An Inverted U-Relationship between Product Market Competition and Growth in an Extended Romerian Model: A Comment

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This paper shows that the results of Bucci (2005) depend critically on the assumption that there are no difference between the intermediate goods share in final output, the returns of specialization and the degree of market power of monopolistic competitors. In this paper, we disentangle the market power parameter from the intermediate goods share in final output and the returns to specialization. The main result of this paper is the death of the inverted-U shape relationship between competition and growth. Indeed, we find a decreasing relationship between competition and growth which is due to the composition of two negative effects on growth: resource allocation and Schumpeterian effects. [JEL Classification: O31; O41]

Nel saggio mostriamo come i risultati di Bucci (Fascicolo IX-X, Rivista di Politica Economica*, 2005) dipendano in maniera chiave dall'assunzione che non ci siano differenze tra la quota dei beni intermedi nella produzione finale, i rendimenti provenienti della specializzazione e il grado di potere di mercato dei competitori monopolistici. Il risultato principale di questa nota è la scomparsa della relazione ad U invertita tra concorrenza e crescita. Troviamo una relazione decrescente tra le due, dovuta alla composizione di due effetti negativi sulla crescita: l'allocazione delle risorse e gli effetti schumpeteriani.*

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1. - Introduction

Bucci (2005) studies the impact of competition in the intermediate goods sector on growth. He uses the Gancia and Zilibotti (2005) model in which he introduces a different assumption concerning the production of intermediate goods. Indeed, unlike Gancia and Zilibotti (2005) who assumes that one need one unit of final good to produce one unit of intermediate good, Bucci (2005) does the hypothesis that the firm has to use one unit of labor. This assumption which is called "resource allocation effect" implies that labor can be allocated between three sectors: final good, intermediate goods and research. The interplay between this effect and the traditional Schumpeterian effect allows to obtain an interesting result. Indeed, Bucci (2005) finds an inverted-U relationship between competition and growth. For low value of competition, more competition is beneficial to growth since it allows a better allocation of resource without hampering that much innovation incentives. In this case, the resource allocation effect is bigger than the profit incentive effect. On the other hand, for high value of competition, more competition reduces strongly growth because of the reduction of profit. In this case, the profit incentive effect is bigger than the resource allocation effect.

Among the assumptions used by Bucci (2005) to derive this result is that there are no difference between the intermediate goods share in final output, the returns to specialization and the degree of market power of monopolistic competitors. This leads to the natural question whether making such a difference to the model changes its predictions. In this note, we show that including this difference into the model developed by Bucci (2005) eliminates the result mentioned above.

2. - The Model

The model developed is based on Bucci (2005).[1] The economy

[1] We use the notations of BUCCI A. (2005) in order to have a direct comparison with his model.

is structured by three sectors: final good sector, intermediate goods sector and R&D sector. The final output sector produces output that can be used for consumption using labor and intermediate goods. These are available in A varieties and are produced by employing only labor. The R&D sector creates the blueprints for new varieties of intermediate goods which are produced by employing labor and knowledge. These blueprints are sold to the intermediate goods sector.

2.1 *The Final Good Sector*

In this sector, atomistic producers engage in perfect competition. The final good sector produces a composite good Y by using all the *jth* type of intermediate goods x_j and labor L_Y.[2] Production is given by:

$$(1) \qquad Y = N^{\gamma - \lambda\left(\frac{1}{\alpha}-1\right)} \left[\int_0^N x_j^{\alpha} dj \right]^{\frac{\lambda}{\alpha}} L_Y^{1-\lambda}$$

where α, λ and $\gamma \in]0, 1]$ are three parameters. This production function allows us to disentangle the degree of market power of monopolistic competitors in the intermediate sector $1/\alpha\, 1$, the intermediate goods share in final output λ and the degree of returns from specialization λ.[3] In this sense, this model is a generalization of Bucci (2005) and Benassy (1998) models.[4] If we normalize to one the price of the final good, the profit of the representative firm is given by:

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[2] Time subscripts are omitted whenever there is no risk of ambiguity.

