Indicators and Tests of Sustainability: The Italian Case

Matteo Formenti*
“Tor Vergata” University of Rome

The issue of sustainability of Italian fiscal policy in the period 1970-2006 is studied with two instruments: indicators and tests. The indicators Primary Gap, Tax Gap developed by Chouraqui et al. (1990) and S2 by the European Commission show a non-sustainable fiscal policy. Tests of sustainability following Trehan and Walsh (1988), Hakkio and Rush (1991) and Bohn (2008) show an opposite result accepting the sustainability path of fiscal variables. According to the strategy proposed by De Luzenberger and Marini (1993) a Chow-type test, that confirms a systematic change of fiscal policy in 1992-1993, permits to discern the two conflicting results in favour of the indicators’ response. JEL Classification: [C12; C22; E60; H60]

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1. - Introduction

The issue of how measures sustainability of fiscal policy is currently one of the most debated question for both policy makers and macroeconomists. Since the early 1980s, when most countries experienced high levels of government debt and primary deficit, macroeconomists were attracted to the uncovered question of how

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* <matteo.formenti@uniroma2.it>, Faculty of Economics – Department of Economics and Institution. I thank my supervisors Prof. G. Marini, Prof. Scaramozzino and Dott. A. Piergallini and the anonymous referees for useful comments. I am grateful to Dott. Vigliotti and Dott. Lizza for the support received during my six months internship at the Ministry of Economy and Finance.
much the government debt can increase without a Ponzi game, while policy makers were attracted to which instruments warn about the future path of fiscal policy. Two possible strategies for an empirical evaluation of fiscal sustainability, based on the government intertemporal budget constraint, appear to be worth pursuing. One strategy is to construct indicators of the sustainability of fiscal policy, along the lines suggested by Buiter (1985), Chouraqui et al. (1990) and Buiter et al. (1993). The other is to implement tests for fiscal solvency, following the seminal works by Hamilton and Flavin (1986), Trehan and Walsh (1988), Hakkio and Rush (1991) and Bohn (1998, 2008). The indicators presented in the literature are either simple measures mainly based on current information, or summary values of model-based projections of future paths of fiscal policy. The testing techniques, so far employed, involve unit root tests (Hamilton and Flavin, 1986; Wilcox, 1989), cointegration analysis of fiscal data (Trehan and Walsh, 1988 and 1991; Hakkio and Rush, 1991), or require the adoption of a model-based sustainability approach (Bohn, 1998 and 2008). The purpose of this thesis is to review indicators and tests for Italian fiscal sustainability and their useful integration: as shown in De Luzenberger and Marini (1993) the simultaneous use of indicators and tests provides additional information on the issue of government solvency. Indicators and tests simply reinforce each other when their predictions are not in conflict. However, when conflicting results exist, indicators may signal the occurrence of a change in policy which may reverse the predictions of tests. This is because indicators respond to a set of current and expected future conditions. Tests, on the other hand, always rely on a sample of past data. For this reason, indicators are faster than tests in responding to changes in fiscal policy regimes. The key point is that a change in regime should cause a structural break in the data generating process, which can be detected empirically. An intuitive strategy to combine the potentially different results of indicators and tests is given by the Chow test for the presence of structural breaks. We apply this strategy to Italy's fiscal policy in the period 1970-2006. The work is structured as follows. Section 2 and 3 briefly review the
literature on sustainability of fiscal policy and the fiscal accounting, Section 4 and 5 describe the most commonly used indicators of fiscal sustainability and the main procedures to test for sustainability. Section 7 presents the empirical results applied to Italian fiscal policy during the period 1970-2006 and Section 8 concludes.

2. - The Literature on Sustainability of Fiscal Policy

The literature on sustainability fiscal policy is debated in both Keynesian and Ricardian economic theory in different manner. The former deals with government debt expansion as instrument to stimulate the aggregate demand in case of obsolete productive resources, where the overall effects depend on which debt’s instruments are chosen by policy makers: higher effects if financed by monetary supply, lower with bonds auction in the government debt market. The Fiscal Optimal Theory based on the Ricardian equivalence, on the other hand, sustain that intertemporal budget constraint always matter in the long run. So an actual increase of debt, financed by all public agents, cannot be considered as an increase of private saving nor as an increase of public spending. Economic agents know that an increase in government debt, or tax, has the same long run effect because it never modify their permanent income. The Keynesian view requires that economic growth is higher than debt growth to repay future debt. This is why the stability instruments are used in case of an enduring effect of debt’s expansion and why deficit, instead, should be only conjectural (only investment spending and not primary spending, because the latter will increase the debt continuously). On the other hand, the Ricardian view, called also debt neutrality view, consider the effect of debt on the accumulation of capital and the economic growth. Since the 1970s, in spite of a large use of this Theory in practice, the main critiques are based on liquidity constraint and economic agent’s limited rationality. Barro (1974, 1979) demonstrated how this two elements can be easily overcome using the family, instead of the individuals as the representative
economic agent. Both Ricardian and Keynesian analysis does not deal with sustainability of fiscal policy, but only on the effects of debt’s expansion on economic growth. Sustainability, on the other hand, refers to how fiscal variables evolve over time and whether government is able to repair its entire debt, according to past and future dynamics of fundamentals of the economy. It never refers to the way to repay it. Literature on sustainability of fiscal policy is divided in two: the approach of General Equilibrium Models (GEMs) and Partial Equilibrium Models (PEMs). GEMs analyze sustainability firstly with Diamond (1965) and later with Ihori (1978), Chalk (2000), De la Croix and Michel (2002) and Marin (1999, 2002). Diamond analyses, in a neoclassical technology framework, the effects of a debt stock on the long run steady state of the economy: an increase in government debt cause a decrease (increase) of consumer’s utility when the economy is efficient (inefficient). This result was the base of other important works. Ihori (1978) analyses the effect on the long run accumulation of capital and the way of how sustainable fiscal policy affects the long run economic growth. Recently, Chalk (2000) and De la Croix and Michel (2002) analyze the impact of the public debt, and its relative sustainability, on the economic growth. The former analyses sustainability when deficit is financed with public bond and defines the necessary condition of sustainability: economic growth must be lower than the interest growth rate. The latter authors analyze sustainability in overlapping dynamic general equilibrium model with two periods. Others theoretical models determines the maximum sustainable government debt using Diamond’s overlapping-generations model, as in the work of Rankin and Roffia (2003). They establish the sustainable level of debt in a steady state and demonstrate that, almost always, exist a maximum sustainable level of debt. Moreover, a slow debt increase may suddenly collapse the economy without any warning. Annicchiarico and Giammarioli (2004) present a two periods overlapping generations model with endogenous growth and a public sector with objectives of convergence for public debt and primary balance. To evaluate the performance of a simple fiscal policy rule, in terms of its capability to guarantee convergence
and sustainability of public finances, they introduce a simple fiscal policy rule. The policy reaction rule links the primary balance ratio to the distance between the current and the target levels of public debt, and primary surplus, to GDP ratios. They conclude that a fiscal rule characterized by time invariant parameters may trigger a non linear process of adjustment towards the objective of convergence. In particular, larger initial public debt ratio causes larger fluctuations in the economic growth. On the other hand, the formal analysis of sustainability in PEMs was firstly studied by Domar (1944). In the celebrated work the economic growth and the interest rate are exogenous variables respect to fiscal policy creation and the author defines the necessary condition for sustainability: primary balance growth must be equal the economic growth times debt, in GDP terms. Remark that this is a necessary condition because, neither debt ratio, nor tax ratio are useful instruments to characterize the fiscal policy’s rule, as confirmed by the work of Balassone and Franco (2000). Later, Buiter (1985) and Blanchard et al. (1990) analyze the sufficient conditions for sustainability: the former argues that constant net worth on GDP is a condition to ensure sustainability. The latter authors, instead, define two fundamental conditions: the convergence of debt on GDP to the initial level and the equality of discounted primary balance with discounted (negative) debt, in GDP terms. Remark that first condition implies the second one because discounted debt goes to zero when time goes to infinite, while the undiscounted debt goes to its initial value, when debt increase is lower than the difference between interest rate and economic growth. Summing up, from the one hand GEMs are able to capture the sustainability of fiscal policy, because consider endogenous debt, deficit level, the accumulation of interest rate and economic growth rate. From the other hand, GEMs empirical test require an huge number of parameters and this may cause some problems of validity. PEMs, instead, are easier to test empirically. They aim to verify whether the sustainability conditions hold, without capture what level of debt can affect the overall economy.
3. - The Fiscal Accounting

