Classical and Bayesian Methods for the VAR Analysis: International Comparisons

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This work deals with the main limits of VAR methodology when applied to macroeconomics: there is a limited number of time series available, and a high number of parameters are to be estimated. The two issues are approached appealing respectively to temporal disaggregation methods and Bayesian inferential techniques. In order to assess the efficacy of the solutions proposed in the literature, a comparative exercise on monetary transmission mechanisms for the United States, the United Kingdom and Japan is presented.

1. - Introduction

The mingling between statistics and economic science has experienced a substantial growth during the last half-century: over time, the importance of measurement and data-description in addition to a purely theoretical economic approach has been widely appreciated.

The tasks macro-econometricians aim to address range from...
data description and forecasting to structural analysis. In that, the econometric model, and along with its correct formulation, can be placed at the heart of numerous policy-related decisions and informative reasoning.

Two main branches of econometrics influence macro-econometric model specification: structural and non-structural econometrics. The former finds its roots in the 1930s and its main justification in specific economic theories, namely the Keynesian theoretical framework. It developed via large-scale models, parameterized over systems of equations and covering an impressive number of variables, each one of them addressing a particular aspect of the theory it was designed to model. Confutation of the Keynesian ideology fuelled by the 1970’s stagflation period and by previously advanced theoretical critiques (the well-known Lucas’ critique) contributed to re-directing academic attention towards a less theoretical framework. Dating back to the 1920s with Slutsky and Yule’s AR processes and subsequent ARIMA generalizations of the 1970s (Box and Jenkins), non-structural econometrics is mainly focused on the forecasting and modelling performance of the model rather than on its ability to give voice to specific theoretical frameworks.

VAR systems can be appreciated as a bridge between structural and non-structural econometrics. It is within Sims’ (1980) critique against the econometric framework popular at that time that the essence of VARs is best expressed. Sims points out how large-scale structural simultaneous equation models too often require the use of “incredible restrictions” — i.e. a large number of a priori restrictions, mirroring specific economic hypotheses — and introduces VAR systems as a valid alternative to them. The VAR specification enables the modeller to embody particular theoretical constructs and, at the same time, to take distance from any a priori restriction on an as-need basis. Easy to use and to interpret, numerically stable, not differentiating between endogenous and exogenous variables and able to model any covariance-stationary stochastic process, reduced-form VAR models have grown a solid reputation in forecasting and data description analysis over the last 20 years, mainly thanks to their generality and
non-reliance on economic theory. In its aim of reducing to a minimum the amount of *a priori* information imposed by the modeller and of exploiting reduced-form correlations in observed time series, reduced-form VAR has proven to generally do particularly well in out-of-sample forecasts. Truly enough, if reduced-form VARs share the merit of avoiding heavy reliance on economic theory, they also preclude interesting structural (conditional) econometric forecasting and factual analysis which require identified structures, as for instance studies focused on monetary-policy transmission mechanisms. The results of estimated models being heavily dependent on the identification technique applied, VARs are not as unanimously accepted in their structural form as in the reduced one. Identified VARs can, and often are, be aligned with other structural models, such as Lucas’ DSGE (*Dynamic Stochastic General Equilibrium*) specifications, modelling economies with fully-articulated preference, technologies and rules of the game. Having become very popular in macroeconomics over the past 25 years¹, DSGE models are often estimated, identified and evaluated in relation to VARs.

The attention VAR systems have been given by both academic literature and international institutions in forecasting, data description, benchmarking, model evaluation and, to a lesser extent, structural inference, justifies a thorough investigation of techniques that can be applied for addressing the main drawback VARs show, that is a high parameter dimensionality. When combined with time series limited in length², the *spectrum* of unknown variables increasing as the square of the number of variables in the system quickly erodes the degrees of freedom in VAR estimations. Model size and dimensionality of the parameter

¹ This mainly thanks to DSGE methodology which did not grow around precise economic theories and thus inherited the same defects the early wave of structural econometrics did (see system of equation approach). DSGE specifications focus on agents’ tastes and available technologies of production rather than on decision rules. In this framework, they manage to nest a variety of popular and useful preference and technology structures.

² Structural modifications of the economic system and modellers’ attention on business cycle rather than on single periods, makes also long time series not completely usable.
space pose a serious challenge for VARs leading to serious issues such as hampering misspecification and over-fitting. This paper reviews and analyzes the literature dealing with over-parametrized VARs in the methodological solutions proposed.

After briefly exposing salient features of VARs (paragraph 1), a first set of solutions active on sample size is investigated: paragraph 3 outlines major temporal disaggregation procedures. The Bayesian inductive inferential method offers relevant tools in reducing the constraints imposed on the number of parameters by the efficiency of the estimation procedure without need to link the model to specific economic beliefs. The inner side of the Bayesian approach is outlined in paragraph 4, where an empirical test juxtaposing classical and Bayesian methods in predictive accuracy is proposed. Paragraph 5 introduces a BVAR macroeconomic model under Litterman prior estimated for three economic systems, i.e. the United States, the United Kingdom and Japan. An impulse-response analysis built on such a specification allows: 1) to prove the model success in capturing relevant links among macro-variables in both sign and timing dimensions and thus offering new material on VAR specifications within general equilibrium analysis and 2) to provide new evidence in one of the classical fields of economic interest, the effects of monetary policy. Paragraph 6 concludes.

2. - Vector Autoregressive Models

In econometrics, VAR stands for a system of equations including \( n \) variables, each one of them regressed over current values, \( p \) lags and a set of deterministic factors. A VAR model, in its bivariate soul as matching point between structural and non-structural econometrics, comes in two varieties, structural and reduced form respectively. The structural form, which attracts statistical and economic interests since it boasts non-correlated white noise errors, can be written as:

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3 The statistical procedure may be misleading in considering some relevant variables even if, in reality, they only model random characteristics or casual links.
The possibility for contemporaneous influence among variables this form enables for can be acknowledged as its strength since it allows for the identification of residuals as pure shocks. On the other hand, the structural specification does not grant the use of traditional techniques at estimation stage (such as OLS solution, method of moments and maximum likelihood technique) insofar as it presents simultaneity between errors and regressors. The key role played by the a-theoretical VAR formulation comes thus to be intuitive as reduced VARs with pre-determined regressors are effectively estimable. The reduced-form VAR is derived through the multiplication of the structural equation by the matrix $B^{-1}$ as to isolate contemporaneous “t”-terms on the left side of the equation:

$$ (1) \quad Bx_t = b_0 + A_t x_{t-1} + \epsilon_t $$

where $c_0 = B^{-1}b_0$, $C_1 = B^{-1}A_t$, $\epsilon_t = B^{-1}\epsilon_t$.

The identification issue applied to the VAR framework entails the establishment of a univocal connection between the reduced (estimable model) and the structural (model bearing verifiable shocks) parametric space via the introduction of a sufficient number of restrictions, i.e. $(n^2-n)/2$. Numerous solutions have been put forward in this regard. Among them, Sims’ approach proposed in 1980 (recursiveness assumption) is commonly recognized as the most established one. Having roots in the rigorous statistical methodology, it is based on the pre-multiplication of the reduced VAR by the inverted Cholesky factor of the associated covariance matrix. In that, a triangular structure is imposed on the matrix $B$ so that the structural model — better known as “recursive VAR” under this assumption — can be identified by outlining a recursive structure for it or, in other words, via a set of zero-type restrictions.

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4 In this respect, it is important to recall that the combination of not-strictly exogenous regressors and correlation among residuals could pose an obstacle in deriving unbiased and efficient estimators.
Other identification techniques come to be more rooted in economic theories and sometimes rely on other models. For instance, (log) linear DSGE share features that can be re-conducted to (non-zero) restrictions on a reduced VAR representation and are thus often applied at identification stage (see Canova, 2007 for details).

The econometric specification discussed in this paragraph is characterized by its principal aim, that is the study of the impact which defined shocks have on the variables in the model, and thus by the tools applied to reach this goal: impulse-response function and variance decomposition analysis. Impulse-response functions (IRFs) are derived from the MA-VAR formulation and they aim at capturing the response of each equation’s dependent variable to structural (orthogonal, if recursive identification applies) shocks. Every analysis implies an amount of \( n^2 \) IRFs for a \( n \)-variable VAR model:

\[
D_h = \frac{\partial x_{i+h}}{\partial \varepsilon_t}.
\]

where the \( ij \) element in matrix \( D_h \) is the response of the \( i \)-th variable to a one-unit increase in the \( j \)-th VAR error at the time \( t+h \).

Forecast error variance decomposition offers an alternative to impulse-response functions towards the investigation of VAR systems’ dynamics. Briefly, via the analysis of the forecast error \( (x_{t+h} - E_r x_{t+h}) \), it allows the researcher to discriminate among variables’ contributions to the \( h \)-step forecast error variance of variable \( i \). Coherent results, irrespective of the identification procedure adopted, are achieved only when considering a relatively long period of time \( h \).\(^5\)

3. - Temporal Disaggregation

Temporal disaggregation (TD) is a statistical procedure which leads to the computation of high-frequency time series (HF data)

\(^5\) Such kind of analysis if conducted over a limited time horizon can be misleading in its results when each shock is allowed to influence the variables with different time-lag.
from data observed at lower frequency (LF data). Such a set of techniques takes importance in relation to the discrepancy between supply, where the National Statistical Institutes face relevant gathering costs in collecting statistical information, and demand, with researchers asking for an always-wider spectra of datasets, one can normally observe in a “time-series market” context. This section reviews existing TD literature in its offering a panacea for misspecified VARs and concludes with an empirical exercise comparing data-fit properties across a set of TD techniques.

3.1 The Methods Proposed so Far

The current methodology puts forward different solutions for the matter at stake. Apart from the more plain and classical mathematical or statistical-based methods (data-based approach), it is essential to focus on methodologies that involve related indicators (model-based approach) in computations, i.e. adjustment procedures and two-stage methods. Related indicators mean time series which are reasonably and actually connected with the variable to be interpolated and which are observed at the desired, higher frequency (HF series).

Adjustment procedures entail a preliminary-estimation stage, centred around reference indicators, and a subsequent adjustment intervention, where the values previously derived are made consistent to the observed LF data. In this consideration, Denton (1971) proposes a set of procedures based on the minimization of a quadratic penalty function in the difference, \( p(y_h, z_h) \), between revised (HF adjusted values, \( y_h \)) and unrevised series (HF preliminary estimates or original values, \( z_h \)). The minimization of \( p(y_h, z_h) \), subject to the aggregation constraint, results in:

\[
(4) \quad \min u = (y_h - z_h)^\top A (y_h - z_h) - 2\lambda (y_l - \bar{C} y_h),
\]

---

6 Notation is as follows. Subscript “l” indicates LF data, whereas subscript “h” denotes HF ones, i.e. series observed at a higher frequency than LF data.

7 Techniques relying on structural time series and the Kalman filter are not discussed.
with a non-singular \((n \times n)\) matrix and solution: \(y_h = z_h + R(y_l - C'z_l)\), where \(R = A^{-1}C(C'A^{-1}C)^{-1}\). Final estimates for the HF values are given by the unrevised values plus an adjustment factor proportional to the discrepancies between LF data and aggregated HF estimates. The definition of \(A\) is of key importance here inasmuch as it directly impacts on the factor driving the distribution of discrepancies on the HF sub-periods. By reformulating this matrix in different ways it is possible to build a variety of alternative specification for \(p(y_h, z_h)\): quadratic in the differences, centred on first and second differences etc.