[3] BENASSY J.-P. (1996) made a simple modification of the DIXIT A.K. - STIGLITZ J.E. (1977) model by disentangling clearly taste for variety and market power. At the same time, BENASSY J.-P. (1998) and DE GROOT H.L. - NAHUIS R. (1998) show that the introduction of this modification in an endogenous growth model with expanding product variety *à la* GROSSMAN G.M. - HELPMAN E. (1991) affects the welfare analysis.

[4] lndeed, we obtain the BUCCI A. (2005) model by introducing the following constraints $\lambda = \alpha, \gamma = 1 - \alpha$ in our model. In the same way, by introducing the constraint $\lambda = 1$, we obtain the BENASSY J.-P. (1998) model.

(2) $$\pi_Y = N^{\gamma-\lambda\left(\frac{1}{\alpha}-1\right)}\left[\int_0^N x_j^\alpha dj\right]^{\frac{\lambda}{\alpha}} L_Y^{1-\lambda} - \int_0^N p_j x_j dj - w_Y L_Y$$

where w_Y is the wage rate in the final good sector and p_j is the price of the *jth* intermediate good. Under perfect competition in the final output market and the factor inputs markets, the representative firm chooses intermediate goods and labor in order to maximize its profit taking prices as given and subject to its technological constraint. The first order conditions are the followings:

(3) $$\frac{\partial\pi_Y}{\partial x_j} = \lambda N^{\gamma-\frac{\lambda}{\alpha}+\lambda}\left[\int_0^N x_j^\alpha dj\right]^{\frac{\lambda}{\alpha}-1} L_Y^{1-\lambda} x_j^{\alpha-1} - p_j = 0$$

(4) $$\frac{\partial\pi_Y}{\partial L_Y} = (1-\lambda)N^{\gamma-\frac{\lambda}{\alpha}+\lambda}\left[\int_0^N x_j^\alpha dj\right]^{\frac{\lambda}{\alpha}} L_Y^{1-\lambda} - w_Y = 0$$

Equation *(3)* is the inverse demand function for the firm that produces the *jth* intermediate good whereas equation *(4)* characterizes the demand function of labor.

2.2 *The Intermediate Goods Sector*

In the intermediate goods sector, producers engage in monopolistic competition. Each firm produces one horizontally differentiated intermediate good and has to buy a patented design before producing it. Following Grossman and Helpman (1991) and Bucci (2005), we assume that each local intermediate monopolist has access to the same technology employing only labor l_j:

(5) $$x_j = l_j$$

We suppose that firms behavior which produce intermediate goods is governed by the principle of profit maximization at given factor prices under a technological constraint. The profit function of firms is the following:

$$\pi_j = p_j x_j - w_j l_j \tag{6}$$

where w_j is the wage rate in the intermediate goods sector. Using the first order condition, we obtain the price of the *jth* intermediate good:

$$p_j = \frac{w_j}{\alpha} \tag{7}$$

At the symmetric equilibrium, all firms produce the same quantity of the intermediate good x, face the same wage rate w and by consequence fix the same price for their production p. The price is equal to a constant mark up $1/\alpha$ over the marginal cost w. Defining by $L_j = \int_0^N l_j d_j$, the total amount of labor employed in the intermediate goods sector and under the assumption of symmetry among intermediate goods producers, we can rewrite the equation *(5)* as follows:

$$x_j = \frac{L_j}{N} \tag{8}$$

Finally, the profit function of the firm which produces the *jth* intermediate good is:

$$\pi_j = \lambda\,(1 - \alpha)\, N^{\gamma-1} L_j^{\lambda} L_y^{1-\lambda} \tag{9}$$