Assuming fixed real ex-post interest rate $r_t = r > 0$, the intertemporal government budget dynamic is in nominal value

$$(1) \quad B_t = (1+r)B_{t-1} + G_t - T_t$$

where $B_t$ is the level of debt, $G_t$ is the government expenditure and $T_t$ is the government revenue. From (1) we derive the primary balance $s_t$, revenue-expenditure difference, in terms of debt increase and in terms of GDP

$$(2) \quad \Delta b = g_t - t_t + (r - \gamma_t) b_t$$

$$(3) \quad \Delta b = (r - \gamma) b_t - s_t$$

where $\gamma_t$ is the economic growth of the economy. The equation (2) describes the debt dynamics. We now assume that in equilibrium it must be $\Delta b = 0$. Plug it in (3) and we get two important equations: the primary balance equilibrium equation

$$(4) \quad s_t = (r - \gamma) b_t$$

and solving for $b_t$ the equilibrium debt ratio

$$(5) \quad b^* = \frac{s_t}{r - \gamma_t}$$

The sign of the equilibrium debt ratio is not determined a priori. Actually it depends on the primary balance sign (surplus if positive, deficit is negative) and on the difference between economic growth rate and interest rate. If signs are different, the government is a creditor to the private sector. To assess sustainability, we assume that government satisfies both the budget constraint in each period and the intertemporal budget constraint (Chalk and Hemming, 2000). Using the budget

1 We can assume that $r_t$ is stationary with $r$ mean.
dynamics for a closed economy as in (1) we solve it forward, always in GDP terms, to get another form of the government budget intertemporal constraint

\[
\begin{align*}
(6) \quad b_t &= \left( \sum_{i=0}^{\infty} \frac{g_{t+i}}{1+r} - \sum_{i=0}^{\infty} \frac{t_{t+i}}{1+r} \right) + \lim_{T \to \infty} \frac{b_{t+T}}{1+r} \\
(7) &= -\sum_{i=0}^{\infty} \frac{s_{t+i}}{1+r} + \lim_{T \to \infty} \frac{b_{t+T}}{1+r}
\end{align*}
\]

Equation (7) shows that a fiscal policy is sustainable when present value of primary discounted balance and present discounted value of final debt exceed the initial stock of debt. In the case of debt greater than zero, equation (7) is satisfied even if government rolls over its debt to repay debt itself and interests. The equations can be compared to the ones presented in Blanchard et al. (1990).

3.1 The Sustainability Condition

The intertemporal budget constraint given by (1) can be written with \( r_t \) time variant at time \( t_0 \)

\[
\frac{dB_t}{dt} = G_t - T_t + r_t B_{t-1}
\]

\[
B(0) = B_0
\]

Solving the differential equation (8) for debt at time \( t \) gives

\[
(9) \quad B_t = e^{\int_{t_0}^{t} r_t d\tau} \left( B_0 - \int_{t_0}^{t} e^{\int_{\tau}^{t} r_u du} S_u d\tau \right)
\]

such that rearranging for \( B_0 \)
and assuming a constant $r \geq 0$ we finally obtain an equation for debt at $B_0$

\[(11) \quad B_0 = e^{-r_0}B_t + \int_0^t e^{-r_0}S_d \, d\tau \]

We now take conditional expectations to derive, in terms of GDP, the conditions that ensure sustainability

\[(12) \quad b_0 = E\int_0^\infty e^{-r_0}S_d \, d\tau \]

\[(13) \quad \lim_{t \to \infty} E(e^{-r_0}b_t) = 0 \]

The first equation (12) affirms that expected discounted value of primary balance must be equal the initial value of debt. Equation (13), called no-Ponzi game condition, ensure no debt’s over accumulation because debt increase at a lower rate with respect to the interest rate\(^2\). Agenor (2000) distinguishes equation (13) in terms of equality or inequality, $\lim_{t \to \infty} E(e^{-r_0}b_t) \leq 0$, because agent taken as individuals cannot rescind debt with the Government.