The second group of model-based methods are meant as a one-stage procedures where the estimation and reconciliation steps come to be unified under a constrained regression model (Friedman, 1962; Chow and Lin, 1971; Bournay and LaRouque, 1979; Fernández, 1981; Litterman, 1983; Salazar et al., 1997; Santos and Cardoso, 2001). Those approaches explicitly model the relationship between variable and related series in regression terms, over the HF horizon. Depending on the function acquiring static or dynamic specification, they can be further classified as static or dynamic methods respectively. Static methods share their roots in the precocious work of Chow and Lin (1971) in which the following simple regression model is deemed description of the relationship between HF series and related series observed at the same time frequency:

\[
y_h = X_h \beta + \vartheta_h
\]

with \(X_h\) grouping \(n\) known observations on \(k\) related series and \(\vartheta_h \sim (0; V_h)\). In this way, the authors derive the following estimates for disaggregated HF series:

\[
\hat{y}_h = X_h \hat{\beta} + \left(V_{\hat{\beta}}V_l^{-1}\right) \hat{\vartheta}_l
\]

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*To be reminded, for seek of completeness, that to the same category do belong also the combination of ARIMA-based regression-based approaches (Wei W.W.S. - Stram D.O., 1990; Guerrero V.M., 1990).*
It is simple to notice how $\hat{\beta}$ is nothing more than the GLS (Generalised Last Squared) estimator of $\beta$ in the LF regression conditional to $X_h$, to be said the most efficient estimator in the class of the unbiased and linear estimators (Aitken’s theorem, generalization of the Gauss-Markov’s theorem). Furthermore, the estimates themselves consist of two fundamental parts. A first component ($X_h\hat{\beta}$) comes from the BLUE estimates for the coefficients, which are computed via the aggregation of the HF related series over the LF sub-period. The second element directly depends on the residuals of the LF model. It conveys the importance that insights on the structure of the HF residuals, and so on the matrix $V_h$, carry within the Chow-Lin TD solution. In most cases, though, it is practically impossible to estimate $V_l$ from the data, and thus $V_h$ via the LF-HF relationship. Considering this limitation together with the strategic importance that such a step holds within Feasible GLS estimation procedures relying on a consistent estimate for $V_h$, the literature offers multiple alternative TD solutions depending on the kind of residual-generating process assumed. Briefly:

— Chow - Lin WN: a white noise process with zero mean and variance $\sigma^2_{\theta_h}$ is supposed to drive the residual movements. This hypothesis leads to an equal allocation of LF discrepancies over all the HF sub-periods at reconciliation stage. Corresponding covariance matrix: $V_h = I_h \sigma^2_{\theta_h}$.

— Chow - Lin AR(1): under this assumption, the estimation of the auto-regressive parameter $\rho$ is central. $\rho$ can be obtained either via iterative procedures (Chow and Lin, 1971), or via maximum likelihood techniques (Bournay and Laroque, 1979), or by minimizing the weighted sum of LF residuals (Barbone et al., 1981). Corresponding covariance matrix: AR(1) standard;

Refer to Di Fonzo T. (2003) and respective authors mentioned in the forehand of the paper for a thorough review of the here-only-intuitively-introduced TD methodologies.
— Fernández RW: by considering a random walk hypothesis as HF residual-generating process, the author implicitly covers the case of non-stationary series with no cointegration between disaggregated and related time series, or with a cointegration vector that differs from $\beta$. Fernández draws a bridge between Denton’s QLF (quadratic loss function approach) and BLUE estimators, by proving how the former can be re-expressed in terms of the latter. Corresponding covariance matrix: $V_h = \sigma^2_{\epsilon_h} (D'D)^{-1}$, where $D'D = A$;

— Litterman RW Markov: it entails a generalization of the Fernández's approach, as one can notice from the structure imposed on HF residuals $\{\vartheta_i^H\}$:

\[
\begin{align*}
\vartheta_i^H &= \vartheta_{i-1}^H + \epsilon_i^H \quad \text{for } i = 1, \ldots, n \\
\epsilon_i^H &= \varphi \epsilon_{i-1}^H + \eta_i^H \quad \text{for } i = 1, \ldots, n; \quad \eta_i^H \sim \text{WN}(0; \sigma^2_{\eta_i}); \quad -1 < \varphi < 1
\end{align*}
\]

Corresponding covariance matrix: $V_h = \sigma^2_{\epsilon_h} (D'H'HD)^{-1}$ where the matrix $H$ has units on its main diagonal and the opposite of the auto-regressive coefficient on the diagonal to the right of main one. In a static framework as in Chow and Lin’s model and relative derivations, the dynamic structure of the series is modelled on a disaggregated basis solely via the information lying in related series and error terms. Those solutions forget to consider how a more realistic representation of the system can be obtained by simply including lags of the dependent variable in the regression specification. With these doubts on the capability of static models to best model the actual time development of data, the literature has progressively turned attention to the specification of the regression model itself.\(^{10}\) Hence, the dynamic specifications proliferate:

\[
y_i^H = \rho y_{i-1}^H + x_i^H \beta + \vartheta_i^H \quad \text{with } \vartheta_i^H \sim \text{WN}(0; \sigma^2_{\vartheta_i}); |\rho| \leq 1
\]

Among the more well-known approaches proposed within the

\(^{10}\) It is to be kept in mind how the more a model can fit the original sample, the less important a correct specification of $V_h$ is.
dynamic environment, the work of Santos and Cardoso (SC) (2001) is central. In reformulating the previous regression relationship by means of the Klein (1958) transformation, they derive a solution to the TD issue in the dynamic context which delivers results not conditional to the first observation and coherent with the standard Chow-Lin, BLUE approach. When the model is re-formulated in a static form with AR(1) residuals\(^{11}\), the Chow-Lin estimation procedure can be applied, even though the spirits of the two models remain substantially different:

\[ \hat{y}_h = Z_h \hat{\gamma} + V_h C V_l^{-1} \left( y_l - Z_l \hat{\gamma} \right) \]

with \( \hat{\gamma} = \left( X_l V_l^{-1} X_l \right)^{-1} X_l V_l^{-1} y_l \)

\[ Z_h = \left[ X_h \ : \ \rho \right] \]

3.2 Exercise: Temporal Disaggregation of Real GDP

Real Gross National Product (GDP) is the ideal target of an empirical study aimed at comparing the performance of various model-based TD techniques. This comes natural when reflecting on the role real GDP plays as a key economic indicator and its typical only-quarterly survey frequency in contrast to the monthly frequency of main macro data. Lastly and within the scope of this work, the choice is coherent with the eventual development of a complete macroeconomic VAR model drown on a monthly frequency as to soften over-parametrization-related issues.

Quarterly real GDP series for the United States (US), the United Kingdom (UK) and Japan (JPN) are disaggregated to a monthly frequency over an almost-20-years sample period (1986:01 to 2006:06). In the exercise, the selection of the related series to consider is a crucial step, as it happens for all model-

\(^{11}\) That is, the basic dynamic model is re-expressed as the sum of the following terms: 1) a weighted sum of past and actual values of the regressors; 2) an error term that follows an AR(1) process with \( \rho \) as auto-regressive coefficients; 3) the so-called “truncation remainder” given by the expected value of the first observation conditioned to the reference indicators of that and the previous periods.
based disaggregation procedures. The final decision was reached by taking into account data availability constraints together with 1) a preferred focus on real GDP in its expenditure form rather than in its production measure, in consideration of the level of detailed information the latter requires; 2) each country’s GDP computational procedure (chain vs. fixed weighted)\(^{12}\); 3) national coincident index components for each country as from the Conference Board; 4) preliminary GDP estimation models applied in each country. In line with Robertson and Tallman (1999), non-agricultural payroll employment, total industrial production index and consumption expenditure are selected for the US. The UK and JPN analysis comprise the first two variables in the US exercise together with, respectively, retail sales index and real retail sales as third component.\(^ {13}\)

The cluster of TD procedures compared embraces all the ones mentioned before together with some evolutions of the Santos and Cardoso’s basic dynamic model that allows for the possibility of AR(1)-type residuals in the dynamic specification with both fixed and non-fixed AR parameter.

As a first step of the comparative exercise, estimates for the log-expressed models are computed.\(^{14}\) Thereafter, the comparison across TD specifications is developed on the simultaneous consideration of both moments and properties of the under-model-assumption WN error terms and goodness of fit statistics, as given by the Akaike and Schwarz criterion (see Table 3). Within the static framework, the standard Chow-Lin AR(1) solution turns out to be inadequate when presented with unit root residuals, which is the case for all three country specifications (Dickey-Fuller test, with more than one lag for the cases of the US and JPN). As a natural step-forward, the random-walk Fernández model is thus analyzed in the behaviours of its residuals. When the correlogram

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\(^{12}\) As for the implications that different GDP computation procedures play when choosing the related series to be considered, refer to Ingenti R. e Terhan D. (1996).

\(^ {13}\) Details and sources for the time series are gathered in the data APPENDIX.

\(^ {14}\) Implications lying in the use of log-transformed data are pointedly considered; Consult in this regard Salazar E.L. et al. (1997); Proietti T. (1998); Aadland D.M. (2000).
shows a relevant first order correlation for $\epsilon_i^H$, as could be observed for all three countries, Litterman specification follows. Specifically for the Litterman solution, two different model estimations are performed in order to account for its poor performance under negative AR(1) parameter (see Litterman, 1983). In such cases, the classical Litterman specification is joined by a twin version constrained in the AR parameter, which is set at the value assumed by the first-lag coefficient in an AR(1) estimation for Fernández residuals (such a solution always delivers positive AR(1) coefficients). The modified Litterman model is estimated for the exercises of the UK and JPN with coefficients fixed at 0.7545 and 0.685065 respectively. In both cases, this second constrained model performs better than the unconstrained Litterman specification in terms of residuals’ statistics (standard deviation, approximation to normality and correlogram) and goodness-of-fit indicators. In parallel to the static framework, modifications of the dynamic SC model prove to be more proper than its original specification is. This conclusion can be drawn based on residuals’ properties: the latter delivers residuals which, once the implicit AR(1) structure of re-formulated SC specification is taken into account, present AR(1) characteristics for all the three country exercises. Goodness-of-fit indicators do in general endorse this conclusion. Final results state the supremacy of the following country-models:15 16

— US: The Fernández solution proves to fit the data the best as from the information lying in correlogram and distribution of the residuals $\epsilon_{i,h}$ together with the non-substantial difference in goodness-of-fit indicators it presents relative to the dynamic specifications.

$$\ln y_i^H = \alpha + \beta_1 \ln IPP_i^H + \beta_2 \ln CR_i^H + \beta_3 \ln ONA_i^H + \vartheta_i^H \text{ with } \vartheta_i^H = \vartheta_{i-1}^H + \epsilon_i^H$$

— UK: Static models seem to be preferable also for the UK

15 It is important to note how none of the under-model-assumption WN residuals resulting from the best-fit models do perfectly comply to the WN hypothesis.

