)/\xi^2 = -\lambda \tag{22}$$

for $\lambda = 0.5\tilde{\lambda}$. The resulting expressions for the optimal comsumption labor wedge are:

$$\frac{U_2(c_1^2, c_2^2, l^2/\xi^2)}{U_3(c_1^2, c_2^2, l^2/\xi^2)} = -1 \tag{23}$$

$$\frac{U_2(c_1^1,c_2^1,l^1/\xi^1)}{U_3(c_1^1,c_2^1,l^1/\xi^1)/\xi^1}=-1+ \tag{24}$$
$$\frac{\mu}{\eta^2}\frac{U_3(c_1^1,c_2^1,l^1/\xi^2)/\xi^2}{U_3(c_1^1,c_2^1,l^1/\xi^1)/\xi^1}\left[\frac{U_2(c_1^1,c_2^1,l^1/\xi^2)}{U_3(c_1^1,c_2^1,l^1/\xi^2)/\xi^2}+1\right]$$

Condition *(23)* reproduces the customary "no distortions at the top". Since type 1 agents do not have an incentive to mimic type 2 agents, there is no need to distort the allocation for type 2. Instead, type 1's will be distorted as long as type 2's incentive compatibility constraint is binding, as implied by *(24)*. This distortion affects the optimal allocation for type 1 in a way that makes it undesirable for type 2 to misreport her type. In particular, the distortion corresponds to a *positive* marginal tax on type 1's labor income when:

$$\frac{U_2(c_1^1,c_2^2,l^1/\xi^2)}{U_3(c_1^1,c_2^1,l^1/\xi^2)/\xi^2}<-1 \tag{25}$$

Since by *(21)* and *(19)*, $c_j^1 < c_j^2$ for $j = 1, 2$ and $l^2 > l^1$ when the incentive compatibility constraint is binding, *(25)* holds if the reduction in labor effort increases the demand for credit good consumption, that is when credit good consumption and labor effort are gross substitutes. By contrast, if credit goods and labor effort are gross complements, and increase in credit good consumption will increase labor effort, and the optimal marginal labor income tax on type 2 agents will be negative.

Let's now consider the implications for the cash-credit wedge. By *(21)* and *(19)*:

$$\frac{U_1(c_1^2,c_2^2,l^2/\xi^2)}{U_2(c_1^2,c_2^2,l^2/\xi^2)}=1 \tag{26}$$

$$\frac{U_1(c_1^1,c_2^1,l^1/\xi^1)}{U_2(c_1^1,c_2^1,l^1/\xi^1)}=1+$$

$$\frac{\mu}{\eta^2}\frac{U_2(c_1^1,c_2^1,l^1/\xi^2)}{U_2(c_1^1,c_2^1,l^1/\xi^1)}\left[\frac{U_1(c_1^1,c_2^1,l^1/\xi^2)}{U_2(c_1^1,c_2^1,l^1/\xi^2)}-1\right] \tag{27}$$

By *(26)*, type 2 agents' marginal rate of substitution between cash and credit goods is not distorted, while there will be a distortion for type 1 agents if the incentive compatibility constraint is binding and the expression in square brackets in *(27)* is not zero. Thus, the rationale for distorting the cash credit margin for type 2 agents is similar than for the labor wedge. The consumption pattern is distorted to make it less appealing for type 2 agents to report to be type 1.

The sign and the extent of the cash-credit distortion for type 1 agents depends on preferences. Clearly, for $U(\cdot)$ weakly separable in consumption and labor the cash-credit wedge is undistorted. More in general, there will be an interaction between the choice of labor and consumption and it will be optimal to distort the cash-credit wedge. Specifically, the distortion will be positive when the marginal rate of substitution between cash and credit good consumption:

$$\frac{U_1(c_1^1,c_2^1,l^1/\xi^2)}{U_2(c_1^1,c_2^1,l^1/\xi^2)}>1 \tag{28}$$

Since by *(21)* and *(19)*, $c_j^1 < c_j^2$ for $j = 1, 2$ and $l^2 > l^1$ when the incentive compatibility constraint is binding, *(28)* corresponds to a utility specification in which a type 2 agent who cheats and reduces her labor effort wishes to increase her cash good consumption more than her credit good consumption. In other words, it will be optimal to increase the relative price of cash goods when cash good consumption and labor effort are gross substitutes. Thus, the marginal tax on cash consumption will reduce the marginal utility benefit from reducing labor effort and relax the incentive compatibility constraint. If the opposite inequality holds, cash goods and and a reduction in labor effort are complements, thus an increase in cash good consumption will generate a decline in the marginal utility value of a reduction in labor effort. In this case, it is optimal to subsidize cash good consumption.

Equation *(27)* violates *(18)* in both cases but it points to the conditions under which a departure from the Friedman rule are optimal. If a decline in cash good consumption relative to credit good consumption increases the marginal utility value from reducing labor effort, $R > 1$ relaxes the incentive compatibility constraint and improves efficiency. Instead, if a rise in cash good consumption relative to credit good consumption induces a decline in the marginal utility value of reducing labor effort, a lower nominal interest rate relaxes the incentive compatibility constraint. In the general government problem, the zero lower bound constraint on the nominal interest rate will be binding in this case and the Friedman rule will be optimal. This is exactly the case considered by da Costa and Werning (2007), who show that when cash goods and labor effort are gross complements, the Friedman rule is optimal.

3. - Discussion

3.1 *Theory*

There is a strong link between the optimality of the Friedman rule and the uniform commodity taxation principle in representative agent economies. Chari, Christiano, and Kehoe (1996) show that when preferences are homothetic in cash and credit goods and weakly separable in labor, the Friedman rule holds. Weak separability is the sufficient condition for optimality of uniform commodity in a representative agent economy (see Atkinson and Stiglitz, 1980). Homotheticity also implies that the income elasticity of money demand is unitary. If the homotheticity assumption is relaxed, the inverse elasticity rule of commodity taxation applies. Cash goods should be subsidized relative to credit goods whenever the income elasticity of money demand is greater than one, that is when an increase in labor supply increases the demand for cash goods. This case corresponds to gross complementarity in preferences between cash goods and labor effort. In a real interpretation of the economy, the optimal tax system would subsidize cash goods relative to credit goods. However, such a subsidy would violate the zero lower bound constraint on nominal interest rates in the monetary version of the economy. Thus, the binding zero lower bound on the nominal interest rate implies that the optimality of the Friedman rule holds for a broader class of preferences than uniform commodity taxation.

The uniform commodity taxation principle and the binding zero lower bound on the nominal interest rate also play a role in the heterogeneous agent economy discussed above. Atkinson and Stiglitz (1976) derive the conditions for optimality of uniform commodity taxation in heterogeneous agent economies. A key insight is that when redistribution is constrained in some way, differential commodity taxation may be optimal to attain the desired distribution of resources. The structure of labor income taxes and the assumptions on preferences are both critical for the properties of optimal commodity taxes. The case with differences in productivities and linear income tax is one in which differential commodity taxation is optimal in general with redistributional objectives. In particular, commodities with high income elasticity should be taxed more heavily, as they will be consumed more by high productivity agents. If instead the labor income tax is nonlinear with private information on individual productivities, differential commodity taxation is optimal only if the pattern of consumption can serve as a screening device. If utility is homothetic in consumption and weakly separable in labor, then the income elasticity of demand is unitary for all commodities, and uniform commodity taxation applies.