3.2 The Necessary and Sufficient Conditions

The budget constraint dynamics given by equation (6) can be written with the inflation rate $\pi_t = \frac{P_t - P_{t-1}}{P_t}$ as

\[^2\text{This condition can be violated if we consider Government as the economic agent playing the game infinitively. This is possible in practice and in overlapping generation models with inefficient economies (DIAMOND P., 1965). BOHN H. (1995) clarifies the issue showing that, in an economy with infinite life agent and perfect financial market, Government must always satisfied the No Ponzi condition.}\]
\[ (14) \quad b_t = - \left( \sum_{i=0}^{\infty} \frac{1 + \pi_{t+i} + \gamma_i}{1 + i_t} g_t - \sum_{i=0}^{\infty} \frac{1 + \pi_{t+i}}{1 + i_t} t_t \right) + \lim_{T \to \infty} \sum_{i=0}^{\infty} \frac{1 + \pi_{t+i} + \gamma_i b_{t+i}}{1 + i_t} \]

where \( b_t = B_t/Y_t \), \( t_t = T_t/P_t Y_t \), \( P_t \) is the nominal price and \( i_t \) the nominal interest rate. Following Jha (2004) we define the necessary and sufficient condition for a sustainable fiscal policy.

**CASE 1:**

\[ i_t - \pi_t \leq \gamma_t. \]

Consider the first term in the inequality as the real interest rate \( r_t = i_t - \pi_t \) on government debt, therefore debt ratio can be stable, and the economy can be solvent, only if the following condition is satisfied:

\[ \lim_{t \to \infty} b_t = 0. \]

If the initial value of the debt to GDP ratio \( b_t \) is strictly positive then two conditions have to be met: \( i_t - \pi_t \leq \gamma_t \) allows primary balance consistency of debt ratio, \( g_t - t_t \leq 0 \) enables debt burden to be eventually liquidated. The government remains solvent if both necessary and sufficient conditions are satisfied because they allow fully debt repayment.

**CASE 2:**

\[ i_t - \pi_t \geq \gamma_t. \]

A real interest rate higher than the economic growth is condition to debt stock diverging towards infinity. In this case, the sequence of primary balance is irrelevant even if the growing debt stock is balanced by an increasing primary balance in the future. Jha (2004) proposes to solve this problem using the iterative dynamics budget constraint:

\[ (16) \quad b_t = - \left( \sum_{i=0}^{\infty} \frac{g_{t+i}}{1 + \vartheta_t} - \sum_{i=0}^{\infty} \frac{t_{t+i}}{1 + \vartheta_t} \right) + \lim_{T \to \infty} \frac{b_{t+i}}{1 + \vartheta_t} \]

where \( \vartheta_t = i_t - \pi_t - \gamma_t \) is the interest real rate minus the growth.
rate and $\frac{1}{1 + \theta_i}$ is the discount factor to adjust the real economic growth rate. In such a way, the non-explosiveness of public debt is ensured by the second term, the no Ponzi game condition: this is another explicit necessary and sufficient condition that ensure sustainability$^3$. It is to be remarked that empirical literature on indicators focuses on the first condition while theoretical literature on tests verify the second one. At the end, as argued in Bohn (2008), we need a definition of sustainability:

**DEFINITION 1:** A fiscal policy satisfies ad hoc sustainability, if it is on a trajectory such that the expected present value of future primary surpluses equals the initial debt.

This is a flawed definition that motivates standard empirical test.

### 4. - The Indicators of Sustainability

As argued by Chouraqui et al. (1990), and subsequently implemented by Blanchard (1990), indicators of sustainability are instruments used by policy makers to understand whether fiscal policy is sustainable or not. OECD defined in 1990 four characteristics they must ensure:

- **Discretionary** element in fiscal policy: of the changes in the fiscal position of the government (taxes, transfer, spending), what part is due to changes in the economic environment and what part is due to changes in policy?

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$^3$ The present value of budget constraint as the sum of current and future discounted surplus is condition sufficient to balance the current value of debt. Chalk N. and Hemming R. (2000) find two particular cases to be mentioned. The first one is when government generates a primary surplus that overcome the interests bearing from public debt, and no-Ponzi game condition is satisfied, but with negative permanent deficit. The second one is when debt tends asymptotically to zero without a sustainable fiscal policy. This occurs when economic growth is greater than interest rate and strongly greater than debt growth. At the same time interest rate is lower than debt growth, such that we have debt ratio decreasing but no-Ponzi game condition satisfaction.
• **Sustainability** of fiscal policy: can the current course of fiscal policy be sustained without exploiting public debt?

• The aggregate *impact* of fiscal policy: at given income, interest and exchange rate and ignoring distortion, what is the effect of fiscal policy on aggregate demand?

• The *allocation* consequence of fiscal policy: what are the microeconomic distortions on investment, saving, labour supply and demand due to the tax/incentive structure?

CAB (cyclically-adjusted-budget balance) was the first indicator, used mainly in the 1980s, able to capture the effects of fiscal policy. It is a measure of discretionary budget changes implemented by fiscal authorities. Indeed, the purpose of the CAB is to distinguish changes in government’s overall actual budget balance, due to policy actions, from changes induced by fluctuation of economic activity. From the other hand, it does not capture the sustainability issue because it does not consider output fluctuations around a trend and it is not forward looking.

4.1 *Primary Gap* e *Tax Gap* Indicators

The most widely used indicators of sustainability are the Primary Gap and the Tax Gap. Primary Gap calculates the sustainable level of the fiscal variable at hand. Tax Gap defines the gap as the difference between the sustainable and the current level of the primary deficit, or the tax ratio. Both ensure that debt ratio converges to a finite value given the overall fiscal variables.