16 Details on the coefficient estimates for each model are given in Table 2.
case and within them the Litterman solution with fixed AR(1) coefficient. Indeed, the Fernández specification shows a first-order correlation for \( \varepsilon^H_i \) on the correlogram of residuals and the Litterman standard solution delivers a negative AR(1) coefficient:

\[
\ln y^H_i = \alpha + \beta_1 \ln IPP^H_i + \beta_2 \ln IVD^H_i + \beta_3 \ln ONA^H_i + \vartheta^H_i
\]

with \( \vartheta^H_i = \vartheta^H_{i-1} + \varepsilon^H_i \) and \( \varepsilon^H_i = 0.7545*\varepsilon^H_{i-1} + \eta^H_i \)

— Japan: The SC specification modified to account for AR(1) error terms with \( \rho \) as first-order coefficient is to be given preference over the best-performing static solution, i.e. Litterman. This can be concluded when considering at the same time goodness-of-fit statistics, residual properties and significance of the coefficients on deterministic terms:

\[
\ln y^H_i = \alpha + \rho \ln y^H_{i-1} + \beta_1 \ln IPP^H_i + \beta_2 \ln VRD^H_i + \beta_3 \ln ONA^H_i + \vartheta^H_i
\]

with \( \vartheta^H_i = \rho \vartheta^H_{i-1} + \varepsilon^H_i \)

As a last step, the data are re-expressed in levels and the Denton’s adjustment algorithm is applied as to guarantee obedience to the aggregation constraint.

4. - Bayesian Methods

In the 1960s, books on development and application of Bayesian methods to the solution of econometric issues start to proliferate. Following the early work of Reverend Bayes, eminent economists and distinguished statisticians have subsequently brought considerable contributions to the Bayesian sphere of statistics. The esteem of these methods and their implied potential is more recent, though, and predominantly due to Harold Jeffreys’ *Theory of Probability*. Jeffreys, in spite of the deep astonishment his beliefs provoked in the framework of that time, succeeded in
defending his just-born concept of probability as a *reasonable degree of belief* and in proposing a complete axiomatic system for scientific learning. At the basis of this learning method lies a reinterpretation of the Bayes’ theorem, able to formally combine prior beliefs (*prior distribution*) and data-information (*likelihood function*) in its final intent to deliver a Kernel for the parameters vector — random variables, according to this inferential technique — summarizing all information available (*posterior distribution*). Consider the Bayes’ theorem:

\[
p(\theta / y) = \frac{p(y / \theta)p(\theta)}{p(y)}
\]

where \( \theta \) and \( y \) are both random, \( \theta \) is the parameter vector and \( y \) denotes a vector of observations from the considered sample. Keeping in mind that \( p(y) \) can be regarded as a constant with respect to \( \theta \) and noticing how \( p(y | \theta) \) is algebraically identical to the likelihood function for \( \theta \), \( l(\theta | y) \), it can be written:

\[
p(\theta | y) \propto l(\theta | y)p(\theta) \leftrightarrow \text{posterior distr.} \propto \text{likelihood function x prior distr.}
\]

The likelihood function is unique with respect to the assumed residual generating process, but different types of prior distributions are feasible. By far, the most classic and intuitive is referred to as *natural conjugate prior* and is characterized for entering the same family of density functions as the one the likelihood function belongs to. For reduced VAR with normal residuals, it is therefore given by a combination of an inverted wishart and a normal distribution:\(^{17}\)

\[
p(C, \Sigma) \propto p(C | \Sigma)p(\Sigma)
\]

with: 
\[
p(C | \Sigma) = \int_{MN}^{k \times m} (C | \tilde{C}; \Sigma \otimes \tilde{V}), \quad p(\Sigma) = \int_{W}^{m} (\Sigma | \tilde{S}; \tilde{v})
\]

that is a normal-wishart density \( p(C; \Sigma) \sim \text{NW} (\tilde{C}; \Sigma \otimes \tilde{V}; \tilde{S}; \tilde{v}) \) under the condition of \( \tilde{v} > m-1 \) e \( \tilde{S} \in \tilde{V} \) and and positive matrix. Such a distribution delivers the following first moments for the

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\(^{17}\) Notation is as follows. A single bar over symbols denotes a value *a-priori* defined by the modeller; a double bar indicates posterior information.
parameters (Drèze and Richard 1983): 

\[ E(C \mid \Sigma) = E(C) = \bar{C}, \quad \text{var}(C \mid \Sigma) = \Sigma \otimes \bar{V}, \]

\[ E(\Sigma) = \frac{1}{\bar{v} - m - 1} \bar{S} \quad \text{with} \quad \bar{v} > m + 1, \quad \text{Cov}(\Sigma) = \frac{1}{\bar{v} - m - 1} \bar{S} \otimes \bar{V} \]

The marginal prior distribution for the parameters matrix can be derived by integrating \( S \) out from \( p(C \mid \Sigma) \). It yields a matrix-variate \( t \)-distribution with \( \bar{v} \) degrees of freedom:

\[ p(C) \propto \left[ \bar{S} + (C - \bar{C})' \bar{V}^{-1} (C - \bar{C}) \right]^{-(\tau + k)/2} \]

(11)

The joint posterior distribution, as it is typical for a natural conjugate prior, will enter the same family of density functions of the prior:

\[ p(C, \Sigma \mid Y) \sim \text{NW}(\bar{C}; \Sigma \otimes \bar{V}; \bar{S}; \bar{v}) \]

(12)

with \( \bar{C} = (\bar{V}^{-1} + X'X)^{-1}(\bar{V}^{-1}\bar{C} + X'X\hat{C}), \bar{S} = \bar{S} + W + (\hat{C} - \bar{C})' [\bar{V} + (X'X)^{-1}]^{-1} (\hat{C} - \bar{C}) \]

\[ \bar{v} = (\bar{V}^{-1} + X'X)^{-1}, \bar{v} = \bar{v} + n \]

and a marginal distribution for \( C \) that is a matrix-variate \( t \).

\[ p(C \mid Y) = \int_{M_t}^{k \times m} (C \mid \bar{S}; \bar{V}; \bar{S}; \bar{v}) \]

The main weakness of the just-described prior has to do with the limits the Kronecker-structure of the prior covariance matrix imposes on the posterior distribution of such parameters.

Generally, all posterior-distribution characteristics, such as tests of different kinds of hypotheses and predictive densities, can be retrieved from the following structure (Koop, 2004):

\[ E\left[ \left[ g(\theta) \mid y \right] \right] = \int g(\theta) | y \rangle p(\theta \mid y) d\theta \]

(13)

where \( g(\theta) \) denotes the function of interest. Usually, those computations are performed via simulations conducted on the
posterior distribution — in reference to the traditional “law of large numbers” and to the central limit theorem — and frequently require the use of high-power data-processors.

Briefly, the main differences between the Bayesian and the more classical, frequentist approach to inference reflect the principles at their basis (Zellner, 1988; Rothenberg, 1963). In econometrics, the frequentist spirit is developed around a concept of probability which is defined as the limit of a specific event’s relative frequency: essential is the attention posed on repeated performance. In a Bayesian framework, probability is defined in terms of degrees of confidence and acquires subjective connotations. The modeller’s belief in how likely or unlikely a given event is to occur, rooted in quantitative and/or qualitative information, is combined with knowledge coming directly from the sample in use and thus making probability not necessarily depending on relative frequency over different data-sets. Secondly, the concept of the unity of science, to be found [...] in its methods, not in its material” (Pearson, 1938, p. 16), is fully reflected in Bayesian econometric techniques, appreciated for their suitability to a variety of applied areas. The opposite can be said for the frequentist method that instead proposes specific principles and techniques for different issues. A third point of opposition between the two inferential alternatives lies in the learning process they imply. Bayes’ theorem is an optimal information processing rule (IRP), and along with it the Bayesian method involves a formal learning model which enables the modeller to empirically test different hypotheses.18 On the contrary, classical inference learns informally, without the use of consistent methods. The higher level of formality that distinguishes the former method from the latter is reflected in the ways prior information enters the investigation, that is in a direct and flexible rather than an implicit and rigid form.

18 It must however be kept in mind Rothenberg T.J. (1963)'s reasoning when evaluating Bayesian inductive learning. He defines it as historically invalid inasmuch as it offers an incremental portrayal of scientific growth unable to fit the non-constant one history depicts. In other words, Bayesian growth process is centred on the quest of confirmation to generally acknowledged theories rather than on the proposition of innovative ideas.
On a more practical/applied level, there are several advantages in preferring Bayesian inferential methods over classical ones. Within the VAR framework, an insightful prior (not too loose and not rejected by the data) can lead to a non-trivial gain in estimation precision when dealing with limited datasets and relatively large model size and parameter spaces. Working with priors instead of constraining parameters to some specific values when coping with over-parametrized models allows the researcher to remain more flexible in the specification and to more directly weigh his confidence in a priori beliefs. Recently, Bayesian methods have gone through a revival of interest in terms of DSGE models estimation (and evaluation) after their initial success within VAR applications. On a general level, this attention is especially due to the acknowledgment of DSGE models being only an approximation to the correct data generating process (DGP) of the actual data and thus “false”. Under those conditions, the classical maximum likelihood (ML) estimation approach is unsuitable in its standard proposition since it is asymptotically justified only when the model is actually the true DGP, up to a set of unknown parameters (see Canova, 2007 for a detailed discussion on misspecified DSGE models). Bayesian posterior inference helps by not requiring the model to be the real DGP. Bayesian analysis also allows for a fully-fledged model parametrization based on the likelihood function generated by the DSGE model — which does not fail in the presence of a singular covariance matrix for the endogenous variables — rather than classical alternatives built on IRFs matching, for instance between DSGE and VAR.\footnote{Likelihood-based methods also find their limits in a DSGE framework, especially at identification stage. If it is true that via prior distribution they can rely on a wider set of information (for instance coming from other datasets) and thus help in under-identification issues, they may also mask identification problems. When not carefully applied, they may deliver well-behaved posteriors based only on well-behaved priors, even in case no information on the parameters is contained in the data. For a more detailed discussion on the role played by Bayesian econometrics with DSGE estimation, identification and evaluation see AN S. - SCHORFHEIDE F. (2007) and the references therein. On a more detailed discussion concerning (different interpretations of) likelihood-based methods applied to DSGE-model estimation and their implications, refer to GEWEKE J. (2007).}
4.1 The Minnesota Prior

Robert Litterman (1979; 1986) first suggests a Bayesian alternative to the more traditional VAR estimation techniques by developing a particular prior nowadays referred to as “Minnesota prior” due to its roots at the University of Minnesota and the Federal Reserve Bank of Minneapolis. Litterman proposes a solution which is capable to pragmatically describe the true state of knowledge and uncertainty regarding the structure of the economy. Indeed his prior does not have roots in a specific theory, but rather “the restrictions it imposes may be referred to as instrumental” (Litterman 1986, page 16).

The basic idea is the random-walk hypothesis, i.e. a RW model as a reasonable approximation of the movements of the variables. This belief is not imposed exactly on the parameters of the model, but rather by assigning to each one of them a prior variance via a set of hyper-parameters:

\[
\text{diagonal}[\text{var}(C)] = \begin{cases} 
\text{var}(c_{0,i}) = \infty \\
\text{var}(c_{i,i,p}) = \sigma_{i,i,p} = \lambda_1 M_{i,j} g_p(\hat{\sigma}_{ii} / \hat{\sigma}_{ij}) 
\end{cases}
\]

On the parameter \(c_{0,i}\), which identifies the deterministic component, a non-informative prior is imposed, i.e. the prior mean and variance are set equal to zero and infinity respectively.