This logic applies squarely to the examples considered here. In the first, the constraint on redistribution stemming from the proportional labor income tax motivate departures from the Friedman rule. This constraint implies that, in a real version of the economy, uniform commodity taxation would only hold with equal Pareto weights on both types of agents. Since in the monetary version, the zero lower bound constraint rules out subsidies to cash goods, the Friedman rule holds for a range of Pareto weights. The incentive compatibility constraint places a limit on redistribution with non-linear labor income taxes. Departures from the Friedman rule may be warranted if they relax such constraint. If cash goods and labor effort are gross substitutes, increasing the relative price of cash goods relaxes the incentive compatibility constraint, while the opposite is true if cash goods and labor effort are gross complements. In a monetary version of the economy, the zero lower bound constraint on nominal interest rate prevents the relative price of cash goods from falling below one. Thus, the Friedman rule is optimal for the class of preferences for which a cash good subsidy would be optimal in a real version of the economy. This explains the result in da Costa and Werning (2007) who show that under weak gross complementarity between cash goods and labor effort the Friedman rule is optimal.

3.2 *Empirical Evidence*

The previous discussion suggests that the income elasticity of monetary holdings is key for the optimality of the Friedman rule when redistributional considerations are present. The empirical evidence on the cross-sectional distribution of currency can be used to discipline assumptions on preferences and transactions technologies to derive realistic implications for the income elasticity of money demand and therefore optimal policies.

The empirical evidence strongly suggests a negative correlation between labor income (and wealth) and cash holdings. Erosa and Ventura (2000) report that low income households use cash for a greater fraction of their total purchases relative to high income households in the US. Mulligan and Sala-i-Martin (2000) estimate the probability of adopting financial technologies that hedge against inflation, based on US data, and find that is positively related to the level of household income and wealth, and to education. Attanasio, Guiso and Jappelli (2001) find that the probability of using an interest bearing bank account increases with educational attainment, income and average consumption, based on cross-sectional household data for Italy. Guiso, Haliassos and Jappelli. (2001) present evidence from a broad set of countries that confirms this pattern.

The fact that low income households hold more cash implies that they are more exposed to inflation. This is consistent with indirect evidence on the distributional consequences of inflation presented by Easterly and Fisher (2000). Based on polling data for 38 countries, they find that the poor are more likely than the rich to mention inflation as a top national concern. This suggests that low income household perceive inflation as being more costly.

This evidence strongly supports models that generate a negative cross-sectional correlation between labor income and monetary holdings, giving rise to less than unitary income elasticity of money demand. In the next section, I present such a model and derive the implications for optimal fiscal and monetary policy.

4. - A Monetary Economy with Heterogenous Agents

The economy is populated by agents, firms and a government. Agents consume, supply labor and trade in assets in each period. They differ in labor productivity but have identical preferences. Purchases are made with currency or with a costly alternative payment technology. Perfectly competitive firms have access to a linear technology that uses labor to produce consumption goods. The government finances an exogenous stream of spending by taxing labor income at a proportional rate, issuing nominal debt and printing money. There is no aggregate or idiosyncratic risk.

I now illustrate the model, a version of the one analyzed in Albanesi (2005), in more detail.

4.1 *Firms*

There are two types of competitive firms. All firms live for one period. Goods firms hire labor to produce a continuum of differentiated consumption goods indexed on the interval [0, 1]. The production technology is linear, and different consumption goods are perfect substitutes in production. Perfect competition implies:

$$P_t\,(j) = W_t \tag{29}$$

for $j \in [0, 1]$, where $P_t\,(j)$ is the price of good j and W_t the nominal wage per efficiency unit of labor at time t. $P_t = W_t$ will denote the price of consumption goods.

Financial firms produce transaction services, enabling agents to purchase goods without the use of cash. A financial firm's profits for providing transaction services for the purchase of good j are:

$$\pi_t\,(j) - W_t\theta\,(j) \tag{30}$$

where $\theta\,(\cdot)$ is measured in efficiency units of labor and satisfies $\theta' > 0$ on the interval $[\underline{z}, 1]$, with $\underline{z} \geq 0$. π_t is the dollar charge for arranging purchases of consumption good j without currency. Profit maximization implies: $\pi_{tt}\,(j) = W_t\theta\,(j)$ for all t and all $j \in [0, 1]$. This specification follows Prescott (1987).

4.2 *Agents*

A unit measure of agents is divided into two types, where $0 < v_i < 1$ is the fraction of type i agents, with $i = 1, 2$ and $\Sigma_i\, v_i = 1$. All agents have identical preferences defined over a consumption aggregator c^i and over hours of work l_t^i given by:

$$\sum_{t=0}^{\infty}\beta^t U(c_t^i, l_t^i) \tag{31}$$

$$c^i = \left[\int_0^1 c^i\,(j)^\rho\, dj\right]^{\frac{1}{\rho}}$$

where $\rho \in (0, 1)$ for a agent of type $i = 1, 2$. I will restrict attention to preferences of the class:

$$U(c^i, l^i) = h(c^i) + v(l^i),$$

where h is strictly increasing and strictly concave, while v is strictly decreasing and concave.

Agents of different types differ in labor productivity, denoted with ξ_i, for $i = 1, 2$. I will assume $\xi_2 > \xi_1$.

In each period, agents choose transaction services and consumption levels, they supply labor, accumulate currency and trade nominal bonds. Given *(31)* and the assumption on transaction costs, agents will optimally choose z^i_t, the fraction of consumption goods purchased without the use of cash, and $c^i_{1,t}$, $c^i_{2,t}$, is the level of consumption of goods purchased with and without currency, respectively. Then:

$$c^i = \left[(1-z^i)(c^i_1)^\rho + z^i(c^i_2)^\rho\right]^{\frac{1}{\rho}}$$

Given M^i_t, an agent beginning of period cash holdings, the cash in advance constraint is:

$$P_t c^i_{1,t}(1 - z^i_t) - M^i_t \le 0. \qquad (32)$$

The asset market meets after trading on goods and labor market has closed. During the asset market session agents receive labor income net of taxes, clear consumption liabilities and trade bonds issued by other agents or by the government. Bonds purchased at time t pay one unit of currency in the $t + 1$ asset market. The government and private agents are committed to debt repayments, so that agents are indifferent between holding privately or government issued bonds. The price of a nominal bond at time t is Q_t. Net new purchases of bonds by agent i at time t are denoted with B^i_{t+1} for $i = 1, 2$.

The asset market budget constraint is:

$$M_{t+1}^{i} + Q_t B_{t+1}^{i} \tag{33}$$
$$\leq M_t^i + B_t^i - P_t c_{1,t}^i (1 - z_t^i) - P_t c_{2,t}^i z_t^i - \int_{\underline{z}}^{z_t^i} \pi_t(j)dj + W_t \xi_i (1 - \tau_t) l_t^i$$

where τ_t is the tax rate on labor income and

$$\int_{\underline{z}}^{z_t^i} \pi_t(j)dj$$

the currency cost of arranging purchases of consumption goods with credit. In addition, a no-Ponzi game condition:

$$0 \leq B_{t+1}^{i} \left(\prod_{s=0}^{+1} Q_{s,s+1} \right)^{-1} \Phi_{t+1} \tag{34}$$
$$+ Q_{t,t+1}^{-1} M_{t+1}^{i} \Phi_{t+1} + \sum_{s=1}^{\infty} \Phi_{t+s} W_{t+s} (1 - \tau_{t+s}^{i}) \xi_i$$

is also required, with $\Phi_t = \Pi_{t'=0}^{t-1} Q_{t'}$, $\Phi_0 = 1$.