**DEFINITION 2**: Primary Gap is the change of the primary surplus needed to stabilize the debt GDP ratio to the current level, given the actual level of government expenditure and revenue.

\[
P_G = s_i - (r - \gamma)b_i
\]

Primary Gap is calculated using the current primary surplus and debt, thus we need to assume how is the long-run expected average values of the interest rate and the growth rate of real GDP. If the current primary balance is lower than the sustainable one
(s – s∗ < 0), the debt ratio will rise without any limits and fiscal policy can be called unsustainable. The sustainable primary balance (s∗) can be used directly as a target guiding the government towards a sustainable deficit path. It is an attractive indicator, since budgetary balance is usually the ultimate object of policy-makers’ interest. Moreover Primary Gap is a measure of the adjustment able to indicate the return of fiscal balance to its sustainable level. To remark that an actual Primary Gap equals to zero does not mean that fiscal policy is automatically sustainable, or without any need for future adjustments. In fact spending ratio may rise (e.g. as a result of ageing) and to have a sustainable level (s∗) of primary balance it requires an adjustment of revenues or expenditures. To take account of future spending pressures the assumption of a constant primary surplus has to be relaxed.

**Definition 3:** Tax Gap is the difference between actual government revenues and the level they should have in order to stabilize the debt GDP ratio, given the actual value of interest spending, interest rate and economic growth.

We obtain this measure considering the intertemporal budget dynamics

\[ b_0 = -\int_o^t (g_s + i_s - t_s) e^{-(r-\gamma)s} ds \]  

that satisfy the constraint for \( t \to \infty \) and solving for constant \( t_s = t \) we obtain the sustainable tax ratio

\[ t^* = (r-\gamma) \left[ \int_o^t (g_s + i_s) e^{-(r-\gamma)s} ds + b_0 \right] \]

such that solving for a \( t_s \), given \( t^* \) constant, we get the actual deviation of the tax rate respect to the sustainable one, a good indicator. Blanchard et al. (1990) imposes that tax rate should return the initial level

\[ t_s = (r-\gamma) \left( b_0 + (1 + e^{-(r-\gamma)n}) \right) ^{-1} \int_o^t (g_s + i_s) e^{(r-\gamma)s} ds \]

In order to understand the fiscal policy effects on economic
variable in the short, medium or long run will be sufficient to choice the time of the integer \((s = 3, 5 \text{ or } 40 \text{ years})\). In fact we know that, for a given choice of time, \(t_{ts}\) converges to \(t\) while for \(t \to 0\) we get a measure of sustainable tax

\[
(21) \quad t^* - t = g + i - t + (r + \gamma)b_0
\]

or

\[
(22) \quad \text{TaxGap}_1 = d_i + (r - \gamma)b_i
\]

where \(d_i\) is deficit. Semmler et al. (2005) propose another Tax Gap, easily quantifiable, to capture the average fiscal adjustment in the medium term

\[
(23) \quad \text{TaxGap}_2 = \left[(5 \text{ years of } g) + (r - \gamma)b_i \right] - t
\]

Tax Gap is positive when fiscal policy have to be adjusted to avoid excessive debt accumulation. The name of the indicator should not lead one to conclude that the right way to correct the current policy would be to increase taxes. The indicator only suggests that the current tax ratio is not high enough to finance future spending and to service the debt. To satisfy the intertemporal budget constraint it will be necessary to reduce spending and/or increase taxes, and the magnitude of the adjustment is given by the value of the indicator. Both Primary Gap \((17)\) and Tax Gap \((23)\) are derived from the condition of sustainability and thus are fully compatible with the definition of fiscal sustainability. Both indicators are calculated at the infinite time horizon. However, computation of the Tax Gap requires spending to be projected over an infinite time horizon and, in practice, requires assumptions about the evolution of spending beyond the horizon of the projection\(^4\).

\(^4\) As a result is convenient to limit the computation of the gap indicators to a finite horizon. On the other hand, we are deviating from the theoretical definition, as the construction of finite gap indicators necessitates determining the target value of debt at the end of the period. This target value of debt is unavoidably arbitrary, and the present discounted value of debt can be non-zero such that condition \((13)\) could not be satisfied.
4.2 The European Commission Indicators of Sustainability

To easily manage fiscal policy sustainability of all European Countries, European Commission defines two indicators of sustainability, $S_1$ and $S_2$, following the Blanchard et al. (1990) approach. Indicator $S_1$ indicates the change in the structural primary balance for every future year that is required to reach a debt ratio in 2050 of 60% of GDP. Indicator $S_2$ indicates the change in the structural primary balance for every future year that ensures that condition (13) is true. The indicators are

\begin{equation}
S_1 = rb_0 - s_0 + \frac{r(b_0 - b_T)}{(1 + r)^{t_0} - 1} - \sum_{t=t_0+1}^{T} \frac{s_t - s_0}{(1 + r)^{t-t_0}} - \sum_{t=t_0+1}^{T} \frac{1}{(1 + r)^{t-t_0}}
\end{equation}

\begin{equation}
S_2 = rb_0 - s_0 + r - \sum_{t=t_0+1}^{\infty} \frac{s_t - s_0}{(1 + r)^{t-t_0}}
\end{equation}

where $t_0$ is the last year before the long term projection. The difference between two indicators is the lag choice, such that often $S_1$ is higher the $S_2$ mainly because the first one does not consider the demography changes in the long run.

5. - Test of Sustainability

5.1 Hamilton-Flavin (1986) e Wilcox (1989)

In the economics literature numerous studies test whether (12) and (13) hold in real economies (firstly Seater and Mariano, 1985; Barro, 1984; Aschauer, 1985 and later Hamilton and Flavin, 1986; Kremers, 1988; Wilcox, 1989; Trehan and Walsh, 1991). The first important contribution was given by the work by Hamilton and Flavin (1986) that suggest to test the presence of a bubble term in series of public net debt. The analysis was based on two important assumptions: constant interest rate and the alternative
hypothesis of test. To avoid constant interest rate they used the average interest rate, as a proxy for constancy, while to face the second assumption they tested equation (6) rewritten in this way:

\[
B_t = -\left[ \sum_{i=0}^{\infty} \frac{G_i}{(1+r)^i} - \sum_{i=0}^{\infty} \frac{T_i}{(1+r)^i} \right] + \text{lim}_{t \to \infty} \left[ \frac{B_t}{(1+r)} \right] + \eta_t
\]

where \( \eta_t = \frac{\nu}{(1+r)^i} \) is a stationary zero mean process. Therefore, they define the following test

\[
H_0 : \text{lim}_{t \to \infty} \left[ \frac{B_t}{(1+r)^i} \right]
\]