The prior distribution on lag coefficients is parametrized on the hyper-parameter \(\lambda_1\) (overall tightness), that is the standard deviation on the first lag of the dependent variable in each equation, the \(ii\)-th element of matrix \(c_1\). Since it is reasonable to assume that a large extent of each variable’s movement is driven by its own lags, Litterman considers a second hyper-parameter, \(\lambda_2\), modelling the relative tightness of the prior (smaller variance of the coefficients) on lags other than the dependent variable’s ones. This factor is meant to shrink that group of coefficients more heavily toward the zero mean of the random walk hypothesis:

\[
M_{i,j} = \begin{cases} 
1 & i = j \\
\lambda_2 & i \neq j
\end{cases}
\]
Furthermore, the so-called “economic certainties” suggest modelling a parameter mean relative to its position in lag terms. A prior variance which diminishes with the increase of the lag order enters the prior by mean of a decay function capable of representing a variety of patterns, among which the harmonic and geometric ones:

\[ g_p = \begin{cases} p^{-\lambda_3} & 0 \leq \lambda_3 < \infty \\ \lambda_3^p & 0 \leq \lambda_3 \leq 1 \end{cases} \]

where \( \lambda_3 \) defines the extent to which coefficients on lags beyond the first one are likely to be different from zero.

Last noteworthy factor is the ratio between \( \sigma_{ui} \) and \( \sigma_{uj} \), that are the estimated standard errors of an univariate auto-regression with \( p \) lags, referring respectively to variable \( i \) and \( j \). It is a scale factor necessary for assuring the prior robustness against different units of measurement in which the data may be expressed\(^{20}\).

Besides such an uncertain portrayal on coefficients characteristics, no informative prior is imposed on the VAR covariance matrix \( \Sigma \). This matrix is supposed to be diagonal with elements drawn directly from the data. It is mainly around such an assumption that the principal critiques against the Minnesota prior have grown. The idea of a perfectly known covariance matrix (in both structure and components) can be easily challenged, especially when weighted against assumed imperfect ignorance on the coefficient vector. If information on covariance terms can be claimed to come directly from a profound analysis of the data, a diagonal-type structure for \( \Sigma \) is not easily justifiable and, most importantly, not supported by empirical evidence. Moreover, the diagonal structure itself implicitly entails a further set of strong assumptions on perfect independence among coefficients across VAR equations\(^{21}\).

\(^{20}\) For instance, if the variability of variable \( i \) is very much smaller than the one of variable \( j \), the coefficients on the first lag of \( j \) in the equation for \( i \) are shrunk toward zero via this scale factor.

\(^{21}\) Along this reasoning, a number of extensions to the original formulation of the Minnesota prior have been proposed covering, among the others, natural-conjugate-style and diffuse-style options (KADIYALA K.R. - KARLSSON S., 1997).
Under the assumption of normal errors, the above leads to a matrix-variate normal posterior distribution for the parameter vector with the following estimators for mean and variance of the parameters:

\[
\begin{align*}
\hat{\bar{C}} &= (\bar{V}^{-1} + (\Sigma^{-1} \otimes X'X))^{-1}(\bar{V}^{-1}\bar{C} + (\Sigma^{-1} \otimes X')X\hat{C}), \\
\bar{V} &= (\bar{V}^{-1} + (\Sigma^{-1} \otimes X'X))^{-1}
\end{align*}
\]

This result echoes what was previously argued on the natural conjugate prior, with the only difference of a \(\Sigma\) matrix that is supposed to be known \textit{a priori}, which causes the priors developed on each equation to be independent among each other.

4.2 Comparing Classical and Bayesian Inference

\textbf{BVAR} models are routinely used in short-term baseline forecasts within central banks and international institutions: univariate random walk models are typically proficient at forecasting macroeconomic time series. Empirical exercises aimed at assessing the features of various estimation methodologies and specifications have been considered extensively by the econometric literature, such as the work of Litterman (1986), McNees (1986) and Fair and Shiller (1990). In the following, an application meant to compare classical and Bayesian approaches to inductive inference is conducted in a VAR environment. The work does not claim to be exhaustive in its conclusions or to come to a final and unassailable merit ordering across models, especially considering the wide spectrum of formulations the Bayesian sphere encompasses.

The section follows the work of Robertson and Tallman (1998). There are two main reasons for choosing their specification scheme. First, it is coherent with the Jeffrey-Wrinch simplicity postulate also known as Zellner’s (1992) \textit{keep it sophisticatedly simple} exalting the good performance of measured models in a scientific framework. Second, the variables included reflect macroeconomic aggregates...
policy makers control and, at the same time, are influenced by. Considering that the specification is conceived to be usable in the development of a second model exploring the consequences to monetary policy shocks, this selection of variables seems particularly appropriate. A monthly VAR with six lags and six variables — real GDP (GDP), unemployment (U), consumer price index (CPI), M2, a short interest rate (I) and commodity price index (CP) — is estimated for three countries, i.e. the United States, the United Kingdom and Japan\textsuperscript{22}. All time series but one are available at a monthly frequency: for quarterly-available real GDP data, the results of the TD exercise are made use of. Apart from interest rate and unemployment, all the variables enter the model in logarithmic terms.

The exercise juxtaposes frequentist unrestricted VAR (UVAR) to Minnesota-type Bayesian VAR. Two Bayesian specifications are considered: one with prior hyperparameters assuming Litterman’s default values (BVAR), \textit{i.e.} $\lambda_1 = \lambda_2 = 0.2$, $\lambda_3 = 1$, $g_p = p^{-\lambda_3}$ and a second one where the hyperparameters are chosen to minimize the log-determinant of the covariance matrix of one-step-ahead, out-of-sample forecast errors computed on the last 24 periods (BVAR1). In this last specification the hyperparameters assume the following values (a harmonic decay function applies): for the US $\lambda_1 = 0.6$, $\lambda_2 = 0.1$, $\lambda_3 = 1$; for the UK $\lambda_1 = \lambda_2 = 0.1$, $\lambda_3 = 1$; for Japan $\lambda_1 = 0.2$, $\lambda_2 = 0.02$, $\lambda_3 = 0.1$. The comparison is developed on the predictive performance of the analyzed models for forecasts computed on the sub-sample period ranging from 2004:07 to 2006:06\textsuperscript{23}.

Table 4 collects root mean squared errors, the U-Theil statistic and the Diebold and Mariano (DM) test of equal predictive accuracy for one-step-ahead and three-step-ahead, out-of-sample forecasts for the two BVAR and the UVAR specifications.\textsuperscript{24} Some

\textsuperscript{22} Details and sources for the time series are given in the data APPENDIX.

\textsuperscript{23} This application and the one of the previous paragraph are both developed in Matlab environment. Codes as from Le\textsc{S}age J.P. (1999) and Quilis E.M. (2004) respectively.

\textsuperscript{24} Theil's U coefficient compares a model forecast RMSE with those obtained from a random walk model by taking the \textit{ratio} of the two. DM statistics tests the null hypothesis of equal forecasting ability across two models.
features deserve comments. First, BVAR models in general outperform both UVAR and random walk specifications. Exceptions to this statement are rare, and in the five cases where UVAR models record a lower RMSE than BVAR models, the DM-test null hypothesis is accepted. Second, the superiority of BVAR specifications over the unrestricted model, given by the fact that the use of a non-tough prior leads to higher efficiency in the estimation, is better reflected in GDP, M2 and I variables and over the one-rather than three-month horizon. The DM test does not allow for model ordering on unemployment and CPI inflation variables: the null hypothesis of equal forecast performance is repeatedly accepted in those cases.25 Lastly, for all countries the highest U-Theil statistics are reported for interest rate series; for these series, unrestricted and Bayesian forecast techniques also differ the most. Such a result can be deemed sound when interest rates are taken as a proxy of monetary policy interventions and are thus supposedly more intensively exposed to exogenous shocks. A technical forecast procedure as VAR cannot account for those facts.

The conclusions of this exercise are in line with Robertson and Tallman’s (1999) ones as well as with those of other authors in finding a better precision of Bayesian Minnesota-type specifications than of classical maximum likelihood techniques. By linking a large number of coefficients to a lower hyper-parameters universe, this class of priors literally reduces the VAR dimensionality, leading to finer predictive performance.

5. - VAR Empirical Application: International Comparisons

VARs have proven to be a suitable tool in benchmark forecasting. The transition of such dynamic auto-regressive structures from the forecasting framework, where they were first applied, to an informative tool for structural analysis and

25 Litterman R.B. (1986) underlines the weakness the prior exhibits in forecasting GDP deflator over the period 1948:01-1979:03 with a six-lag, seven-variable VAR.
macroeconomic policymaking was not straightforward. Initially criticized for their extensively focusing on sample correlations, VARs have lately been appreciated for that specific peculiarity as a convenient device in summarizing first and second moment properties of data.

In the literature, a lot of studies have adopted VAR-type specifications to document the behaviour of macro-variables following monetary policy innovations via the analysis of impulse-response functions.²⁶ This section follows the work of Sims (1992) in applying VAR models to monitor the responses of macro-variables to monetary policy shocks across different countries. The main differences between the two analyses lie in the reference to Bayesian methodologies made here and in the countries considered.

5.1 Describing the Model

This third application entails a model which evokes the previously proposed six-variable, six-lag VAR system in its Bayesian formulation. The Minnesota-type solution is given space²⁷ since it implies a prior which allows for a lessening of the trade-off between the number of parameters and the efficiency of the estimation²⁸ and simultaneously introduces restrictions abstracting from specific economic theories. At the identification stage, a similar spirit of keeping a priori restrictions minimal inspired the final choice of a recursive structure. The Cholesky-style identification is imposed on the model with the following causality order: interest rate (i), M2, commodity price index (CP),


²⁷ The hyperparameters of the prior take values as originally assigned by Litterman (i.e. $\lambda_1 = \lambda_2 = 0.2, g_p = p^1$) for all three countries specifications in order to keep consistency across the analyses.