4.3 *Government*

The government finances an exogenous stream of consumption $\bar{g}$ and is subject to the budget constraint:

$$P_t \bar{g}_t + M_t + B_t = Q_t B_{t+1} + M_{t+1} + W_t T_t \tag{35}$$

where M_t, B_t are the supply of currency and bonds, respectively, and:

$$T_t = \sum_i \nu_i \tau_t \xi_i l_t^i \tag{36}$$

4.4 *Competitive Equilibrium*

The timing of events in each period is as follows:

1. Agents enter the period with holdings of currency and debt given by M_t^i and B_t^i for $t = -1,0, 1...$ and choose z_t^i.

2. Agents, firms and the government trade on the goods and labor markets. The agents' purchases of cash goods are subject to *(32)*. Clearing on the goods market requires:

$$\sum_{i=1,2} v_i \left(c_{1,\,t}^i (1 - z_t^i) + c_{2,t}^i z_t^i + \int_0^{z_t^i} \theta(j) dj - \xi_i l_t^i \right) + g_t = 0 \tag{37}$$

3. Asset markets open. Agents purchase bonds and acquire currency to take into the following period subject to the constraint *(33)*. Clearing on the asset market requires:

$$\begin{gathered} \sum_{i=1,2} v_i B_t^i = B_{t,} \text{ for } s > 0 \\ \sum_{i=1,2} v_i M_{t+1}^i = M_{t+1} \end{gathered} \tag{38}$$

DEFINITION 1. A competitive equilibrium is given by a government policy $\{g_t, \tau_t, M_{t+1}, B_t\}_{t \geq 0}$, a price system $\{P_t, W_t, Q_t, \pi_t\ (j)\}_{t \geq 0, j \in [0,1]}$ and an allocation $\{c_{1,\,t}^i, c_{2,\,t}^i, l_t^i, z_t^i, B_t^i\}_{i=1,\,2,\,t \geq 0}$ such that:

1. given the policy and the price system agents and firms optimize;
2. government policy satisfies *(35)* and *(36)*;
3. markets clear.

The following proposition characterizes the competitive equilibrium.

PROPOSITION 2. An allocation $\{c_{1,t}^i, c_{2,t}^i, l_t^i, z_t^i, B_t^i\}_{i=1,\,2,\,t \geq 0}$ and a price system $\{P_t, W_t, Q_t, \pi_t\ (j)\}_{t \geq 0, j \in [0,1]}$ constitute a competitive equilibrium if and only if, for a given government policy $\{g_t, \tau_t, M_{t+1}, B_t\}_{t \geq 0}$, *(37)*, *(35)* and the following conditions are verified:

$$Q_t = \beta \frac{P_t}{P_{t+1}} \frac{\hat{u}_{2,t+1}^i}{\hat{u}_{2,t}^i} \tag{39}$$

(40)
$$0 < Q_t \leq 1$$
$$\frac{-u^i_{l,\,t}}{\hat{u}^i_{2,\,t}} = \xi_i(1-\tau_t) \text{ for } t \geq 0$$

(41)
$$W_t = P_t$$
$$R_{t+1} \equiv \frac{\hat{u}^i_{1,\,t+1}}{\hat{u}^i_{2,t+1}} = Q_t^{-1}$$

$$(R_{t+1}-1)(P_{t+1}c^i_{1,\,t+1}(1-z^i_{t+1})-M^i_{t+1}) = 0$$
$$P_{t+1}\,c^i_{1,\,t+1}(1-z^i_{t+1}) \leq M^i_{t+1}$$

(42)
$$\left[\left(\frac{1}{\rho}-1\right)\left(1-R_s^{\frac{\rho}{\rho-1}}\right)-\frac{\theta(z^i_s)}{c^i_{2,s}}\right]\begin{cases}\leq & 0 \text{ for } z^i_s = \underline{z} \\ = & 0 \text{ for } z^i_s \in (\underline{z},\overline{z}) \\ \geq & 0 \text{ for } z^i_s = \overline{z}\end{cases}$$

for $t \geq 0$, and:

(43)
$$P_0 c^i_{1\text{-}0}\ (1 - z^i_0) \leq M^i_0$$

(44)
$$\hat{u}^i_{1,0}\frac{M^i_0}{P_0}+\hat{u}^i_{2,0}\frac{B^i_0}{P_0}$$
$$=\sum_{t=0}^{\infty}\beta^t\left[u^i_{1,\,t}c^i_{1,\,t}+u^i_{2,\,t}\hat{c}^i_{2,\,t}+u^i_{l,\,t}l^i_t\right]$$

for i =1, 2, with

$$C(z^i_t) = \int_{\underline{z}}^{z^i_t}\theta(j)dj$$

Here,

$$u^i_{j,t} = \partial U(c^i_t, l^i_t)/\partial c^i_{j,t}, u^i_{l,\,t} = U_2(c^i_t, l^i_t) \text{ and}$$

$$\hat{c}^i_2 = c^i_2 + \frac{C(z^i)}{z^i}, \hat{u}^i_1 = u^i_1/(1-z^i), \hat{u}^i_2 = u^i_2/z^i \text{ for } i, j = 1,2$$

Equation *(44)* is the agents' intertemporal budget constraint and it incorporates the transversality condition. The proof of this proposition is in Appendix A.

5. - Ramsey Equilibrium

The Ramsey equilibrium is the competitive equilibrium that maximizes social welfare from the standpoint of time 0. The government selects policies once and for all at time 0 for all future periods and is committed to following these plans. The social welfare function is simply a weighted sum of the agents' lifetime utility. The Pareto weight on type i agents is η_i, with $\eta_1 + \eta_2 = 1$. Pareto weights are time-invariant and the case $\eta_i = v_i$ corresponds to a utilitarian government.

As the one period example, I solve for the Ramsey equilibrium by deriving the Ramsey allocation problem, where the government chooses an allocation at time 0 subject to the constraint that it constitutes a private sector equilibrium. This problem's choice variables are $\{c^i_{1,t}, c^i_{2,t}, l^i_t, z^i_t\}_{i=1,\,2,\,t\geq 0}$.

PROPOSITION 3. An allocation $\{c_{i1,t}, c_{i2,t}, l_{it}, z_{it}\}_{i=1,\,2,\,t\geq 0}$ and values of $\{R_t\}_{t\geq 0}$ constitute a Ramsey equilibrium if and only if they solve the primal problem:

$$\max_{\left\{c^i_{1,t}, c^i_{2,t}, l^i_t, z^i_t\right\}_{i=1,2,t\geq 0}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1,2} \eta_i U(c^i_t, l^i_t)$$

subject to:

$$\frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} = R_t, \text{ for } i = 1,2 \tag{45}$$

$$R_t \geq 1 \tag{46}$$

$$\frac{-u^2_{l,t}}{\xi_2 \hat{u}^2_{2,t}} = \frac{-u^1_{l,t}}{\xi_1 \hat{u}^1_{2,t}} \tag{47}$$

(42) and *(37)* for all t, as well as *(44)* and *(43)*, for given P_0.