(27)

\[
H_1 : \text{lim}_{t \to \infty} \left[ \frac{B_t}{(1+r)^i} \right] = A > 0
\]

where \( A \) is a constant. In this way, fiscal policy is non-sustainable when discounted debt tends to a positive and finite value. Later, Wilcox (1989) applied the same methodology and data of Hamilton and Flavin (1986) removing the constant interest rate. In their analysis, if no-Ponzi condition holds, sustainability takes place only if undiscounted series of expected debt is a stationary process with unconditional zero mean. It is important to remark that research of stationarity assumptions is a necessary condition for all time series analysis. Test of unit root on fiscal policy time series aims to detect the stationarity as a fundamental condition for the following analysis. If this holds the intertemporal government budget constraint is fulfilled because discounted debt converges to zero. The strong assumptions about future states of the world and the type of discount rate have given rise to criticism (see Bohn, 1995, 1998). Moreover, Kremers (1988) criticizes the use of ADF test without serial autocorrelation correction of errors and the instability of results with different lags. Balassone and Franco (2000) harshly criticize the small sample of data, the constant interest rate hypothesis and the alternative hypothesis, extremely subjective, to test sustainability.
5.2 Trehan-Walsh Test Analysis (1988, 1991)

In order to face the last doubts Trehan and Walsh demonstrate that when budget identity (1) is not valid, for a constant interest rate, a sustainability sufficient condition is the cointegration of primary balance and debt with a $(1, r)$ vector or the cointegration of total revenue, total expenditure and public debt with a $(1, -1, r)$ vector. Alternatively, they show that an unit root behaviour of debt, revenues, expenditures and a stationary process of primary balance is a sufficient condition for sustainability\textsuperscript{5}. In such a case, a test of the intertemporal budget constraint, or an unit root test to primary balance or a cointegration analysis of total revenue and total expenditure, is condition necessary to have sustainability. The estimates follows this equation:

\begin{equation}
T_t = \alpha + \beta (G_t - I_t) + \epsilon_t
\end{equation}

where $I_t$ is the interest spending on debt. Their result is a positive cointegration coefficient, hence a sustainable US fiscal policy during the period 1890-1986. Trehan and Walsh (1991) generalize and refine this methodology in two directions: they show that as long as the public debt series is close to an ARIMA process, it can be avoid to test the alternative hypothesis (27). Secondly, they demonstrate that it is a sufficient condition to test whether deficit is stationary, or to test whether the primary balance and the public debt series are cointegrated and the process $(1 - \lambda L)s_t$ is stationary, with $0 < 1 + r_t$. In such a way, they are testing the no-Ponzi condition. It is to be remarked that a rejection of the null hypothesis in favour of stationarity of lagged variables is not \textit{per se} a sustainability problem. Finally, Uctum and Wickens (2000) extended the latter analysis introducing a stochastic and time-variant interest rate without any change in the necessary and sufficient condition for sustainability (\textit{i.e.} discounted debt series must be stationary with zero mean). Artis and Marcellino (2000)

\textsuperscript{5} They also proposed a test when primary balance follows a non stationary process.
criticize this result because, theoretically, it does not imply debt convergence to zero at time infinite. It follows that both series (discounted and undiscounted) must be tested with or without trends.

5.3 The (1988) Bohn Analysis of Sustainability

To avoid the main assumptions of tests is convenient to analyze directly whether government debt ratio is sustainable, as suggested by Bohn (1995, 1998). If debt is sustainable, and the economy is dynamic efficient, the intertemporal budget constraint is fulfilled and a given fiscal policy is said to be sustainable. Let \( c_1 = B/Y \) be the constant debt ratio and plug it in equation (13) (29)

\[
\lim_{t \to \infty} c_t e^{(r-\gamma)t}
\]

The condition \( \gamma < r \) characterizes a dynamic efficient economy\(^6\) and is likely to hold in real economies. Therefore, in the following we limit our considerations to the case of dynamic efficient economies and assume that discount rate of government debt exceeds the GDP growth. The new test seems to be easier because it is a simple unit root test for the government debt time series. Instead, as argued by Semmler \textit{et al.} (2005) «testing for stationarity of the debt ratio is characterized by some shortcomings, so it is difficult to distinguish between a time series which is stationary about a positive intercept and one that shows a trend. This holds because standard unit root regressions have low power against autoregressive alternatives if the AR coefficient is close to one. As a consequence, the hypothesis that a given fiscal policy is sustainable has been rejected too easily». Bohn (1998) argues his new test-analysis considering the main critiques to test: discounting future government spending and revenues by constant interest rate is uncorrected in stochastic economy. In fact, the

\(^6\) Assuming that capital productivity is equal to interest rate, an economy where the productivity rate of the capital is higher (lower) than the economic growth is dynamically efficient (inefficient).
discount factor depends on the distributions of the variables discounted. He proposes to test whether primary balance is a positive linear function of debt ratio. If this holds, a given fiscal policy is called sustainable\(^7\). Assuming a deterministic economy such that primary surplus \(s_t = t_t - g_t\), we test debt ratio with this simple regression:

\[
s_t = \frac{T_t - G_t}{Y_t} = \alpha + \beta b_t
\]

where \(\alpha\) and \(\beta\) are positive or negative constant. The coefficient \(\alpha \geq 0\) is a systematic component that determine the primary balance reaction to variations of GDP. The coefficient \(\beta\) is the primary balance reaction to an increase in the debt ratio. Plug equation (30) in the differential equation (8) to have public debt dynamics:

\[
\frac{db_t}{dt} = b_t (r - \gamma - \beta) - \alpha
\]

and solve the differential equation to obtain:

\[
b_t = \left( \frac{\alpha}{r - \beta - \gamma} \right) + e^{r-\beta-\gamma} C_1
\]

where \(C_1 = b_0 - \alpha (r - \beta - \gamma)\) is a constant and \(b_0\) is the debt GDP ratio at time zero. Bohn (1998) demonstrates that a sufficient condition for debt boundness is \(r = \beta > \gamma\), or an economy dynamic efficient, such that the first term of (32) converges to zero and that the second term converges (diverges) for \(\beta > 0\) (\(\beta < 0\)). Now the conditions for debt ratio boundness are:

**PROPOSITION 1:** In an economy this conditions turn out to be true.

- For \(\beta > 0\) is a sufficient condition for the debt-GDP ratio to remain bounded if \(r < \gamma\).