²⁸ A 246-observation dataset when compared to a six-variable, six-lag VAR specification can result in being not wide enough to avoid over-parametrization problems and important reductions in terms of degrees of freedom.
CPI, unemployment rate (U) and real GDP (GDP). This recursive ordering seems the most appropriate for the monthly-frequency VAR proposed here since it precludes the Central Banks (CB) to contemporaneously observe variables which are hardly available under such schedule in reality (as for example real output) when deciding on monetary policy interventions.29

TABLE 1
CORRELATION MATRIX OF BVAR RESIDUALS

<table>
<thead>
<tr>
<th>United States</th>
<th>i</th>
<th>M2</th>
<th>CP</th>
<th>CPI</th>
<th>U</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>-0.1048</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>0.1819</td>
<td>-0.1014</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>0.0648</td>
<td>0.0471</td>
<td>0.0304</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>-0.1504</td>
<td>0.0339</td>
<td>-0.0945</td>
<td>0.0241</td>
<td>1</td>
<td>-0.1457</td>
</tr>
<tr>
<td>GDP</td>
<td>0.1018</td>
<td>-0.01</td>
<td>0.0958</td>
<td>-0.2075</td>
<td></td>
<td>-0.1457</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>United Kingdom</th>
<th>i</th>
<th>M2</th>
<th>CP</th>
<th>CPI</th>
<th>U</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>-0.1689</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>0.0066</td>
<td>-0.1067</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>-0.0568</td>
<td>-0.008</td>
<td>0.1106</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>-0.032</td>
<td>0.0881</td>
<td>0.0242</td>
<td>-0.075</td>
<td>1</td>
<td>-0.0186</td>
</tr>
<tr>
<td>GDP</td>
<td>-0.0117</td>
<td>-0.0425</td>
<td>-0.0602</td>
<td>0.0229</td>
<td>-0.0186</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Japan</th>
<th>i</th>
<th>M2</th>
<th>CP</th>
<th>CPI</th>
<th>U</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>0.0046</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>0.0231</td>
<td>-0.0709</td>
<td>-0.1607</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>0.029</td>
<td>0.0087</td>
<td>-0.1607</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>0.0035</td>
<td>0.0515</td>
<td>-0.0948</td>
<td>-0.0496</td>
<td>1</td>
<td>-0.1363</td>
</tr>
<tr>
<td>GDP</td>
<td>0.0812</td>
<td>-0.0297</td>
<td>0.0952</td>
<td>-0.1502</td>
<td>-0.1363</td>
<td>1</td>
</tr>
</tbody>
</table>

29 This solution is in line with SIMS C.A. (1992). Considering that different assumptions were followed in other studies applying Cholesky-type identification structures, the impact of variables ordering on IRFs patterns was analyzed and only a slight impact identified (Graph 7B). Same conclusions can be drawn from looking at the correlation matrix of innovations driving IRFs (in general very low, see Table 1) and initial heights of off-diagonal IRFs (close to zero in the first period, see Graphs 2 to 6). Such conclusions implicitly assess the robustness of findings coming from IRFs analysis across alternative orthogonalization assumptions.
It is important to briefly recall the advantages a Bayesian specification has over a classical unrestricted VAR when working with over-parametrized models, as in this case. The issue can be approached via an informal economic analysis on the IRFs consistency to stylized empirical macroeconomic facts and/or to economic literature. Chart 7A juxtaposes IRFs derived from a Minnesota-type Bayesian estimation (BVAR) and a classical unrestricted VAR (UVAR). The two sets of IRFs differ heavily both in magnitude and in the direction of variables’ responses to the shocks considered. One important pattern can be put forward when questioning the plausibility of UVAR IRFs: UVAR IRFs produce “perverse” real output responses to restrictive monetary policy shocks (chart 7A, third row). Other country-specific features stand out in favouring BVAR modelling over UVAR. First, the shock on interest rates seems to be over-persistent in US and UK data for UVAR specification, lasting longer than the four-year period considered (chart 7A, first row); second, in the US a rise in the commodity price index (mirroring an increase in expected inflation) results into an almost instant reduction of the CPI for the unrestricted estimation (chart 7A, forth row; see the following for a discussion on the UK case of stationary CPI).

In reference to standard practice in VAR analysis to report the response of a system in equilibrium to exogenous innovations — and not endogenous shocks, since they would be predictable by agents with rational expectations —, the choice of which variable to choose for exemplifying monetary policy interventions lies at the heart of the model building process. Indeed, shocks to such a variable will be a proxy of Central Bank (CB)’s unanticipated policy interventions, changes or mistakes\textsuperscript{30}. Economics teaches how monetary policy institutions have the power to influence, more or less directly, the official interest rates and the money supply. In principle, the choice of an option over the other as more appropriate should be driven by considerations on CB’s

\textsuperscript{30} There is plenty of literature on the impact such choice may play on the results of the analysis; see for instance Eichenbaum M. (1992).
institutional details. There are little doubts on the post-1982 Federal Reserve’s focus on the federal fund rate as target parameter for its interventions (Bernanke and Mihov, 1998) as well as on the Bank of England’s particular attention toward short-term interest rates (King, 1997). The picture is less clear for the post-2001 Japanese situation, with the CB adopting a quantitative-easing policy alongside an interest rate targeting (Blenk et al., 2001). In the model, short-term interest rate is attributed the role of modelling monetary authorities’ interventions in view of those institutional peculiarity and two additional factors, that are a) the influence globalization and deregulation play on the share of the money supply that is not controlled by CBs and b) the features of the model itself which relates interest rate movements to money market supply factors as the opposite-sign response of M2 to an innovation in interest rate tells. Such a choice implies the following structure for the CB’s reaction function31:

\begin{equation}
\begin{align*}
    i_t &= b_{10} + a_{11}i_{t-1} + a_{21}i_{t-1} + \ldots + a_{61}i_{t-6} + a_{12}M2_{t-1} + \ldots + a_{62}M2_{t-6} + \\
        &+ a_{13}CP_t + \ldots + a_{63}CP_{t-6} + a_{14}IPC_t + \ldots + a_{64}IPC_{t-6} + a_{15}U_t + \ldots + a_{65}U_{t-6} \\
        &+ a_{16}PNL_t + \ldots + a_{66}PNL_{t-6} \\
\end{align*}
\end{equation}

(16) with \( i_t = \Psi_i + \epsilon^i_t \)

It is a time-invariant linear structure which has often been criticized for being unsuitable in modelling the CB’s feedback function and thus incapable of correctly identifying monetary policy shocks matching authentic economic events (Rudebusch, 1998a; Rudebusch, 1998b). This critique, which may have a say within a historical evaluation of policy shock/business cycle causality testing, does not hold in an impulse-response analysis framework where the main focus remain on the system’s reaction to policy shocks (see Sims, 1998 for a thorough discussion).

31 Literature has considered almost uniformly interest rates as measure of monetary policy interventions and shocks.
5.2 The Effects of Monetary Policy

The understanding of key macroeconomic variables’ reactions to monetary policy interventions and, in general, an insight into the influence that policy decisions may bear on business cycles has always been one of the hottest topics in Economics. Business cycle-style theorists defend the random or real nature of economic cycles; on the contrary, monetarists and Keynesians attribute a non-trivial role to monetary policymakers in steering the economy over the short run.

On this matter, time-pattern-type comparisons between real and nominal variables, as first proposed by Friedman and the monetarists, can be tricky when conducted within a VAR framework and may lead to inexact conclusions (see Rudebusch, 1998a; Rudebusch, 1998b). Nevertheless, a brief time-pattern analysis seems adequate, even though the limits of working with identified residuals, or monetary policy shocks, which directly depend on model specification need to be kept in mind. Graph 1 juxtaposes estimated monetary policy shocks and business cycle peaks and troughs. First, from the US and UK charts it can be noticed how all the troughs but 1999:09 and 2005:09 for the US and 1995:08 for the UK follow interest rate reductions, which in turn come after monetary policy tightenings. The Japanese experience of the late 1980s/early 1990s is a further proof of the CBs having a bear on business cycle path. A loosen monetary policy, persisting throughout 1987, is followed by strong economic growth which starts to lighten with monetary policy tightenings aimed at readdressing the biggest financial bubble Japanese markets have ever experienced. The business cycle reverses again in May 1989 in concomitance to positive interest rate shocks.

Considering the aforementioned risks of drawing conclusions on time-pattern comparisons and being aware of how, in a bivariate system, the mere fact of one variable leading another in

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32 Estimated exogenous policy shocks are given by the three-month centred moving average of structural residuals in the BVAR system, interest rate equation: \( \sigma_\varepsilon (\varepsilon_{t-1} + \varepsilon_t + \varepsilon_{t+1})/3 \). Instead, approximations of business cycle peaks (dotted line) and troughs (solid line) are ECRI's (Economic Cycle Research Institute) official ones.
GRAPH 1
CENTRED, THREE-MONTH MOVING AVERAGE OF MONETARY POLICY SHOCKS AND BUSINESS CYCLES

**UNITED STATES**

**UNITED KINGDOM**

**JAPAN**
timing does not necessarily imply causality, a more profound and rigorous analysis is requisite. Here, a VAR system, via its IRFs, reveals to be a very effective tool. The following part of this section elaborates on VAR impulse-response analysis for the US, the UK and Japan. Remarks on this last country are considered separately and subsequently to the US and UK cases due to the singular situation it underwent during the 1990s.

Monetary policy tightenings, or positive innovations in the interest rate\textsuperscript{33}, follow a homogenous path in both the US and the UK. A first adjustment stage lasting from two to four months is followed by a second phase when the shock reduces towards zero. The zero-level is eventually reached around the thirty-eighth month, later the landing on some negative values (Graph 2, first row). Theorists claiming monetary policy to be capable of affecting real output (classics, Keynesians, monetarists, neo-Kenynesians and neo-classics) identify specific directions through which the monetary authority may exerts its influence, together with manifold transmission channels. Most of those channels are demand-sided and centred around the interest rate, the exchange rate, asset prices and the credit market as key transmission variables. The interest-rate-based, the exchange-rate-based and the asset-prices-based transmission mechanisms are rooted in the concept of short-run sticky prices. Keynes with its liquidity-preference theory and Taylor (1995) who broadens the Keynesian analysis to consider its price side rather then the quantity side — financial market framework — and three types of prices — short term interest rate, long-term interest rate (the expectations model of the term structure) and exchange rate — and distinguishes between the real and the nominal sides of the issue, deem investment spending and durable expenditure major factors within the transmission process. The monetarist paradigm approaches the issue from a broader prospective by embracing the entire universe of assets: under this interpretation, a CB's

\textsuperscript{33} Assuming that monetary policy actions can be reflected in movements of short-term interest rate implies this last variable must be considered effect-free from shifts on the demand for money. The fact that interest rate shocks can be driven also by factors not related to the CB's decisions is accounted for informally.
intervention influences the marginal utility of all portfolio assets and thus their relative prices, leading to amplified consequences in real variables, *i.e.* consumer spending and economic agents’ overall wealth.\(^{34}\)

From Graph 2 (third row) one can see the way a positive innovation in monetary policy does affect real GDP: in the US, GDP starts declining from the fifth month, reaching its minimum

---

\(^{34}\) This broad view of monetary policy influence is based on the neo-classic Tobin’s *q* as for the investment side and on the Modigliani’s life-cycle model as for the consumption one.
around the twenty-fifth period; same timing characterizes the UK although the response proves to be more persistent in this case. Noteworthy is the timing of the output reaction: the response of real GDP reaches its maximum when the interest rate happens to be almost back to its baseline level, or even when it undergoes negative shocks. A plausible reason for this anomaly may be found within the *credit channel* concept (Bernanke and Gertler, 1995). As a set of factors amplifying conventional monetary policy transmission mechanisms, this concept is based on the notion of imperfect financial markets, *i.e.* markets where asymmetric information cause the appearance of the external finance premium, that is a difference in costs between funds raised externally and funds generated internally. The weakness that a monetary policy tightening causes on firms’ balance sheets and private expenditure (balance sheet and bank lending channel) enables that premium to persist once the interest rate starts steering back to its baseline level. This may help explaining a declining GDP concomitant to negative interest rate shocks. To conclude on monetary policy influence on real output, a variance decomposition analysis is put forward. Disregard to real GDP shocks, interest rate innovations count the most in explaining real GDP forecast variance for both countries (these values are: 25.19% for the UK and 12.41% in the US).