The proof of Proposition 3 parallels the one for a representative agents economy in Chari, Christiano and Kehoe (1996). Constraints *(45)-(47)* are the analogue of *(10)* and *(11)* in the first example in Section 2. The level of P_0 should also be a choice variable, since it is not pinned down by the competitive equilibrium conditions and competitive equilibria are indexed by P_0. However, the qualitative properties of Ramsey policies for $t > 0$ do not depend on the value of P_0, so I take it as given and treat it as an initial condition for the purpose of this analysis.

5.1 *Optimal Policies*

The key properties of Ramsey policy for $t > 0$ are described in the following proposition.

PROPOSITION 4. Assume:

(A1) $U(c, l) = h(c) + v(l)$, with $h(\cdot)$ strictly increasing and strictly concave and $v(\cdot)$ strictly decreasing and concave;

(A2) $\theta(j)$ is strictly increasing for $j \in [\underline{z}, 1]$, with $\underline{z} \geq 0$, and $\lim_{z \downarrow \underline{z}} \theta(z) = 0$.

Then, $R_t = 1$ for $t \geq 1$ in the Ramsey equilibrium, if and only if constraint *(47)* is not binding. Moreover, if *(47)* is non-binding, $\eta_1 \geq \bar{\eta}_1$ with $0 < \bar{\eta}_1$.

The proof of this proposition is an application of Albanesi (2005) and is omitted for brevity. It is similar to the proof of the optimality of the Friedman rule for a representative agent economy in Christiano, Chari and Kehoe (1996) and relies on the homotheticity of the consumption aggregator and separability of utility in consumption and leisure imposed in (A1). It holds irrespective of the functional form of $v(\cdot)$ or initial conditions[2].

Proposition 4 demonstrates that the Friedman rule is not optimal when high productivity agents enjoy a relatively high Pareto weight, that is η_1, is low. The intuition is similar to that

[2] The proof of this proposition implicitely also shows that in a homogeneous agent version of this economy the Friedman rule would be optimal. In such an economy, the constraints *(45)* and *(47)* would not be part of the Ramsey problem.

for the first example in Section 2. The optimal labor wedge for each type of agent i is decreasing in η_i and in the value of the multiplier on the implementability constraint, a measure of the cost of raising distortionary revenues. Linear labor taxation forces this wedge to be equalized across agents in any equilibrium, which results in constraint *(47)*. As in the one period economy, this constraint will be generically be binding. A departure from the Friedman rule can relax this constraint with heterogeneity in transaction patterns.

The redistributional effects of monetary policy and the optimal nominal interest rate hinge critically on the negative cross-sectional correlation between cash holdings and labor productivities, an implication of the equilibrium income elasticity of money demand in this model. One can define a short run elasticity, corresponding to the sensitivity of money holdings to consumption for given transaction pattern, that is z_t^i. This depends on the properties of the utility function only and is equal to one in this model, given the homotheticity in consumption of $U(\cdot)$ for given z. The long run elasticity incorporates the effect on z^i. Since by *(42)* the average cost of transaction services is decreasing in the level of consumption, the long run income elasticity of money demand is smaller than one. This property implies that high productivity agents hold less cash as a fraction of consumption. Consequently, a departure from the Friedman rule is optimal when the government wishes to redistribute in their favor, that is when the Pareto weight on low productivity agents is low enough.

What would be the predictions of this model with non-linear taxes and private information on labor productivities? da Costa and Werning (2007) consider a general utility specification of the form $U(m, c, l)$ and show that the Friedman rule is optimal if money and labor are gross complements. For the model in this paper, solving the sub-optimization problem in which for given real balances, credit good consumption and labor effort, agents choose z and cash good consumption delivers an indirect utility function of the form $U(m, c, l)$, with U separable in $\{m, c\}$ and l. This separability implies that the level of monetary holdings does not influence labor effort and thus the Friedman rule would be optimal with non-linear income taxes. More in general, if utility is allowed to be non-separable in consumption and labor, it is straightforward to show that m and l are gross substitutes for given c in the implied specification of U, if z is increasing in c. Gross substitutability between money holdings and labor follows from the fixed cost of using alternatives to cash to make payments. In turn, this feature is essential for generating an empirically plausible cross-sectional distribution of currency. Gross substitutability between money holdings and labor effort implies that a rise in the nominal interest rate relaxes the incentive compatibility constraint in da Costa and Werning's model, as illustrated in the simple example discussed in Section 2. da Costa and Werning assume a utilitarian government and do not consider redistributional motives. This discussion suggests that with assumptions on preferences that generate an empirically plausible cross-sectional distribution of money, even with non-linear labor income taxes, the Friedman rule is optimal only if the government wishes to redistribute in favor of the low productivity agents. Otherwise, efficiency considerations would render positive nominal interest rates optimal.

Battacharya, Haslag and Martin (2005) also analyze departures from the Friedman rule in a variety of heterogeneous agent models with *ad hoc* restrictions on fiscal instruments and show that the optimal monetary policy is sensitive to those restrictions.

5.2 *A Calibrated Example*

It is interesting to analyze the properties of optimal policies as a function of the distribution of Pareto weights for a calibrated example to quantitatively evaluate the extent to which departures from the Friedman rule are optimal.

I consider the following specification for utility and transactions technology:

(48)
$$U(c^i, l^i) = \frac{(c^i)^{1-\sigma} - 1}{1-\sigma} + v(l^i), \text{ for } i = 1,2, \sigma > 0$$

(49)
$$v(l^i) = \gamma_0 \frac{(1-l^i)^{1-\gamma_1}}{1-\gamma_1}, \gamma_0, \gamma_1 > 0$$

(50)
$$\begin{aligned} \theta(j) &= 0 \text{ for } j \leq \underline{z} \\ &= \theta_0 \left(\frac{j - \underline{z}}{\overline{z} - j} \right)^{\theta_1} \text{ for } j \in (\underline{z}, \overline{z}) \\ &= \infty \text{ for } j \geq \overline{z} \end{aligned}$$

where $0 \leq \underline{z} < \overline{z} \leq 1$.

I set $\beta = 0.97$ and $\sigma = 0.8$. Other parameters are chosen so that in a steady state with $\tau_1 = \tau_2 = 0.30$ and $R = 1.05$ the model matches corresponding averages for the US economy. The fraction of low productivity agents in the population is set to 0.6 and their productivity is set to $\xi_1 = 1$, while ξ_2 is set so that the Gini coefficient for consumption in the model is equal to 25.5%[3]. The properties of money demand depend on ρ, θ_0 and θ_1. I fix $\rho = 0.5$ and set θ_0 and θ_1 to approximate the interest elasticity and the average velocity of transactions accounts (currency plus checkable deposits, plus time and savings deposits) as a fraction of personal consumption expenditures. These two statistics are equal to –5.11% and 1.37, respectively, based on Flow of Funds data for the post-war period. Initial real and nominal debt holdings are set to 0 and the distribution of currency is symmetric. The parameters are summarized in Table 1.

