Corrigendum to:

Optimal Information Releasing in Multidimensional Public Procurement

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In Doni (2006) the case of the secrecy of the information (S_1) has been incorrectly analysed. According to our assumption, the sellers' strategy were equal to:

$$p^1(c_i) = c_i + \frac{\int_{c_i}^{c} (1 - F^c(s))ds}{1 - F^c(c_i)}, \forall i = \mathrm{A, B}$$

This hypothesis was wrong and the mistake was due to a misunderstanding[1]. In fact such a strategy would be optimal if the procurer (P) adopted a first price auction as a selection mechanism.

The aim of this note is twofold: in the first place, by taking advantage of our mistake, we want to analyse a new selection policy (S_0), in which the P awards the contract on the basis of a reverse, first price, auction. We then compare it with the other two policies analysed in the original paper: multidimensional auction with a private (S_2) or public (S_3) revelation of the P's quality evaluation. Subsequently, we give an intuition of how the bidders' optimal strategy in the S_1 game differs from the one indicated in the original paper.

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[1] Luckyly, the main results of the paper were not affected by this mistake. The only proposition that is no longer valid is proposition 3, p. 260. Moreover, graph 2 is wrong and observation 1 is not clearly supported in the paper.

If the P adopts the S_0 strategy, he then selects the seller with the lowest cost, whatever her quality level may be. Therefore, a small advantage in the cost level would allow a bidder to win the auction even if her quality level were really lower than that of her opponent. This observation can be synthesised as follows:

Observation 0: if the P uses the S_0 strategy, then the auction mechanism is not necessarily efficient, since it can assign the contract to the rival bidder who has a lower valuation.

It is possible to compare S_0 and S_2 by calculating the P's expected utility in the two cases. Assuming N bidders we know that if the P adopts a first price auction (S_0) we obtain:

$$E[U(S_0)] = E[q_i - p(c_i)] \qquad \text{where } c_i < c_j \ \forall i,\ j = 1,\dots,\ N;$$

since q_i and c_i are assumed to be independent, we get:

$$E[U(S_0)] = E(q) - E[p(c_{min})]$$

Thanks to the well known properties of first price auctions, we can replace the expected price of the winning bidder with the expected cost of the second lowest cost seller. Using the classical notation of order statistics, we can write:

$$E[U(S_0)] = E(q) - E[c_{(2)}]$$

As far as strategy S_2 is concerned, in the original paper we have shown that the sellers' equilibrium strategies resemble the ones that would be adopted in a first price auction, where the generic bidder i valuation is equal to $v_i = (q_i - c_i)$ and her bid is equal to $s_i = (q_i - p_i)$. Consequently, the P's expected utility coincides with the expected value of the score offered by the winning bidder. As the bidders' valuations are IPV, we can be sure that this value is equivalent to the expected value of the second most efficient seller. Therefore we can write:

$$E[U(S_2)]= E[v_{(N\text{-}1)}]$$

Thanks to a recent paper of Engelbrecht *et* al. (2007) we can easily compare S_0 and S_2:

Proposition 3: *i)* If $N = 2$, then $E[U(S_0)] > E[U(S_2)]$; *ii)* if N is sufficently large, then $E[U(S_0)] < E[U(S_2)]$.

Proof: see Engelbrecht *et* al. (2007), propositions 1 and 2, p. 632.

Thanks to our propositions 1 and 2, we also know that if $N = 2$, S_3 strictly dominates S_2, given mild conditions on $F_c(c)$. Unfortunately, the current literature does not contain any results that permit us to compare S_2 and S_3 for a higher number of bidders. At the same time, we have not been able to reach a general conclusion with respect to the comparison between S_0 and S_3. In the case of only two bidders, by assuming that their costs are extracted by a random variable uniformly distributed, it is possible to show[2] that S_0 dominates S_3, whatever the value of $\Delta = q_H - q_L$ may be. Nevertheless, we cannot exclude the possibility that the result might change in the case of different distributions.

To summarise, if competition is low (N=2), the P may find it unprofitable to take into account the sellers' quality during the awarding phase. Obviously, this extreme conclusion holds only if quality is an exogenous variable. Conversely, if this element depends on the sellers effort, then the first price auction contains a serious drawbacks, because it does not create any incentive to the provision of quality.

Finally, we want to analyse the sellers' equilibrium strategy in game S_1. Let us assume that a generic bidder i has cost equal to the maximum level: $c_i = \bar{c}$; she will choose the price to bid by solving the following maximization problem:

$$\operatorname*{Max}_{p_i}\left(p_i - \bar{c}\right)\mathrm{Prob}\left(q_i - p_i \geq q_j - p_j\right)$$

by taking into account every possible realization of the couple (q_i, q_j), we can rewrite the problem as:

$$\operatorname*{Max}_{p_i}(pi - \bar{c})\{[\alpha^2 + (1-\alpha)^2]\mathrm{Prob}(p_j \geq p_i) + \alpha(1-\alpha)$$
$$\mathrm{Prob}(p_j \geq p_i + \Delta) + \alpha(1-\alpha)\,\mathrm{Prob}(p_j \geq p_i - \Delta)\}$$

[2] The Author can provide the handwritten calculations upon request.

If the seller i hypothesises that her opponent will bid as in a first price auction, then she knows that $p_j \leq \bar{c}$. On the other hand, as the sellers' expected profit can never be negative, $p_i(\bar{c})$ must be at least as large as $\bar{c}$. More specifically, in a first price auction, we know that $p_i(\bar{c}) = \bar{c}$ and the expected profit of this type of bidder is equal to 0. Conversely, if the P runs a multidimensional auction, also this type of bidder can gain a positive expected profit. In fact, by bidding $p_i(\bar{c}) > \bar{c}$, she can win with positive probability every time that her quality will be higher than that of her opponent. Therefore, if the P adopts S_1, the sellers' equilibrium strategy cannot be equal to the one used in a first price auction. This kind of policy probably induces the sellers to bid in a less aggressive way. They try to earn a rent that is due not only to the asymmetric information on costs, but also to sellers' differentiation according to the P's preferences.

Unfortunately, it is not possible to calculate the bidders' equilibrium strategy exactly in this kind of model. Doni *et* al. (2008) succeed in arriving at more precise conclusions in a very similar setting; the main differences are that they represent *i)* q_i and q_j by means of continuous, i.i.d. random variables, and *ii)* c_i and c_j as random variables known by both sellers but unknown to the P.

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