\(^7\) The intuitive reasoning behind this argument is the follow: a government raise in the primary surplus as in the public debt is a corrective action which stabilizes the debt ratio and makes public debt sustainable.
• For $\beta > 0$ and $r > \gamma$ the debt-GDP ratio remains bounded if and only if $r - \gamma < \beta$.
• For $\beta < 0$ a necessary and sufficient condition for the debt-GDP ratio to remain bounded is $r - \beta < \gamma$.

This proposition demonstrates that:

**DEFINITION 4:** A necessary but not sufficient condition for debt ratio boundness is a linear increase in the primary surplus ratio as a result of an increase in the debt ratio (i.e., $\beta > 0$), in a contest of deterministic economy with a constant real interest rate and a constant growth rate of real GDP.

If economy is dynamically efficient, debt ratio remains bounded if $\beta$ exceeds the difference between the interest rate and the GDP growth. If the latter inequality does not hold debt ratio does not converge. Finally the response of the primary balance to a rise in the debt ratio must be sufficiently larger than $r - \gamma$ to have debt boundness. On the other hand, a negative $\beta$ may directly imply a bounded debt. In conclusion, a necessary and sufficient condition is that economy growth must be sufficiently large to exceed the interest rate plus the absolute value of $\beta$. This condition holds also for dynamic inefficient economies. Proposition 1 ensures debt ratio boundness but it is incomplete. The proper intertemporal budget constraint requires that discounted stream of government debt converges to zero. This imply that equation (32) must be in nominal terms

$$B_t e^{-rt} = \left(\frac{\alpha}{r - \beta - \gamma}\right)Y_0 e^{(r-\gamma)t} + e^{-\beta t}C_2$$

where $C_2 = B_0 - \left(\alpha r - \beta - \gamma\right)Y_0$. This expression must be fulfilled such that the intertemporal budget constraint is satisfied. Now it is possible to show which are the new conditions that bounded the discounted debt ratio:

**PROPOSITION 2:** For our model economy the following turns out to hold true.
• For $\alpha \geq 0$, the intertemporal budget constraint of the government holds if $\beta > 0$.
• For $\alpha \leq 0$, the intertemporal budget constraint of the government is fulfilled for $\beta > 0$, and $r > \gamma$. 


• For $\beta < 0$ the intertemporal budget constraint of the government is not fulfilled except for $B_0 = \left(\frac{\alpha}{r - \beta - \gamma}\right)Y_0$ and $r > \gamma$

Proposition 2 shows that discounted value of public debt converges to zero if the primary balance positively reacts to increases in the debt (i.e. if $\beta > 0$) without an autonomous decrease (i.e. for $\alpha \geq 0$). This implies that the level of the primary balance must not decline in case of an increase in GDP. If the reverse holds ($\alpha < 0$), only $\beta > 0$ guarantees the sustainability of intertemporal budget constraint when interest rate exceeds the economic growth.

**Definition 5:** As long as economies are dynamically efficient, $\beta > 0$ guarantees that discounted public debt converges to zero and, therefore, is a sufficient condition for sustainability of a given fiscal policy.

To remark that sustainability may be satisfied also in the case of not constant debt (i.e. for $0 < \beta < r - \gamma$). Finally remark that intertemporal budget constraint is not fulfilled if government reduces its primary balance as the debt ratio rises, except for the hairline case when $B_0 = \left(\frac{\alpha}{r - \beta - \gamma}\right)Y_0$.

These theoretical considerations demonstrate that, in a deterministic economy, an increase of primary balance as a consequence of a rise in the debt ratio guarantees the intertemporal budget constraint sustainability of dynamic efficient economies.

### 6. - A Combining Approach

According to Blanchard *et al.* (1990) sustainability of fiscal policy is the condition of no debt accumulation, expressed mathematically via equations (12) and (13). Indicators and tests aim to verify whether this conditions hold giving a positive or negative response to policy makers or macroeconomists.
Indicators easily report when the overall fiscal policy given by debt, primary balance, revenue and expenditure dynamics is direct to an over accumulation of debt. Test, on the other hand, are based on debt stability at $t_0$ and are direct to explore with stationarity and cointegration analysis whether debt dynamics is potentially explosive. In this sense it is possible to compare the two instruments: they are direct to verify the same object, the sustainability of fiscal policy. As it was argued by De Luzenberger and Marini (1993) the simultaneous use of tests and indicators may provides different responses on the sustainability of fiscal policy and an useful instrument to discern this results is given by the Chow-type test. This is because indicators are constructed to be forward looking and to react to current and future fiscal changes. Tests, instead, are backward looking although they always rely on historical data. For these reasons, indicators may respond faster than tests to changes in the fiscal policy and therefore may signal the possible occurrence of a systematic change in the fiscal policy regime. The two approaches could usefully be integrated. A possible strategy is shown in Graph 1.

The idea is that a structural break, happened in fiscal policy, might change the tests and indicators responses. Indicators react immediately to a systematic change in fiscal policy, because are forward looking, while tests are not influenced. A positive (negative) presence of a structural break in the primary balance gives reliability to the indicator (test) response in case of conflicting results. So when both tests and indicators do not show the existence of a solvency problem, no correction in the fiscal stance is required. A Chow-type test is direct to verify the presence of a structural break.

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8 The level of debt is not a priori a problem per se. In fact, higher debt makes only easier that fiscal policy path exceed the sustainability condition.