Graph 1 displays the results for the case with linear labor taxation. The optimal nominal interest rate and labor tax rate are plotted as a function of η_1, for $t > 0$. The Friedman rule is optimal for $\eta_1 \geq v_1 = 0.6$. The tax rate on labor is increasing in η_1, even for $\eta_1 > v_1$. This result emerges since the multiplier on the

[3] More details on the calibration are available in ALBANESI S. (2005).

TABLE 1

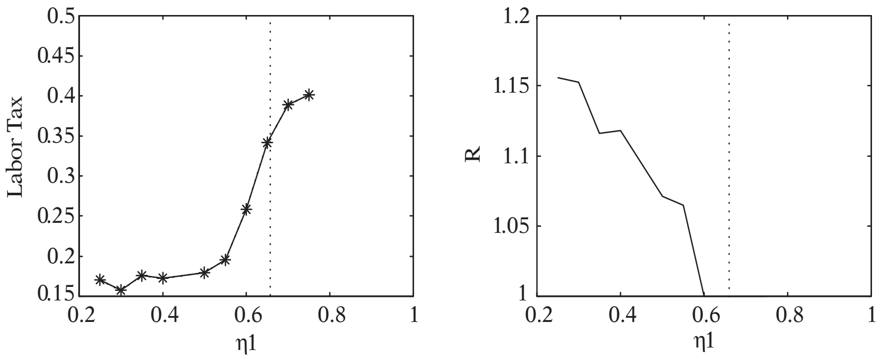
PARAMETER VALUES

Fixed					
β	$\underline{z}$	$\overline{z}$	ν_1	ξ_1	ρ
0.97	0.10	0.75	0.6	1	0.5
Calibrated					
σ	γ_0	γ_1	ξ_2	θ_0	θ_1
0.8	1.6	1	2.5	0.053	0.2

implementability constraint on type 2 falls (and the one for type 1 increases) as η_1 rises. In other words, the marginal value of transferring resources to type 2 falls, which induces a rise in the optimal labor tax rate. The tax rate on labor varies from 0.17 to 0.41, while the net nominal interest rate from 16% to 0. The value of η_1 need not be extremely small to motivate empirical plausible departures from the Friedman rule. For example, for η_1 = 0.5, the net nominal interest rate is equal to 7%.

GRAPH 1

PROPORTIONAL LABOR TAX



6. - Time Inconsistency

What are the implications for time consistency of Ramsey policies? It is well known that Ramsey policies are time inconsistent in a representative agent framework (Kydland and Prescott, 1977)[4]. With distortionary taxes, a "surprise" revision of the intertemporal path of taxes that depreciates the present value of outstanding government liabilities can effectively reproduce the missing lump sum tax.

Lucas and Stokey (1983) show that in a real economy this incentive can be eliminated by appropriately restructuring outstanding claims on the government. This option is not available in a monetary economy. To ensure time consistency, both real and nominal government debt must be non-zero for a given path of prices. But any positive level of nominal debt generates the temptation to inflite it away with a one time rise in the price level. Lucas and Stokey conclude that commitment to a path for nominal prices is necessary for time consistency in a monetary economy. Alvarez, Kehoe and Neumeyer (2004) also examine this issue and prove that optimality of the Friedman rule is a necessary and sufficient condition for time consistency. The key step in their argument is that, under the Friedman rule, a monetary economy is equivalent to a real economy. Then, setting the present value of nominal government liabilities to zero at all dates and states removes the incentive to change the path of prices, while an appropriate choice of the maturity structure of real debt can remove the incentive to change the path of taxes.

With heterogeneous agents, since agent specific lump sum taxes are typically not available, there could be two motives for deviating from previously announced policies: increasing efficiency and improving redistribution. This would seem to exacerbate the time inconsistency problem.

Albanesi (2005) analyzes time consistency of Ramsey policies in a version of the model presented in Section 4 and shows that

[4] In economies with capital, there is an additional incentive to deviate from previously announced plans, since the capital tax does not distort past investment decisions.

Ramsey policies are time consistent. The proof follows the simple strategy laid out by Lucas and Stokey (1983), based on the following operational definition of time consistency. For any $t \geq 0$, define the Ramsey problem at period t analogously to the Ramsey problem for period 0. Then, the Ramsey problem at period t is time consistent for period $t + 1$, if the solution of the Ramsey problem at period t solves the Ramsey problem at $t + 1$. The Ramsey equilibrium is *time consistent* if the Ramsey problem at time t is time consistent for the Ramsey problem at $t + 1$ for $t \geq 0$. In practice, since the Ramsey equilibrium allocation is a stationary function of the state for $t \geq 1$, it is sufficient to identify a set of initial conditions for the time 1 problem exist that would induce the government at time 1 to continue with the allocation that solves the Ramsey problem at time 0.

The time consistency of Ramsey policies with heterogeneous agents is that a one time change in the price level has redistributional effects. The redistributional costs can offset any efficiency gains for an appropriate distribution of nominal debt[5]. Since agents are indifferent with respect to their portfolio composition in equilibrium, it is always possible to identify a distribution of debt that guarantees the government will stick to Ramsey policies in future periods.

What happens with non-linear taxes? In this case, there are no incentives to deviate for the purpose of reducing the deadweight burden associated with government consumption, since lump sum taxes are allowed. However, there will be an incentive to revise policies to ameliorate the distribution of resources. With a utilitarian social welfare function, it would be optimal to equalize consumption across agents with different productivities. However, thus policy violates incentive compatibility constraints. More consumption must be promised to high ability agents to induce them to reveal their type, rather than mimic low ability agents.

[5] This argument was first explored by ROGERS C.A. (1986), who studies optimal wage and capital taxes in a two-period, multiple consumer economy. She finds that the incentive to raise capital taxes may be moderated by redistributional concerns.

Once abilities have been revealed, however, the government has the incentive to equalize consumption, giving rise to time inconsistency. This problem was first laid out by Roberts (1984). How is the argument affected with heterogeneous monetary holdings? In the case considered by da Costa and Werning (2007), the complementarity of money holdings and labor effort implies a positive correlation between monetary holdings and ability. Then, a surprise increase in the price level equalizes the distribution of consumption. By contrast, when monetary holdings and labor effort are substitutes in preferences, high ability agents hold less money, as in the data. The negative correlation between monetary holdings and ability may give rise to the incentive to engineer a one time decline in the price level. Thus, fiscal policy is time inconsistent and so is monetary policy.

7. - Shocks

The previous analysis completely abstracts from the presence of aggregate and idiosyncratic shocks. Still, the results suggest a few informed conjectured. Let's consider the case with aggregate shocks first. With non-linear taxes aggregate shocks can be smoothed by changing the level of the lump-sum component of taxes, so they do not affect the analysis. With linear taxes, a fundamental result in representative agent economies is that inflation exhibits very high volatility (Chari, Christiano and Kehoe, 1991). Innovations in inflation are used to impart the optimal amount of state contingency to the real value of nominal bond returns, thus allowing to maintain labor income taxes. This is optimal since innovations in inflation act as a lump sum tax and distortions associated with the labor taxation are convex in the tax rate.