Graph 2, second line, illustrates the response of inflation to a positive innovation in the interest rate. In the US and the UK, aggregate-price-level responses align with the set of “*stylized facts*” generally characterizing empirical analysis and largely recognized within transmission mechanism theories. CPIs react very slowly to an exogenous shock in monetary policy, responding only around the end of the fourth year and the twenty-ninth period respectively. At the same time, a mild increase in the CPI over the short run can be spotted: this response is considered as one of the most controversial findings in the VAR-based empirical literature in its being theoretically-unjustifiable within a demand-driven transmission mechanism framework. Briefly, two main explanatory strands to the so-called “price puzzle” (as from Eichenbaum, 1992) have been put forward, one exploring methodologi-
cal issues and the other one focusing on supply-side theories of monetary policy transmission. On the methodological side, it is argued that the prime cause for the puzzle lies within a mis-specification of the CB reaction function for nascent inflation which therefore leads to misidentified policy shocks. In parallel, solutions entail an extension of the model for latent variables modelling changes in inflation expectations. This proposed solution/justification and the branch of empirical work centred on it confirms the possibility for VAR models to correctly identify a monetary policy shock.

Barth and Ramey (2000) challenge the demand-only view of monetary transmission and present the “cost channel” as key factor in resolving the price puzzle. Given the necessity for firms to pre-finance production and the essential role of working capital in supply, interest rates can affect firms’ short-run ability to produce (and thus deliver output) by varying marginal production costs. On the supply side, an active cost channel may allow for an initial concave, hump-shaped and possibly positive response of inflation to a contradictory monetary shock by causing industry prices to move much more than the aggregate price level. In this interpretation, the price puzzle is linked to a worsening in credit conditions instead of misspecification concerns. In their paper, Barth and Romey find strong support in favour of an active supply-side transmission mechanism within some industries by specifying an industry equilibrium model. Many successive studies evaluating such theoretical solution find controversial results but empirical evidence of a

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36 They present industry-level evidences on opposite-sign co-movements between industry output and prices (relative to industry wages and production). In their study, they also help explain the magnitude of monetary policy’s effects on output by accounting for this new channel as an active and power collaborator to the demand-based one.
direct relation between a relevant cost channel and the structure of the financial system.\textsuperscript{37}

From this reasoning on the price puzzle comes the insertion of the commodity price index in the proposed BVAR model. As already noted, both the UK and the US applications suffer from a slight positive shift in inflation (more persistent in the US) over the short run even with the CP explicitly accounted for in the VAR system. On the role played by the CP within the VAR specification, such an index performs properly in the US: positive innovations are followed by an increase in inflation - a positive variation in expected inflation causes an almost instant increase in prices- (Graph 3). Something different happens in the UK where the sign of the CPI response cannot be defined and tends to be negative. The reason might be a commodity index that, just for this country, considers only one production factor, petrol, which is also a factor of primary importance in English exports.

\textbf{GRAPH 3}

\textbf{CPI RESPONSE TO A SHOCK ON THE COMMODITY PRICE INDEX: US AND UK}

\begin{center}
\begin{tabular}{|c|c|}
\hline
 & United States & United Kingdom \\
\hline
CPI & \begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{cpi_us_graph.png}
\end{subfigure} & \begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{cpi_uk_graph.png}
\end{subfigure} \\
\hline
\end{tabular}
\end{center}

\textsuperscript{37} CHRISTIANO L.J. \textit{et al.} (2005) conclude that the cost channel only helps explain inflation inertia \textit{via} the estimation of a VAR general equilibrium model on US data; in an extended methodological framework which covers Bayesian DSGE models, RABANAL P. (2007) observes a zero-probability event of increased prices after a positive monetary policy shock recalling the role of misspecification in price-puzzle-type of economic behaviour. Stronger evidences of an effective cost channel are drawn for bank-based financial systems (HENZEL S. \textit{et al.}, 2007 for the Euro area; CHOWDHURY I. \textit{et al.}, 2006 for Japan).
Japan needs to be treated separately considering the economic situation of the last fifteen years (covering around 4/5 of the VAR sample period). Since 1992, the country underwent an economic slump and encountered deflation starting from 1995 (1999) as from GDP deflator (CPI). Bank of Japan’s action in trying to stimulate the economy has been thorough, even if not effectively combined with fiscal policy decisions (Svennson, 2003). Since 1995 Japan experienced interest rates below 1%; in the period 1999-2000:09 and starting from 2001:03 the Bank of Japan implemented a zero-interest-rate policy, joined by a quantitative easing strategy as from 2001:03. A condition of slow growth combined with decreasing prices and ineffective monetary policy, like the one just described, falls within the Keynesian theoretical construct of the “liquidity trap”. In an ISLM system, when the IS curve intersects the horizontal axis of production, the equilibrium in the goods market is reached with a negative interest rate. Under such conditions, each increase in money supply will only result in a mere substitute for assets without affecting the real output. The vainness of any policy interventions in the money market experienced by a country under liquidity trap conditions is reflected in the Japanese IRFs (Graph 4). A positive interest rate shock, re-entering as from the eighth month, plays no role in determining real variables: real GDP happens to be indifferent to an interest rate shock up to the twenty-third month when it undergoes some turbulence (the error bands of the IRF remain around the zero level). Identical conclusions can be drawn by looking at the variance decomposition: interest rate shocks count the least on real output forecast error variance (1.15%). Graph 4 displays the CPI steadiness following an interest rate shock: a temporary policy shock unable to act on long-run expectations cannot influence the general price level and thus real variables. Once again, variance decomposition analysis is a good conveyor of the effects associated to a CPI variation within a liquidity trap context: the second contributor (in size) to the forecast error of real GDP is the CPI (22.58%), right after real GDP.

Lastly, a money market analysis of the relationship between the interest rate and the quantity of money, *i.e.* the *liquidity*
is also conducted. The US and the UK IRFs (Graph 5) obey the well-accepted literature in that regard. A positive innovation in the interest rates causes a decline in M2 (more relevant for the US), which persists also when the disturbance turns to negative due to the impact a reduction of real GDP has on the transaction demand for money. The situation is different for Japan, where M2 adjusts with a delay of about eleven months when it approaches a sharp drop (Graph 5). The monetary authority of a country under liquidity trap conditions is unable

---

38 That is, the drop in money demand that follows an interest rate increase and is caused by asset price reductions (demand for money as reserve of value) and a fall in real GDP (transactions demand for money).

39 It is essential to underline how this analysis is difficult to conduct in a BVAR model as the one presented here that does not explicitly consider some key variables, such as exchange rate and fiscal factors, that may guarantee a clear distinction between supply and demand shocks affecting the money market.
to exert any kind of control over broad monetary aggregates. Policy actions result in an equi-proportional increase in both reserves and money held by economic agents only, independent of the role played by financial intermediaries (Krugman, 1998).

5.3 The Real Side of the Economy

Macroeconomics identifies two main types of disturbances that may hit an economy: demand shocks and supply shocks. In a BVAR framework, the discrimination between the two can be based on a juxtaposition of CPI and output behaviour. Generally, a negative correlation between real GDP and the CPI suggests the observed disturbance to have roots in supply factors; on the opposite, a positive correlation is symptom of a shock possibly associated to demand aspects. An exhaustive and coherent study on the nature of the business cycle must consider IRFs, specifically, the response of the CPI and real GDP to innovations in the latter, besides a mere correlation analysis (graph 6). A correlation analysis alone would indeed indicate supply side shocks as the prime cause for the slow economic path undergone by the three countries during some of the years considered. In an economy characterized by sticky prices over the short run, demand shocks allowing for an initial over-the-trend expansion in production and for a lagged increase in prices can be hidden behind a negative correlation.

A positive disturbance affecting real output demonstrates a sort of persistency in both the UK and the US, while a less linear path characterizes Japan (Graph 6). Over the four years considered, the CPI reacts to a positive shock on real GDP only with some lags in all the countries: for the US and Japan the price level starts expanding around the twelfth and eighth month respectively, while in the UK the CPI remains steady at its baseline level. Thus, the VAR model is replying to shocks on output as if they were demand shocks.

This set of model conclusions is in line with thorough studies on specific developments in this period and stands as evidence of
VAR effectiveness in describing and summarizing relationships of variables. Independent empirical studies have confirmed the recession that in the early nineties involved all three countries, i.e. the only relevant disturbance in the sample period, having roots in demand factors. Blanchard (1993) shows how the 1990-1991 US economic slump had consumption as its preeminent cause. Pure shocks on consumption, in particular related to the uncertainty concerning the domestic political evolution and the joining of the European Union of that time (animal spirit), combined with monetary policy interventions are to be counted as principal reasons for the slow growth hitting the UK in the period 1990-1992 (Catão and Ramaswamy, 1995). On the Japanese 1990s-down-turn, Powell (2002) and Iwaisako (2000) find tighter monetary policy targeting the unsustainable boom of the late 1980s as prime cause for the recession.

40 In particular, the fall in the consumer confidence index following the Iraq invasion of Kuwait (1990:08) played a major role in the 1990-1991 US economic recession than the simple consumers’ anticipation of a future negative cycle did.

41 That is, rapid growth of the monetary base, artificial reduction of the interest rates and the consequent distortion of the interest rate role as signal between consumers and producer that led the former group to spend and the latter group to invest more than ever before.
6. - Final Remarks

This work has tried to depict a general portrayal of the actual state of the art in temporal disaggregation techniques and Bayesian inference when applied to VARs as to mitigate the effects of over-dimensionality those models are usually confronted with. From an applied perspective, the exercise proposed in paragraph 3.2 compared the suitability of various temporal disaggregation procedures applied to three different data sets and showed how the classical Chow-Lin solution may not always be the best fit to the data. A second exercise (paragraph 4.2) juxtaposed classical and Bayesian (Litterman-type) estimation approaches within a VAR framework in order to assess the performance of the two models in out-of-sample forecasts. As expected, better precision is achieved under a simple Minnesota-prior Bayesian estimation than under unrestricted VARs thanks to the dimensionality reduction of the former.

In paragraph 6, structural analysis came into play. Minnesota-prior Bayesian VARs were estimated for three countries, and the effect of a monetary policy shock was analyzed. This specification showed: 1) in its findings, to keep coherence with previous studies and well-established literature; 2) via informal economic analysis, simple Bayesian specifications to be a good basis for comparing experiences of some countries concerning the efficacy of monetary policy; 3) in the ability of VARs to correctly discriminate among different types of shock that may hit the economy, VARs to be an effective descriptive tool and a powerful source of information for policymakers.