9 Chow test is commonly used for searching time series structural brakes as Semmler W. et al. (2005) used for fiscal sustainability but not as an instrument of choice between the indicator and test's response.
7. - The Italian Case

7.1 The Indicators

We examine the sustainability of Italian fiscal policy from 1970 to 2006, using OCSE and Bank of Italy database. Time series\(^{10}\) include GDP, general government total public debt, total revenue, total expenditure, primary balance, government interest spending, social security paid and received by government, long term interest rate on government bond, as a proxy for constant

\(^{10}\) As argued by Balassone F. and Franco D. (2000) and Artis M. and Marcellino M. (2000) the final result can very if you use different data. Public debt should include public administration indebtedness and public participation although they are difficult to calculate it.
interest rate: all data are taken at market price with annual frequency. Indicators are calculated following the equation (17), (23) and (25) computing the real growth as a difference between the log GDP and log deflator. They show two alternative trends during the years 1970-2006: a first period that we call pre-Maastricht (1970-1992) and a second one post-Maastricht (1992-2006). Primary Gap in Graph 2 show that in the first period Italy’s primary balance would have been increased by a nine-eighteen percent to stabilize the debt to GDP ratio, while in the second period the indicators was positive, caused by a policy change. In the last five years it appears a reverse change. In the overall period it shows a no sustainability path in the overall period. Tax Gap in Graph 3\textsuperscript{11} shows a similar result and confirms our ideas of a supposable structural break caused by the goal to be part of the European Monetary Union. The graph shows a negative result in the first period and positive line in the second one. This means

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{primary_gap_1970-2006.png}
\caption{PRIMARY GAP 1970-2006}
\end{figure}

\textsuperscript{11} We graph the Tax gap with five years average of revenue and expenditure, as equation (23).
Graph 3

TAX GAP ANNUAL 1970-2006

Graph 4

S2 1970-2006
Indicators and Tests of Sustainability: The Italian Case

Graph 5
DEBT AND PRIMARY BALANCE, % GDP, 1970-2006

Graph 6
REVENUE-EXPENDITURE-NET INTEREST EXPENDITURE (% GDP)
that in the pre-Maastricht period tax would have been increased by a two-five percent to stabilize the debt to GDP ratio. A necessary increase appears in the last five years. According to this indicator, fiscal policy in overall period was non-sustainable. The $S_2$ indicator of European Commission is shown in Graph 4. Despite of the sharply changes in the first years, the indicator indicates a necessary a six percent change on average for every future year of the primary balance to ensure the solvency condition. It decreases sharply in 1992 but it confirms what the others two indicators have already shown: a global no sustainable path of fiscal policy. In conclusion, indicators confirm that fiscal policy was non-sustainable in spite of a consistently reaction in year 1992, due to a systematic change in fiscal policy caused by the need to adopt the Treaty of Maastricht.

7.2 Test of Sustainability and Bohn (1998) Analysis

Tests are controversial but show an overall different response with respect to indicators. Remark that a test on public debt stationarity provides a sufficient conditions for public finance sustainability and for this reasons we compute unit root test. We use ADF and Philipp Perron$^{12}$ test with serial correlation using Newey-West standard errors and KPSS$^{13}$ test. A constant is included in the regression and the lag length is chosen based on the SIC information criteria. The detailed results for all variables are presented in Tables 1-4. All test reject the stationarity at five percent level. In conclusion, Trehan Walsh (1991) tests evidence that debt and deficit involves a unit root and since primary balance is found not stationary, neither in real terms nor in GDP terms, this concludes that Italian fiscal policies is not ad hoc sustainable in the considered sample period. To remark that a

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$^{12}$ ADF – Augmented Dickey Fuller – e PP – Phillips Perron – test the null hypothesis of unit root against stationarity.

$^{13}$ KPSS – Kwiatkowski Phillips Shmidt Shin – test the null hypothesis of stationarity against unit root. ADF includes autoregressive errors without taking care of heteroskedasticity, PP is robust to heteroskedasticity but it ignores the errors autocorrelation. KPSS is correct for autocorrelation.
### Table 1

**ADF, PP, KPSS Test**

#### ADF Test 1970-2006

<table>
<thead>
<tr>
<th>Variable</th>
<th>No Intercept: No trend</th>
<th>Intercept: No trend</th>
<th>Intercept: Trend</th>
<th>Unit Root/Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.6</td>
<td>0.52</td>
<td>-3.47**</td>
<td>Unit root</td>
</tr>
<tr>
<td>Public Debt</td>
<td>0.32</td>
<td>-0.38</td>
<td>-2.84</td>
<td>Unit root</td>
</tr>
<tr>
<td>State Balance</td>
<td>-0.04</td>
<td>-1.34</td>
<td>-1.31</td>
<td>Unit root</td>
</tr>
<tr>
<td>Primary Balance</td>
<td>-1.19</td>
<td>-1.2</td>
<td>-1.18</td>
<td>Unit root</td>
</tr>
<tr>
<td>Interest Spending</td>
<td>-0.18</td>
<td>-1.35</td>
<td>-1.12</td>
<td>Unit root</td>
</tr>
<tr>
<td>Critical Values (5%)</td>
<td>-1.94</td>
<td>-2.86</td>
<td>-3.41</td>
<td></td>
</tr>
<tr>
<td>Critical Values (1%)</td>
<td>-2.6</td>
<td>-3.43</td>
<td>-3.96</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** *, ** are ADF significance at 5%, 1%. Optimal lag is one given by SIC.

#### PHILLIPS-PERRON Test 1970-2006

<table>
<thead>
<tr>
<th>Variable</th>
<th>Z(rho)</th>
<th>Z(tau)</th>
<th>Unit Root/Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>-5.48</td>
<td>-4.88</td>
<td>Unit root/Stationary</td>
</tr>
<tr>
<td>Public Debt</td>
<td>-4.24</td>
<td>-2.48</td>
<td>Unit root</td>
</tr>
<tr>
<td>State Balance</td>
<td>-3.69</td>
<td>-1.34</td>
<td>Unit root</td>
</tr>
<tr>
<td>Primary Balance</td>
<td>-3.65</td>
<td>-1.17</td>
<td>Unit root</td>
</tr>
<tr>
<td>Interest Spending</td>
<td>-1.27</td>
<td>-0.51</td>
<td>Unit root</td>
</tr>
<tr>
<td>Critical Values (5%)</td>
<td>18.73</td>
<td>-3.55</td>
<td></td>
</tr>
<tr>
<td>Critical Values (1%)</td>
<td>-23.90</td>
<td>-4.27</td>
<td></td>
</tr>
</tbody>
</table>