2.3 *The R&D Sector*

There are competitive research firms undertaking R&D. Following Romer (1990) and Grossman and Helpman (1991), we assume that new blueprints are produced by using old blueprints N and an amount of R&D labor L_N:

$$\frac{\partial N}{\partial t} = \frac{1}{\eta} N L_N \tag{10}$$

where $1/\eta > 0$ represents the productivity of the R&D process. Because of the perfect competition in the R&D sector, we can obtain the real wage in this sector as a function of the profit flows associated to the latest intermediate in using the zero profit condition:

$$w_N L_N = \frac{\partial N}{\partial t} P_N \tag{11}$$

where w_N represents the real wage earned by R&D labor. P_N is the real value of such a blueprint which is equal to:

$$P_N = \int_t^\infty \pi_j e^{-r(\tau - t)} d\tau, \quad \tau > t \tag{12}$$

where r is the real interest rate. Given P_N, the free entry condition leads to:

$$w_N = \frac{NP_N}{\eta} \tag{13}$$

2.4 *The Consumer Behavior*

The demand side is characterized by the representative household who consumes and supplies labor. Following Grossman and Helpman (1991), we assume that the utility function of this consumer is logarithmic[5]:

$$U = \int_0^\infty e^{-\rho t} \log(C) dt \tag{14}$$

where C is private consumption, $\rho > 0$ is the rate of pure time preference. The representative household is endowed with fixed quantities of labor L that are supplied inelastically. The flow budget constraint for the household is:

$$\frac{\partial W}{\partial t} = wL + rW - C \tag{15}$$

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[5] This specification of the utility function does not alter the results.

where W is the total wealth of the agent (measured in units of final good), w is the wage rate per unit of labor service. From the maximization program of the consumer, the necessary and sufficient conditions for a solution are given by the Keynes-Ramsey rule:

(16) $$g_C = r - \rho$$

and the transversality condition:

(17) $$\lim_{t \to \infty} \mu_t W_t = 0$$

where μ_t is the co-state variable.

3. - The Equilibrium and the Steady State

In this section, we characterize the equilibrium and give some analytical characterizations of a balanced growth path.

3.1 *The Equilibrium*

It is now possible to characterize the labor market equilibrium in the economy considered. On this market, because of the homogeneity and the perfect mobility across sectors, the arbitrage ensures that the wage rate that is earned by employees who work in the final good sector, intermediate goods sector or R&D sector is equal. As a result, the following three conditions must simultaneously be satisfied[6]:

(18) $$L = L_Y + L_j + L_N$$

(19) $$w_j = w_Y$$

(20) $$w_j = w_N$$

[6] We assume without loss of commonalty that the total labor force is constant.

Equation *(18)* is a resource constraint, saying that at any point in the time the sum of the labor demands coming from each activity must be equal to the total available fixed supply. Equation *(19)* and equation *(20)* state that the wage earned by one unit of labor is to be the same irrespective of the sector where that unit of labor is actually employed. We can characterize the product market equilibrium in the economy considered. Indeed, on this market, the firms produce a final good which can be consumed. Consequently, the following condition must be satisfied:

$$Y = C \tag{21}$$

Equation *(21)* is a resource constraint on the final good sector.

3.2 *The Steady State*

At the steady state, all variables as Y, C, N grow at a positive constant rate. Obviously, it is easy to show from equations (*21*, *1* and *8*) the following relationship between the economic growth rate, the consumption growth rate and the knowledge growth rate:

$$g_Y = g_C = \gamma g_N \tag{22}$$

Using the previous equations, we can demonstrate the following steady state equilibrium values for the relevant variables of the model[7, 8]:

$$r = \frac{L(1-\alpha)\gamma\lambda - \eta((\alpha-1)\lambda\gamma + \gamma - 1)\rho}{\eta} \tag{23}$$

$$L_j = \alpha\lambda(L + \eta\rho) \tag{24}$$

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[7] Results *(23)* through *(27)* are demonstrated in the *APPENDIX*.