The forces shaping the stochastic properties of optimal inflation with linear taxes are very similar to those that lead to time inconsistency. This suggests that in a heterogeneous agent version of the model optimal inflation volatility may be substantially reduced. Innovations in inflation redistribute resources across agents with different levels of nominal asset holdings. The high inflation volatility that would be required to optimally smooth distortionary taxes in the face of aggregate shocks necessarily involves systematic redistribution in alternative directions. For example, if money is the only asset, since low ability households hold more cash as a fraction of total purchases, positive innovations in inflation redistribute against them and negative innovations redistribute in their favor. Depending on the strength of redistributional consideration, the government may find it optimal to limit inflation volatility to prevent the negative redistributional effects of inflation innovations.

The effect of idiosyncratic shocks is more subtle. Levine (1991) is perhaps the first to point out that the Friedman rule may be suboptimal with idiosyncratic shocks. His argument is based on the fact that implementing the Friedman rule requires a stationary rate of monetary contraction. This is suboptimal when financed with lump sum taxes since it weighs more heavily on agents that, due to adverse idiosyncratic shocks, have low income. On the other hand, Erosa and Ventura (2002) show that an increase in the average inflation rate can generate adverse distributional consequences in a version of the model in Section 4. In their economy, agents'-hold money, nominal bonds and physical capital. Fixed costs of asset participation imply that low income agents hold a greater share of money in their portfolio, which in turn implies that they can reap lower rate of returns with positive inflation. Since idiosyncratic shocks generate a precautionary motive for asset accumulation, the rate of return differential associated with positive inflation implies that low ability households have smaller opportunities for self-insurance, leading to a large increase in inequality in wealth and welfare for revenue neutral increases in average inflation. These results suggest that optimal monetary policy with idiosyncratic shocks depends crucially on the set of available fiscal instruments.

Let's consider a version of the model in Section 4 where agents are *ex ante* identical and are subject to idiosyncratic productivity shocks, and assume the government is utilitarian. High labor income taxes provide insurance in this case. Moreover, lucky agents that have enjoyed a long series of good productivity shocks will hold little money, high levels of nominal government bonds and have high consumption relative to agents that experienced a long series of adverse shocks. Innovations in inflation will ameliorate the distribution of resources if the differences in government debt holdings are sufficiently greater than those in cash holdings. On the other hand, low nominal interest rates on the margin will redistribute to unlucky agents. If the government can also provide (uniform) lump sum transfers, it may be optimal to increase marginal taxes to finance positive lump sum transfers, given that there will be a positive social value to transferring resources to those agents that receive adverse shocks in the current period.

In dynamic models with private information and idiosyncratic productivity shocks, monetary holdings can play multiple roles. As in da Costa and Werning (2007), they can be seen as just another commodity with a relative price that corresponds to the nominal interest rate. Alternatively, money can play the role of an asset. Let's consider these possibilities in order.

If money is just another commodity, optimal monetary policy is tied once again to the optimality of uniform commodity taxation. Golosov, Kocherlakota and Tsyvinski (2003) show that with weakly separable preferences in consumption and labor, uniform commodity taxation is optimal in this class of models, consistent with the findings in Atkinson and Stiglitz (1976). If preferences are allowed to be non-separable, it may be desirable to depart from the Friedman rule if this relaxes incentive compatibility constraints, as previously discussed.

If money is an asset, that is it is held mainly as a store of value rather than for transaction purposes, the picture is quite different. Agents who experience a long history of good shocks will end up accumulating high levels of money. Thus, money holdings can serve as a statistic for an agent history. Moreover, agents can use money to self-insure. This is desirable from an individual standpoint, since the optimal allocation in this class of models exhibits incomplete insurance due to the incentive problem. However, it is suboptimal from a social standpoint, since the additional insurance provided by monetary holdings undermines the provision of incentives.

The results in Kocherlakota (2005) and Albanesi and Sleet (2006) can be extrapolated to derive implications for optimal monetary policy in this context. Both papers show that it is optimal to tax assets with idiosyncratic ability shocks and that the marginal tax on assets should be higher for low income agents. This makes the after tax return on assets between t and $t + 1$ negatively correlated with labor income in $t + 1$, thus discouraging agents from accumulating assets for self-insurance purposes. This form of asset taxation relaxes the incentive compatibility constraint at time $t + 1$ and requires that marginal taxes on monetary holdings should be agent specific. Since optimal allocations are history dependent in this class of economies, the properties of asset taxes will also depend on the degree to which individual asset holdings convey information on past histories of shocks. Kocherlakota (2005) allows labor and asset taxes to depend on the entire history of labor supply and shows that in this case the average or expected marginal tax on assets is zero. If the asset is money, this would correspond to a zero inflation rate. Albanesi and Sleet (2006) allow taxes to be conditioned only on current labor income and outstanding asset holdings. Thus, asset holdings serve as a statistic for the past history of shocks. They show that in this case the optimal marginal average or expected tax on assets should be positive. This would correspond to positive inflation if the asset is money.

Green and Zhou (2005) also consider economies with privately observed idiosyncratic taste or ability shocks and evaluate the efficiency of monetary mechanisms, that is implementations in which money holdings serve as a summary of an agent history. They interpret linear updating rules on the history as inflationary or contractionary mechanisms. Their treatment is considerably more abstract and concentrates on the history encoding role of monetary holdings.

8. - Directions for Future Research

The main purpose of this article is to emphasize the rich set of policy questions raised by the introduction of heterogeneity in monetary economies. Recent work suggests that the implications for optimal policies might be starkly different than for representative agent economies. Albanesi (2005) examines the case with linear income taxation and shows that heterogeneity breaks the link between high inflation and lack of commitment. First, the Friedman rule need not hold under commitment. With a negative cross-sectional correlation between money holdings and labor income, as in the data, optimality of the Friedman rule requires a high Pareto weight on low income agents. Moreover, optimal fiscal and monetary policies are time consistent. The redistributional costs of *ex post* deviations from Ramsey, policies offset the corresponding efficiency gains —. da Costa and Werning (2007) analyze a model with non-linear income taxes and private information on individual abilities. They show that the Friedman rule is optimal if cash holdings and labor effort are gross complements. However, this assumption would lead to a cross-sectional distribution of money holdings that is inconsistent with empirical evidence. In their framework, optimal policies would typically be time inconsistent.

Both papers abstract from aggregate or idiosyncratic shocks and stop short of analyzing sequentially optimal policies. The previous discussion should provide a compelling argument that extending the analysis in this direction is both interesting and important.

One question that remains completely unexplored is optimal stabilization policy in the presence of heterogeneity. It is well known that low skill workers are disproportionally hit by job loss in recessions and that these episodes of unemployment may generate persistent declines in lifetime earnings. To the extent that monetary policy can reduce business cycle volatility and in particular the intensity of recessionary episodes, it may have a significant impact on welfare and play an important role in mitigating earnings inequality though this channel. The analysis of monetary policy with endogenous earnings inequality is a very promising topic for future research.

APPENDIX

A: Characterization of Private Sector Equilibria

Assume that an allocation $\{c^i_{1,t}, c^i_{2,t}, l^i_t, z^i_t, b^i_t\}_{i=1,\,2,\,t\geq 0}$ with $l^i_t > 0$ for $i = 1, 2$ and $t \geq 0$, and a price system $\{P_t, W_t, Q_t, \pi_t\,(j)\}_{t\geq 0, j\in[0,1]}$ constitute a private sector equilibrium for a given policy $\{\overline{g}_t, \tau^i_t, M_{t+1}, B_{t+1}\}_{t\geq 0}$. Then, conditions *(29)* and *(30)* derive from optimality of firm behavior, conditions *(37)* and *(38)* from clearing in the goods and assets markets. The other conditions follow from household optimization.