In spite of those results, some drawbacks of VAR systems exist. Especially in structural inference, VARs present a series of limitations which need to be accounted for. Performance of structural VARs directly and heavily depends on the plausibility of the identification schemes (for example, one can consider the importance of correctly identified monetary policy shocks and some of its counter-effects, such as the price puzzle.) Moreover, its intrinsic soul of modelling past data at best as to reproduce the future in its consequence may limit its attractiveness (remind
the scarce performance of VARs forecasting short-term interest rates). If surprise monetary policy intervention can be analyzed with identified VARs (recursive VARs or finer structural specifications allowing for more complex monetary policy rules), different can be said when the point under question comes to be the effect of changing monetary policy rules.
### Table 2

**TEMPORAL DISAGGREGATION: PARAMETERS AND COEFFICIENTS ESTIMATION**

#### UNITED STATES

<table>
<thead>
<tr>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Chow-Lin</td>
<td>( \hat{\rho} = 0.9240 )</td>
<td>-60.2385</td>
<td>0.2404</td>
<td>18.6317</td>
<td>0.2143</td>
</tr>
<tr>
<td>Fernández</td>
<td>-44.9156</td>
<td>0.2737</td>
<td>13.7804</td>
<td>0.3286</td>
<td></td>
</tr>
<tr>
<td>Litterman</td>
<td>( \hat{\phi} = -0.8690 )</td>
<td>-53.4056</td>
<td>0.2261</td>
<td>16.4199</td>
<td>0.2901</td>
</tr>
<tr>
<td>SC</td>
<td>( \hat{\rho} = 0.8640 )</td>
<td>-8.9418</td>
<td>0.0354</td>
<td>2.8290</td>
<td>0.0049 ( b )</td>
</tr>
<tr>
<td>SC1</td>
<td>( \hat{\rho} = 0.6360 )</td>
<td>-25.4543</td>
<td>0.0504</td>
<td>7.8290</td>
<td>0.0832</td>
</tr>
<tr>
<td>SC2</td>
<td>( \hat{\rho}_1 = -0.1600 ), ( \hat{\rho}_2 = 0.9400 )</td>
<td>-65.7485</td>
<td>0.2600</td>
<td>20.8017</td>
<td>0.0357 ( b )</td>
</tr>
</tbody>
</table>

#### UNITED KINGDOM

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Chow-Lin</td>
<td>( \hat{\rho} = 0.990 )</td>
<td>4.5290</td>
<td>0.3895</td>
<td>0.4955</td>
<td>0.2533</td>
</tr>
<tr>
<td>Fernández</td>
<td>5.7638</td>
<td>0.3857</td>
<td>0.4098</td>
<td>0.1713 ( b )</td>
<td></td>
</tr>
<tr>
<td>Litterman</td>
<td>( \hat{\phi} = -0.736 )</td>
<td>5.6636</td>
<td>0.3810</td>
<td>0.4532</td>
<td>0.1662 ( c )</td>
</tr>
<tr>
<td>Litterman mod. ( a )</td>
<td>( \hat{\phi} = 0.7545 )</td>
<td>8.6486</td>
<td>0.3386</td>
<td>0.1926</td>
<td>0.0064 ( c )</td>
</tr>
<tr>
<td>SC</td>
<td>( \hat{\rho} = 0.8970 )</td>
<td>0.7729</td>
<td>0.0587</td>
<td>0.0640</td>
<td>-0.0145</td>
</tr>
<tr>
<td>SC1</td>
<td>( \hat{\rho} = 0.7500 )</td>
<td>1.6663</td>
<td>0.1182</td>
<td>0.1583</td>
<td>-0.0076 ( b )</td>
</tr>
<tr>
<td>SC2</td>
<td>( \hat{\rho}_1 = -0.77 ), ( \hat{\rho}_2 = 0.72 )</td>
<td>1.5475</td>
<td>0.1107</td>
<td>0.1454</td>
<td>-0.0090 ( b )</td>
</tr>
</tbody>
</table>

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### TABLE 2

**TEMPORAL DISAGGREGATION: PARAMETERS AND COEFFICIENTS ESTIMATION**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Chow-Lin</td>
<td>$\hat{\rho} = 0.9900$</td>
<td>-</td>
<td>0.1366</td>
<td>0.2180</td>
<td>0.8451</td>
</tr>
<tr>
<td>Fernández</td>
<td></td>
<td>-</td>
<td>0.1156</td>
<td>0.2215</td>
<td>0.8418</td>
</tr>
<tr>
<td>Litterman</td>
<td>$\hat{\rho} = -0.8160$</td>
<td>-</td>
<td>0.1413</td>
<td>0.2531</td>
<td>0.8054</td>
</tr>
<tr>
<td>Litterman mod.</td>
<td>$\hat{\rho} = 0.6851$</td>
<td>12.498</td>
<td>0.075$^a$</td>
<td>0.1644</td>
<td>-0.2408$^b$</td>
</tr>
<tr>
<td>SC</td>
<td>$\hat{\rho} = 0.9$</td>
<td>-</td>
<td>0.0449</td>
<td>-0.0001$^b$</td>
<td>0.0904</td>
</tr>
<tr>
<td>SC1</td>
<td>$\hat{\rho} = 0.6810$</td>
<td>-1.1682</td>
<td>0.1121</td>
<td>0.0042$^b$</td>
<td>0.4034</td>
</tr>
<tr>
<td>SC2</td>
<td>$\hat{\rho} = 0.9100$</td>
<td>-</td>
<td>0.0399</td>
<td>0.0006$^b$</td>
<td>0.0810</td>
</tr>
<tr>
<td></td>
<td>$\hat{\rho} = -0.5500$</td>
<td>-</td>
<td>0.4433</td>
<td>0.0071$^b$</td>
<td>0.9000</td>
</tr>
</tbody>
</table>

*Note:* The parameters of the dynamic models on the first line are those relevant over the short run; the others refer instead to the long run, i.e. $\beta_j/(1-\rho)$.

$^a$ The “Litterman mod.” method denotes the Litterman procedure with $\varphi$ parameter fixed to the value of the AR(1) coefficient estimated on Fernández residuals.

$^b$ The estimated parameter is equal to zero at 95% confidence level.

$^c$ The estimated parameter is equal to zero at 95% confidence level and it differs at 90%.

---

### TABLE 3

**TEMPORAL DISAGGREGATION PROCEDURES: PROPERTIES OF RESIDUALS AND GOODNESS-OF-FIT STATISTICS**

<table>
<thead>
<tr>
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<th>SC</th>
<th>FER</th>
<th>SC1</th>
<th>SC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stand. dev.</td>
<td>0.007412</td>
<td>0.012152</td>
<td>0.008962</td>
<td>0.001571</td>
<td>0.001422</td>
<td>0.001844</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0153</td>
<td>0.0443</td>
<td>0.0169</td>
<td>0.0058</td>
<td>0.003501</td>
<td>0.005604</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0156</td>
<td>-0.0124</td>
<td>-0.0205</td>
<td>-0.0067</td>
<td>-0.004553</td>
<td>-0.00831</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.00031</td>
<td>0.014506</td>
<td>-0.0002</td>
<td>0.000233</td>
<td>9.38E-06</td>
<td>1.61E-05</td>
</tr>
</tbody>
</table>

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### UNITED KINGDOM

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<thead>
<tr>
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<th>CL</th>
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<th>FER</th>
<th>SC1</th>
<th>SC2</th>
<th>LITM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard dev.</td>
<td>0.02626</td>
<td>0.036037</td>
<td>0.010385</td>
<td>0.00136</td>
<td>0.00099</td>
<td>0.00103</td>
<td>0.00076</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0504</td>
<td>0.1152</td>
<td>0.0211</td>
<td>0.0045</td>
<td>0.00291</td>
<td>0.002856</td>
<td>0.00302</td>
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<tr>
<td>Minimum</td>
<td>-0.043</td>
<td>-0.0027</td>
<td>-0.0308</td>
<td>-0.003</td>
<td>-0.00356</td>
<td>-0.003712</td>
<td>-0.0018</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0098</td>
<td>0.037015</td>
<td>-3.41E-04</td>
<td>0.00058</td>
<td>-2.47E-05</td>
<td>-2.27E-05</td>
<td>0.00032</td>
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### JAPAN

<table>
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<th>FER</th>
<th>SC1</th>
<th>SC2</th>
<th>LITM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard dev.</td>
<td>0.00366</td>
<td>0.005144</td>
<td>0.014191</td>
<td>0.003702</td>
<td>0.002819</td>
<td>0.005318</td>
<td>0.002379</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.012267</td>
<td>0.020386</td>
<td>0.0249</td>
<td>0.0128</td>
<td>0.007522</td>
<td>0.017514</td>
<td>0.008178</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.01228</td>
<td>-0.017011</td>
<td>-0.0407</td>
<td>-0.012</td>
<td>-0.10714</td>
<td>-0.021896</td>
<td>-0.00626</td>
</tr>
<tr>
<td>Mean</td>
<td>0.000778</td>
<td>0.001163</td>
<td>-0.000498</td>
<td>0.00069</td>
<td>2.21E-05</td>
<td>-1.15E-06</td>
<td>0.000394</td>
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### Table 4

**Comparing Models: RMSE, U-Theil Statistics and DM Test for One and Three-Steps-Ahead, Out-of-Sample Forecasts**

<table>
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<tr>
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<th>1 Month</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>UVAR RMES</td>
<td>U-Theil</td>
<td>BVAR RMES</td>
<td>U-Theil</td>
<td>BVAR1 RMES</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(pct change to UVAR)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.1487</td>
<td>0.86</td>
<td>0.1256 (-16%)a</td>
<td>0.72</td>
<td>0.1246 (-16%)b</td>
<td>0.72</td>
</tr>
<tr>
<td>UK</td>
<td>0.2736</td>
<td>3.24</td>
<td>0.2091 (-24%)a</td>
<td>2.48</td>
<td>0.1426 (-48%)b</td>
<td>1.69</td>
</tr>
<tr>
<td>JPN</td>
<td>0.0841</td>
<td>17.04</td>
<td>0.0721 (-14%)b</td>
<td>14.61</td>
<td>0.004 (-95%)b</td>
<td>0.81</td>
</tr>
<tr>
<td>M2 (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.2212</td>
<td>0.56</td>
<td>0.188 (-15%)b</td>
<td>0.48</td>
<td>0.1799 (-19%)b</td>
<td>0.46</td>
</tr>
<tr>
<td>UK</td>
<td>0.3304</td>
<td>0.44</td>
<td>0.2534 (-23%)a</td>
<td>0.33</td>
<td>0.2472 (-25%)b</td>
<td>0.33</td>
</tr>
<tr>
<td>JPN</td>
<td>0.2405</td>
<td>1.11</td>
<td>0.2042 (-15%)a</td>
<td>0.95</td>
<td>0.1889 (-21%)b</td>
<td>0.87</td>
</tr>
<tr>
<td>CP (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>2.5972</td>
<td>1.11</td>
<td>2.3417 (-10%)a</td>
<td>0.99</td>
<td>2.3438 (-10%)b</td>
<td>0.99</td>
</tr>
<tr>
<td>UK</td>
<td>8.6886</td>
<td>0.98</td>
<td>8.7314 (0%)</td>
<td>0.98</td>
<td>9.5886 (10%)</td>
<td>1.08</td>
</tr>
<tr>
<td>JPN</td>
<td>3.0615</td>
<td>1.12</td>
<td>2.5726 (-16%)a</td>
<td>0.94</td>
<td>3.0251 (-1%)</td>
<td>1.11</td>
</tr>
<tr>
<td>IPC (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.3892</td>
<td>0.74</td>
<td>0.4039 (4%)</td>
<td>0.76</td>
<td>0.3874 (0%)</td>
<td>0.73</td>
</tr>
<tr>
<td>UK</td>
<td>0.3122</td>
<td>0.83</td>
<td>0.2852 (-9%)</td>
<td>0.76</td>
<td>0.2861 (-8%)</td>
<td>0.76</td>
</tr>
<tr>
<td>JPN</td>
<td>3.0615</td>
<td>1.12</td>
<td>2.5726 (-16%)a</td>
<td>0.94</td>
<td>3.0251 (-1%)</td>
<td>1.11</td>
</tr>
<tr>
<td>U</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.1095</td>
<td>0.87</td>
<td>0.1139 (4%)</td>
<td>0.91</td>
<td>0.1079 (-2%)</td>
<td>0.86</td>
</tr>
<tr>
<td>UK</td>
<td>0.0759</td>
<td>0.96</td>
<td>0.0725 (-4%)</td>
<td>0.91</td>
<td>0.0774 (2%)</td>
<td>0.98</td>
</tr>
<tr>
<td>JPN</td>
<td>0.2603</td>
<td>0.99</td>
<td>0.2611 (%)</td>
<td>0.99</td>
<td>0.244 (-6%)</td>
<td>0.93</td>
</tr>
<tr>
<td>Real GDP (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.3532</td>
<td>0.92</td>
<td>0.289 (-18%)a</td>
<td>0.75</td>
<td>0.2995 (-15%)b</td>
<td>0.78</td>
</tr>
<tr>
<td>UK</td>
<td>0.2933</td>
<td>0.88</td>
<td>0.2678 (-9%)a</td>
<td>0.80</td>
<td>0.2885 (-2%)</td>
<td>0.86</td>
</tr>
<tr>
<td>JPN</td>
<td>0.2533</td>
<td>0.86</td>
<td>0.219 (-14%)b</td>
<td>0.74</td>
<td>0.2237 (-12%)b</td>
<td>0.76</td>
</tr>
</tbody>
</table>