[8] In order to have all variables positive, we assume that

$$0 < \eta < \frac{L\lambda - L\alpha\lambda}{\alpha\lambda\rho - \lambda\rho + \rho}$$

(25) $$L_Y = (1 - \lambda)\,(L + \eta\rho)$$

(26) $$L_N = (1 - \alpha)\,\lambda\,(L + \eta\rho) - \eta\rho$$

(27) $$g_Y = \frac{\gamma((1-\alpha)\lambda(L+\eta\rho)-\eta\rho)}{\eta}$$

4. - The Relationship Between Product Market Competition and Growth

In this section, we study the long run relationship between competition and growth in the model presented above. Following most authors, we use the so-called Lerner Index to gauge the intensity of market power within a market. Such an index is defined by the ratio of price P minus marginal cost CM over price. Using the definition of a mark up $Markup = P/CM$ and Lerner Index $LernerIndex = (P - CM)/P$ we can use *(7)* to define a proxy of competition as follows[9]:

(28) $$(1 - LernerIndex) = \alpha$$

We show that our simple generalization of Bucci (2005)'s model that consists in having the monopolistic mark-up in the intermediate goods sector, the intermediate goods share in the final output and the returns to specialization treated separately, the inverted U relationship between competition and growth no longer exists.

PROPOSITION 1. The relationship between competition and growth is negative for all positive values of ρ, η, L and γ and $\lambda \in]\,0, 1]$.

[9] This is the same measure of product market competition used by AGHION P. - BLOOM N. - BLUNDELL R. - GRIFFITH R. - HOWITT P. (2005); AGHION P. - GRIFFITH R. (2005); AGHION P. - HOWITT P. (2005) - BIANCO D. (2006), contrary to BUCCI A. - PARELLO C. (2006) who link the competition to two components the input shares in income and the parameter of substitution between intermediates.

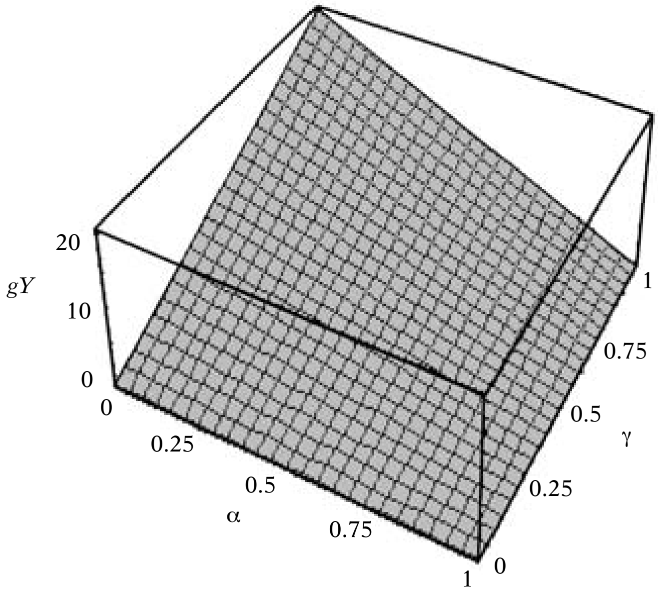
PROOF. The proof is obtained by differentiating the equation *(27)* with respect to α:

$$\frac{\partial g_Y}{\partial \alpha} = -\frac{\gamma\lambda(L+\eta\rho)}{\eta} < 0 \tag{29}$$

In order to illustrate this result, we plot the equation *(27)* for different values of competition α, and returns to specialization γ[10]:

GRAPH 1

RELATIONSHIP BETWEEN COMPETITION α, RETURNS TO SPECIALIZATION γ AND GROWTH g_Y



According to the profit incentive effect, an increase of competition reduces the price of the intermediate good and profit, what determines the incentives to innovation. Therefore, the profit incentive effect seems to predict an unambiguously negative

[10] In drawing Graph 1, we take the same value of parameters like BUCCI A. (2005) in order to be as close as possible to his model: $\rho = 0.03$, $\eta = 1$, $L = 35$ and $\lambda = 0.75$.

relationship between product market competition and growth along the entire range of competition intensity. Contrary to Bucci (2005), an increase of competition reduces the amount of labor devoted to the research sector L_N along the entire range of competition intensity. Moreover, an increase of competition has no effect on the amount of labor allocated to the final good sector L_Y and increases the amount of labor in the intermediate goods sector L_j. This means that the resource allocation effect seems also to predict an unambiguously negative relationship between product market competition and growth. Finally, we always have as we can see on the above figure a decreasing relationship between competition and growth.