The Lagrangian for the household problem is given by:

$$L = \sum_{t=0}^{\infty} \beta^t \Big\{ U(c^i_t, l^i_t) - \delta^i_t (P_t c^i_{1,\,t}(1 - z^i_t) - M^i_t)$$
$$- \phi^i_t \Big[M^i_{t+1} + Q_t B^i_{t,t+1} - M^i_t - B^i_{\hat{t}}$$
$$- W_t (1 - \tau^i_t) \xi_i l^i_t + P_t c^i_{1,\,t}(1 - z^i_t) + P_t c^i_{2,\,t} z^i_t + \int_0^{z^i_t} \pi_t(j)\, dj \Big] \Big\}$$

where c^i_t is defined in *(31)* and δ^i_t, ϕ^i_t are the multipliers on the cash in advance constraint and the wealth evolution equation, respectively.

The necessary conditions for household optimization are given by:

$$u^i_{1,t} = P_t\,(\delta^i_t - \phi^i_t)\,(1 - z^i_t) \tag{51}$$

$$\delta^i_t\,(P_t c^i_{1,t}\,(1 - z^i_t) - M^i_t = 0,\ \delta^i_t \geq 0 \tag{52}$$

$$u^i_{2,t} = P_t\,\phi^i_t\,z^i_t \tag{53}$$

$$- u^i_{1,t} = W_t\,(1 - \tau^i_t)\,\xi^i\,\phi^i_t \tag{54}$$

$$u^i_{z,t} + P_t c^i_{1,t}(\delta^i_t + \phi^i_t) - P_t c^i_{2,\,t} \phi^i_t - \pi_t(z^i_t)\,\phi^i_t \begin{cases} < & 0 \text{ for } z^i_t = \underline{z} \\ = & 0 \text{ for } z^i_t \in (\underline{z}, \overline{z}) \\ > & 0 \text{ for } z^i_t = \overline{z} \end{cases} \tag{55}$$

(56) $$\phi_t^i = \beta\,(\phi_{t+1}^i + \delta_{t+1}^i)$$

(57) $$\phi_t^i Q_t = \beta^s\,\phi_{t+1}^i$$

(58) $$\lim_{T\to\infty} \beta^T \left[(\phi_T^i + \delta_T^i) M_T^i + \phi_T^i B_T^i \right] = 0$$

as well as *(32)* and *(33)*. To see that *(58)* is a necessary condition for household optimization, suppose it does not hold and

$$\lim_{T\to\infty} \beta^T \left[(\phi_T^i + \delta_T^i) M_T^i + \phi_T^i B_T^i \right] > 0$$

(The strictly smaller case is rule out by *(34)*). Then, it is possible to construct a consumption sequence such that the budget constraint is satisfied in each period and utility for each type of household is greater, violating optimality. Combining *(51)-(53)* yields *(41)*, while *(53)* and *(54)* determine *(40)*. The expression in *(39)* follows from *(54), (57)* and *(29)*, while *(43)* follows from *(51)-(53)* at $t = 0$. To derive *(44)*, multiply *(33)* by ϕ_t^i and apply *(51)* and *(56)*. Use *(51), (53)-(55)*, multiply by β^t and sum over t from 0 to T. Let T go to infinity and apply *(58)*. This yields:

(59) $$\sum_{t=0}^{\infty} \beta^t \left(u_{1,t}^i, c_{1,t}^i + u_{2,t}^i \left(c_{2,t}^i + \frac{C(z_t^i)}{z_t^i} - \frac{B_0^i}{P_t z_t^i} \right) + u_{l,t}^i l_t^i \right) = \frac{u_{1,0}^i}{1 - z_0^i} \frac{M_0^i}{P_0}$$

From *(56)-(57)*:

$$P_t = \beta_t \frac{\hat{u}_{2,t}^i}{\hat{u}_{2,0}^i} P_0 \prod_{j=1}^{t} R_j \text{ for } t > 1$$

with $\Pi_{j=1}^1 R_j \equiv R_1$, $\Pi_{j=1}^0 R_j \equiv 1$, where $\hat{u}_{1,t}^i = u_{1,t}^i / (1 - z_t^i)$ and $\hat{u}_{2,t}^i = u_{2,t}^i / z_t^i$. Substitute into *(59)*, to obtain *(44)*.

Now assume that an allocation $\{c_{1,t}^i, c_{2,t}^i, l_t^i, z_t^i, B_t^i, M_{t+1}^i\}_{i=1,2,t\geq 0}$ and a price system $\{P_t, W_t, Q_t, q_{t,}(j)\}_{t\geq 0, j\in[0,1]}$ satisfy *(29)-(44)* and *(37)* for a given policy $\{\bar{g}_t, \tau_t, M_{t+1}, B_t\}_{t\geq 0}$ for which *(35)* holds. Then, goods and financial firms optimize. To see that household optimization conditions are satisfied consider an alternative candidate plan $\{c_{i1t}^{!}, c_{i2t}^{!}, l_{it}^{!}, z_{it}^{!}\}_{i=1,2,t\geq 0}$ which satisfies the intertemporal budget constraint for the price system $\{P_t, W_t, Q_t, q_{t,} (j)\}$. This implies that:

$$\Delta \equiv \lim_{T\to\infty} \beta^t \left\{ u^i_{1,t}\left(c^i_{1,t} - (c^i_{1,t})'\right) + u^i_{2,t}\left(c^i_{2,t} + \frac{C(z^i_t)}{z^i_t} - (c^i_{2,t})' - \frac{C\left((z^i_t)'\right)}{(z^i_t)'}\right) - \gamma\left(l^i_t - (l^i_t)'\right)\right\} \geq 0$$

using *(39)* and the fact that $\{c^i_{1,t}, c^i_{2,t}, l^i_t, z^i_t\}_{i=1,\,2,\,t\geq 0}$ satisfies *(41)*-*(44)* and that the intertemporal budget constraint holds as a weak inequality using *(34)* and *(33)* for the price system $\{P_t, W_t, Q_t, q_{t,} (j)\}_{t\geq 0, j\in[0,1]}$. By concavity of u^i:

$$D \equiv \lim_{T\to\infty} \sum_{t=0}^{T} \beta^t \left(U(c^i_t, l^i_t) - U\left((c^i_t)', (l^i_t)'\right)\right) \geq \Delta$$

where $c^{!}_{it}$ is defined by *(31)*. This establishes the result since *(38)* and *(37)* guarantee market clearing.

B: Solving the Ramsey Problem

To solve the Ramsey allocation problem, it is useful to define the function $Z\,(R,\,c) = \max\,\{\underline{z},\,\min\,\{z^*,\,1\}\}$ where z^* solves:

$$c\left(\frac{1}{\rho} - 1\right)\left(1 - R^{\frac{\rho}{\rho-1}}\right) - \theta(z^*) = 0$$

By assumption A3, $Z_c > 0$ and $Z_R > 0$ for $c > 0$ and $R \geq 1$. The constraint $z^i_t = Z\,(R_t, c^i_{2,t})$ needs to be imposed on the Ramsey allocation problem to ensure that the government chooses the same value of z^i_t that would be chosen by the agents in a private sector equilibrium. This constraint is substituted in the Ramsey allocation problem.