(continued on next page)
### Table 4

**COMPARING MODELS: RMSE, U-THEIL STATISTICS AND DM TEST FOR ONE AND THREE- STEPS-AHEAD, OUT-OF-SAMPLE FORECASTS**

<table>
<thead>
<tr>
<th></th>
<th>3 Months</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UVAR RMES</td>
<td>U-Theil</td>
<td>BVAR RMES (pct change to UVAR)</td>
<td>U-Theil</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.4379</td>
<td>0.89</td>
<td>0.4386 (0%)</td>
<td>0.89</td>
</tr>
<tr>
<td>UK</td>
<td>0.3556</td>
<td>1.80</td>
<td>0.346 (-3%)(^a)</td>
<td>1.75</td>
</tr>
<tr>
<td>JPN</td>
<td>0.0803</td>
<td>10.09</td>
<td>0.0773 (-4%)</td>
<td>9.71</td>
</tr>
<tr>
<td>M2 (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.7503</td>
<td>0.68</td>
<td>0.7352 (-2%)(^a)</td>
<td>0.66</td>
</tr>
<tr>
<td>UK</td>
<td>1.4133</td>
<td>0.65</td>
<td>1.4099 (0%)</td>
<td>0.65</td>
</tr>
<tr>
<td>JPN</td>
<td>0.3872</td>
<td>0.82</td>
<td>0.3876 (0%)</td>
<td>0.82</td>
</tr>
<tr>
<td>CP (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>4.7978</td>
<td>0.97</td>
<td>4.7571 (-1%)</td>
<td>0.96</td>
</tr>
<tr>
<td>UK</td>
<td>14.7113</td>
<td>1.02</td>
<td>14.311 (-3%)(^a)</td>
<td>0.99</td>
</tr>
<tr>
<td>JPN</td>
<td>5.3438</td>
<td>0.92</td>
<td>5.3195 (0%)</td>
<td>0.91</td>
</tr>
<tr>
<td>IPC (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.9893</td>
<td>0.84</td>
<td>0.9869 (0%)</td>
<td>0.84</td>
</tr>
<tr>
<td>UK</td>
<td>0.6852</td>
<td>0.78</td>
<td>0.6774 (-1%)</td>
<td>0.77</td>
</tr>
<tr>
<td>JPN</td>
<td>5.3438</td>
<td>0.92</td>
<td>5.3195 (0%)</td>
<td>0.91</td>
</tr>
<tr>
<td>U</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.1489</td>
<td>0.93</td>
<td>0.151 (1%)</td>
<td>0.94</td>
</tr>
<tr>
<td>UK</td>
<td>0.1664</td>
<td>0.94</td>
<td>0.1652 (-1%)</td>
<td>0.94</td>
</tr>
<tr>
<td>JPN</td>
<td>0.4140</td>
<td>0.92</td>
<td>0.4166 (1%)</td>
<td>0.93</td>
</tr>
<tr>
<td>Real GDP (mtm growth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.7770</td>
<td>0.84</td>
<td>0.7716 (-1%)</td>
<td>0.83</td>
</tr>
<tr>
<td>UK</td>
<td>0.5274</td>
<td>0.86</td>
<td>0.5217 (-1%)</td>
<td>0.85</td>
</tr>
<tr>
<td>JPN</td>
<td>0.6533</td>
<td>0.92</td>
<td>0.6456 (-1%)(^b)</td>
<td>0.91</td>
</tr>
</tbody>
</table>

\(^a\) DM test accepted at 95% confidence level.
\(^b\) DM test accepted at 90% confidence level.
GRAPH 7A

IMPULSE-RESPONSE FUNCTIONS: UVAR vs. BVAR

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>JPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>I vs. I</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>CPI vs. I</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>GDP vs. I</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>CPI vs. CP</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>M2 vs. I</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>GDP vs. GDP</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>

Note: The charts present UVAR IRFs in light color and BVAR IRFs in dark color.


**IMPULSE-RESPONSE FUNCTIONS: RECURSIVE ORDERING**

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>JPN</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I vs. I</strong></td>
<td><img src="chart1.png" alt="Chart" /></td>
<td><img src="chart2.png" alt="Chart" /></td>
<td><img src="chart3.png" alt="Chart" /></td>
</tr>
<tr>
<td><strong>CPI vs. I</strong></td>
<td><img src="chart4.png" alt="Chart" /></td>
<td><img src="chart5.png" alt="Chart" /></td>
<td><img src="chart6.png" alt="Chart" /></td>
</tr>
<tr>
<td><strong>GDP vs. I</strong></td>
<td><img src="chart7.png" alt="Chart" /></td>
<td><img src="chart8.png" alt="Chart" /></td>
<td><img src="chart9.png" alt="Chart" /></td>
</tr>
<tr>
<td><strong>CPI vs. CP</strong></td>
<td><img src="chart10.png" alt="Chart" /></td>
<td><img src="chart11.png" alt="Chart" /></td>
<td><img src="chart12.png" alt="Chart" /></td>
</tr>
<tr>
<td><strong>M2 vs. I</strong></td>
<td><img src="chart13.png" alt="Chart" /></td>
<td><img src="chart14.png" alt="Chart" /></td>
<td><img src="chart15.png" alt="Chart" /></td>
</tr>
<tr>
<td><strong>GDP vs. GDP</strong></td>
<td><img src="chart16.png" alt="Chart" /></td>
<td><img src="chart17.png" alt="Chart" /></td>
<td><img src="chart18.png" alt="Chart" /></td>
</tr>
<tr>
<td><strong>CPI vs. GDP</strong></td>
<td><img src="chart19.png" alt="Chart" /></td>
<td><img src="chart20.png" alt="Chart" /></td>
<td><img src="chart21.png" alt="Chart" /></td>
</tr>
</tbody>
</table>

*Note:* The charts present in light color IRFs for a model under recursive order with interest rate as a first variable and in dark color IRFs with real GDP as a first variable.
In the following, a description of the time series (with relative sources) used in the VAR-related applications and in the temporal disaggregation exercise is presented. All the data but real GDP are published at monthly frequency.

**VARIABLES INCLUDED IN THE VAR**

**UNITED STATES**

**UNITED KINGDOM**
- UNEMPLOYMENT RATE: Percentage of unemployed civilian
labour force; seasonally adjusted. Source: OECD Statistical Compendium.

CPI: Non-harmonized Consumer Price Index (2000 = 100); not seasonally adjusted. Source: IMF.

M2: M2 monetary aggregate; seasonally adjusted. Source: Board of Governors of the Federal Reserve System.

INTEREST RATE: Sterling overnight interbank average (SONIA) lending rate. Source: OECD Statistical Compendium.

COMMODITY PRICE INDEX: Sterling spot price index (1967 = 100) of Brent; not seasonally adjusted. Source: IMF.

JAPAN

REAL GDP: Value of the real Gross National Product, derived via the information contained in CPI and nominal GDP series; seasonally adjusted. Source: Datastream.

UNEMPLOYMENT RATE: Percentage of unemployed civilian labour force; seasonally adjusted. Source: Datastream.

CPI: Non-harmonized Consumer Price Index (2005 = 100); not seasonally adjusted. Source: Datastream.

M2: M2 monetary aggregate plus certificates of deposit (CDs); seasonally adjusted. Source: OECD Statistical Compendium.

INTEREST RATE: Japan overnight call money rate (uncollateralized). Source: Datastream.

COMMODITY PRICE INDEX: Index (2005 = 100) derived via the multiplication of the corresponding US series by the $/yen spot exchange rate and the eventual division of the result by the Japanese CPI; not seasonally adjusted.

RELATED SERIES IN THE TEMPORAL DISAGGREGATION PROCEDURE

UNITED STATES

NON AGRICULTURAL PAYROLL EMPLOYMENT: Number of workers employed in all the sectors of industry but the agricultural one; seasonally adjusted. Source: Bureau of Labor Statistics.

INDUSTRIAL PRODUCTION INDEX: Index computed from
the physical output produced within the overall industrial sectors (2002 = 100); seasonally adjusted. Source: Board of Governors of the Federal Reserve System.

REAL PERSONAL CONSUMPTION EXPENDITURE: The real value (2000 = 100) of total expenditure in consumption; not seasonally adjusted. Source: Bureau of Economic Analysis.

UNITED KINGDOM
NON AGRICULTURAL PAYROLL EMPLOYMENT: Number of workers employed in all the sectors of the economy but agriculture and forestry; seasonally adjusted. Source: Datastream.
INDUSTRIAL PRODUCTION INDEX: Index computed from the physical output produced within the overall industrial sectors (2003 = 100); seasonally adjusted. Source: Office for National Statistics.
RETAIL SALES INDEX: Index of overall retailing sales (2000 = 100); not seasonally adjusted. Source: Office for National Statistics.

JAPAN
NON AGRICULTURAL PAYROLL EMPLOYMENT: Number of workers employed in all the sectors of the economy but the agricultural one (computed on the difference between total number of employees and amount of people employed in that sector); seasonally adjusted. Source: Datastream.
INDUSTRIAL PRODUCTION INDEX: Index of the physical output produced within the mining and manufacturing industrial sectors except publishing (2003 = 100); seasonally adjusted. Source: Datastream.
REAL RETAIL SALES: Value of real retail sales, computed on the nominal series via the CPI; not seasonally adjusted. Source: Datastream.


Geweke J., Models, Computational Experiments and Reality, Iowa City (IA), University of Iowa, Department of Economics and Statistics, 2007.