5. - Conclusion

In this paper, we presented a generalization of Bucci (2005) model in which we disentangle the monopolistic mark-up in the intermediate goods sector, the intermediate goods share in the final output and the returns to specialization. Our main finding is that the result of the Bucci (2005) model that close in an inverted U relationship between competition and growth depends critically on the assumptions that there are no differences between these three parameters. Indeed, for all values of parameters except to $\lambda = \alpha$, we could remove the inverted-U relationship between competition and growth. This result is due to the interplay of two effects: Schumpeterian and resource allocation effects. In our model, we find that the resource allocation effect is always negative which reinforces the Schumpeterian effect on growth. Consequently, we find a decreasing relationship between competition and growth.

APPENDIX

In this appendix, we describe the way followed in order to obtain the main results of this paper (*23* through *27*). Using the equations (*3, 4, 7, 8* and *19*), we obtain

$$L_Y = \frac{(1-\lambda)L_j}{\alpha\lambda} \tag{30}$$

Plugging this equation into equation *(18)* yields

$$L_j = \frac{\alpha\lambda(L-L_N)}{1+(\alpha-1)\lambda} \tag{31}$$

Consequently, the equation *(30)* can be re-written as:

$$L_Y = \frac{(1-\lambda)(L-L_N)}{1+(\alpha-1)\lambda} \tag{32}$$

In order to compute the wage in the research sector, we need to have the value of the blueprint in the steady state (equations *9* and *12*) which is:

$$P_N = \frac{(1-\alpha)\lambda L_Y^{1-\lambda} L_j^{\lambda} N^{\gamma-1}}{\frac{(1-\gamma)L_N}{\eta} + r} \tag{33}$$

Given P_N and using equation *(13)*, we obtain:

$$w_N = \frac{N^{\gamma} L_j^{\lambda} L_Y^{1-\lambda} L_j^{\lambda}(1-\alpha)\lambda}{L_N(1-\gamma)+r\eta} \tag{34}$$

In equating w_j to w_N, we find:

$$L_j = \frac{\alpha(L_N(1-\gamma)+r\eta)}{1-\alpha} \tag{35}$$

Using the equations (*31* and *35*), we obtain:

(36) $$L_N = \frac{L(1-\alpha)\lambda - r\eta(1+(\alpha-1)\lambda)}{1-\gamma(1+(\alpha-1)\lambda)}$$

Consequently:

(37) $$L_j = \frac{\alpha(L(1-\gamma)+r\eta)\lambda)}{1-\gamma+(1-\alpha)\lambda\gamma}$$

In order to determine the equilibrium interest rate, we use these two equations:

(38) $$g_Y = g_C = r - \rho$$

(39) $$g_Y = \gamma g_N$$

After some computations, we obtain:

(40) $$r = \frac{L(1-\alpha)\gamma\lambda - \eta((\alpha-1)\lambda\gamma + \gamma - 1)\rho}{\eta}$$

Finally, from the previous equations, we obtain the rest of variables of the model:

(41) $$L_j = \alpha\lambda\,(L + \eta\rho)$$

(42) $$L_Y = (1 - \lambda)\,(L + \eta\rho)$$

(43) $$L_N = (1 - \alpha)\,\lambda\,(L + \eta\rho) - \eta\rho$$

(44) $$g_Y = \frac{\gamma((1-\alpha)\lambda(L+\eta\rho)-\eta\rho)}{\eta}$$

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