The Lagrangian for the Ramsey problem is:

$$\Lambda = \sum_{t=0}^{\infty} \beta^t \sum_i \Big\{ W^i \left(c^i_{1,t}, c^i_{2,t}, l^i_t, z^i_t, \eta_i, \lambda_i \right)$$
$$- \omega_t \left(\bar{g}_t + \sum_i v_i (c^i_{1,t}(1 - z^i_t) + z^i_t \hat{c}^i_{2,t} + C(z^i_t) - \xi_i l^i_t) \right) \Big\}$$
$$- \sum_{t=1}^{\infty} \beta^t \left[\mu_t (1 - R_t) + \sum_i \mu^i_t \left(\frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} - R_t \right) \right] - \sum_{t=0}^{\infty} \beta^t \zeta_t \left(\frac{-u^2_{l,t}}{\xi_2 \hat{u}^2_{2,t}} - \frac{-u^1_{l,t}}{\xi_1 \hat{u}^1_{2,t}} \right)$$
$$- \sum_i \lambda_i \left(\hat{u}^i_{1,0} \frac{M^i_0}{P_0} + \hat{u}^i_{2,0} \frac{B^i_0}{P_0} \right) - \sum_i \mu^i_0 \left(c^i_{1,0}(1 - z^i_0) - \frac{M^i_0}{P_0} \right)$$

where

$$W^i(c^i_{1,t}, c^i_{2,t}, l^i_t, z^i_t, \eta_i, \lambda_i) = \eta_i U(c^i_t, l^i_t) + \lambda_i \left[u^i_{1,t} c^i_{1,t} + u^i_{2,t} \left(c^i_{2,t} + \frac{C(z^i_t)}{z^i_t} \right) + u^i_{l,t} l^i_t \right]$$

The Ramsey allocation problem is to maximize Λ with respect to $c^i_{1,t}$, $c^i_{2,t}$, l^i_t, z^i_t R_t, P_0, and minimize Λ with respect to μ^i_t, λ_i, ζ_t, μ_t, ω_t, for $i = 1, 2$ and $t \geq 0$, subject to $z^i_t = Z(R_t, c^i_{2,t})$. I will characterize the solution to the Ramsey allocation problem by deriving the first order necessary conditions for this problem. Since the second order necessary conditions for this problem involve third derivative of U, the task of verifying that they hold is intractable but for very specific assumptions on U. As Lucas and Stokey (1983), I simply assume that a solution of the system of equations resulting from the first order necessary conditions exists and constitutes an optimum for the Ramsey allocation problem.

It is convenient to introduce the following notation:

$$\hat{u}^i_{11} = \frac{u^i_{11}}{(1 - z^i)^2}, \; \hat{u}^i_{22} = \frac{u^i_{22}}{(z^i)^2}$$
$$\hat{u}^i_{12} = \frac{u^i_{12}}{(1 - z^i) z^i} = \hat{u}^i_{21} = \frac{u^i_{21}}{(1 - z^i) z^i}$$

$$\hat{W}^i_{1,t} = (\eta_i + \lambda_i)\hat{u}^i_{1,t} + \lambda_i(\hat{u}^i_{11,t}(1 - z^i_t)c^i_{1,t} + \hat{u}^i_{12,t} z^i_t \hat{c}^i_{2,t}$$
$$\hat{W}^i_{2,t} = (\eta_i + \lambda_i)\hat{u}^i_{2,t} + \lambda_i(\hat{u}^i_{12,t}(1 - z^i_t)c^i_{1,t} + \hat{u}^i_{22,t} z^i_t \hat{c}^i_{2,t}$$
$$W^i_{1,t} = (\eta_i + \lambda_i)u^i_{1,t} + \lambda_i u^i_{ll,t} l^i_t$$
$$W^i_{z,t} = \eta_i u^i_{c,t} c^i_{z,t} + \lambda_i \frac{1}{\rho} c^i_{2,t}\left(1 - R_t^{\frac{\rho}{\rho-1}}\right)\hat{u}^i_{2,t}$$
$$+\lambda_i[R_t(1 - z^i_t)c^i_{1,t} + z^i_t c^i_{2,t} + C(z^i_t)]c^i_{z,t}\hat{u}^i_{2,t}\left(\frac{u^i_{cc,t}}{u^i_{c,t}} + \frac{(1-\rho)}{c^i_t}\right)$$

where

$$c^i_{z,t} = \left[(c^i_{2,t})^{\rho} - (c^i_{1,t})^{\rho}\right](c^i_t)^{1-\rho}$$

The first order conditions for the Lagrangian $i = 1, 2$ are:

$$\text{(60)} \quad v_i\omega_t = \hat{W}^i_{1,t} - \mu^i_t\left(\frac{\hat{u}^i_{11,t}}{\hat{u}^i_{2,t}} - \frac{\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}}\frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}}\right) - \zeta_t(-1)^i \frac{-u^i_{l,t}}{\xi_i \hat{u}^i_{2,t}}\frac{\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}}, \text{ for } t > 0$$

$$\text{(61)} \quad \begin{aligned} v_i\omega_t &= \hat{W}^i_{2,t} \\ &+[W^i_{z,t} - \omega_t v_i(c^i_{2,t} - c^i_{1,t} + \theta(z^i_t))]Z_c(R_t, c^i_{2,t}) \\ &-\mu^i_t\left(\frac{\hat{u}^i_{12,t}}{\hat{u}^i_{2,t}} - \frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}\frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}}\right) - \zeta_t(-1)^i \frac{-u^i_{l,t}}{\xi_i \hat{u}^i_{2,t}}\frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}, \text{ for } t > 0 \end{aligned}$$

$$\text{(62)} \quad 0 = W^i_{l,t} - \zeta_t(-1)^i \frac{-u^i_{ll,t}}{\xi_i \hat{u}^i_{2,t}} + \xi_i v_i \omega_t, \text{ for } t \geq 0$$

$$\text{(63)} \quad \begin{aligned} &\sum_i [W^i_{z,t} - \omega_t v_i(c^i_{2,t} - c^i_{1,t} + \theta(z^i_t))]Z_R(R_t, c^i_{2,t}) + \mu_t + \sum_i \mu^i_t \\ &= \sum_i \frac{\lambda_i \hat{u}^1_{2,0}}{R_t}\frac{B^i_0}{P_0}, \text{ for } t > 0 \end{aligned}$$

$$\mu_t(1-R_t)=0, \mu_t \geq 0,\ R_t \geq 1, \text{ for } t>0$$

(64)
$$\mu_t^i\left(\frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i}-R_t\right)=0, \text{ for } t>0$$

$$\zeta_t\left(\frac{-u_{l,t}^2}{\xi_2\hat{u}_{2,t}^2}-\frac{-u_{l,t}^1}{\xi_1\hat{u}_{2,t}^1}\right)=0$$

(65)
$$\frac{-u_{l,t}^2}{\xi_2\hat{u}_{2,t}^2}-\frac{-u_{l,t}^1}{\xi_1\hat{u}_{2,t}^1}\leq 0, \zeta_t \geq 0, \text{ for } t \geq 0$$

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