**KPSS Test 1970-2006**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Level</th>
<th>Trend</th>
<th>Lag</th>
<th>Unit Root/Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.91</td>
<td>0.37</td>
<td>i</td>
<td>Unit root</td>
</tr>
<tr>
<td>Public Debt</td>
<td>1.89</td>
<td>0.32</td>
<td>i</td>
<td>Unit root</td>
</tr>
<tr>
<td>State Balance</td>
<td>0.97</td>
<td>0.32</td>
<td>i</td>
<td>Unit root</td>
</tr>
<tr>
<td>Primary Balance</td>
<td>0.86</td>
<td>0.21</td>
<td>i</td>
<td>Unit root</td>
</tr>
<tr>
<td>Interest Spending</td>
<td>1.51</td>
<td>0.28</td>
<td>i</td>
<td>Unit root</td>
</tr>
<tr>
<td>Critical Values (5%)</td>
<td>0.46</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical Values (1%)</td>
<td>0.73</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** *, ** are KPSS significant 5%, 1%. Optimal lag is one given by SIC.
### Table 2

**ADF, PP, KPSS Test for Variable in Real Terms**

#### ADF Test 1970-2006

<table>
<thead>
<tr>
<th>Variable</th>
<th>No Intercept; No trend</th>
<th>Intercept; No trend</th>
<th>Intercept; Trend</th>
<th>Unit Root/Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Debt</td>
<td>0.76</td>
<td>-1.42</td>
<td>-1.28</td>
<td>Unit root</td>
</tr>
<tr>
<td>State Balance</td>
<td>-0.41</td>
<td>-1.37</td>
<td>-1.79</td>
<td>Unit root</td>
</tr>
<tr>
<td>Primary Balance</td>
<td>-1.16</td>
<td>-1.01</td>
<td>-1.56</td>
<td>Unit root</td>
</tr>
<tr>
<td>Interest Spending</td>
<td>-0.49</td>
<td>-1.78</td>
<td>-0.91</td>
<td>Unit root</td>
</tr>
</tbody>
</table>

Critical Values (5%)  
-1.94  -2.86  -3.41  
Critical Values (1%)  
-2.56  -3.43  -3.96

*Note: *, ** are ADF significance at 5%, 1%. Optimal lag is one given by SIC.*

#### PHILLIPS-PERRON Test 1970-2006

<table>
<thead>
<tr>
<th>Variable</th>
<th>Z(rho)</th>
<th>Z(tau)</th>
<th>Unit Root/Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Debt</td>
<td>-1.1</td>
<td>-0.45</td>
<td>Unit root</td>
</tr>
<tr>
<td>State Balance</td>
<td>-4.17</td>
<td>-1.77</td>
<td>Unit root</td>
</tr>
<tr>
<td>Primary Balance</td>
<td>-8.49</td>
<td>-2.11</td>
<td>Unit root</td>
</tr>
<tr>
<td>Interest Spending</td>
<td>-0.52</td>
<td>-0.28</td>
<td>Unit root</td>
</tr>
</tbody>
</table>

Critical Values (5%)  
-18.73  -3.55  
Critical Values (1%)  
-23.90  -4.27

#### KPSS Test 1970-2006

<table>
<thead>
<tr>
<th>Variable</th>
<th>Level</th>
<th>Trend</th>
<th>Lag</th>
<th>Unit Root/Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Debt</td>
<td>1.76</td>
<td>0.3</td>
<td>1</td>
<td>Unit root</td>
</tr>
<tr>
<td>State Balance</td>
<td>0.47*</td>
<td>0.41</td>
<td>1</td>
<td>Unit root</td>
</tr>
<tr>
<td>Primary Balance</td>
<td>1.49</td>
<td>0.18*</td>
<td>1</td>
<td>Unit root</td>
</tr>
<tr>
<td>Interest Spending</td>
<td>1.89</td>
<td>0.42</td>
<td>1</td>
<td>Unit root</td>
</tr>
</tbody>
</table>

Critical Values (5%)  
0.46  0.14  
Critical Values (1%)  
0.73  0.21

*Note: *, ** are KPSS significant 5%, 1%. Optimal lag is one given by SIC.*
public debt stationarity is not a necessary condition. Therefore, accepting a unit root process of public debt can be interpreted as a situation in which a given fiscal policy is non-sustainable. This response is similar to that of indicators but suffer of the main critiques argued in section 5. A different response is obtained using the empirical analysis of Bohn (1998): we assume stationarity and ergodicity in the regression:

$$ s_t = \beta b_t + \theta Z_t + \epsilon_t $$

$$ = \beta b_t + \theta_1 I_t + \theta_2 Y_t + \theta_3 s_{soc}^t + \epsilon_t $$

where $\epsilon_t \sim N(0, \sigma^2)$ and $Z_{t soc}$ is a vector considering variables related to primary balance, such as interest payment, GDP and $S_{soc}^t$, called to measure the difference between social security paid and received by government. The variables are in GDP terms. To avoid endogeneity problem we use lagged value for debt although in theory an higher level of debt (as the Italy case) has a immediate impact on primary balance. Finally, the variable social surplus is subtracted from the primary balance and it is considered as an exogenous variable to catch possible effects of transfers between the social insurance system and the government. Equations (34)-(35) become

$$ s_{soc}^t = \alpha_0 + \beta b_{t-1} + \theta Z_t + \epsilon_t $$

$$ = \alpha_0 + \beta b_{t-1} + \theta_1 I_t + \theta_2 Y_t + \epsilon_t $$

Estimating equations (34)-(35) with OLS may suffer of biased standard errors and lower t-Statistics because of possible heteroskedasticity and autocorrelation in the residuals. To solve this problem we use OLS estimation but calculate heteroskedasticity and autocorrelation consistent t-Statistics to get robust estimates. Regression results of equation (35) and (37) are in Table 5 and Table 6. Both show a negative systematic part $\alpha < 0$ and a positive coefficient $\beta$. According to Bohn methodology, fiscal policy was sustainable in the period 1970-2006.
7.3 *The Cointegration Analysis*

The presence of unit root processes induce to compute the cointegration analysis of sustainability of fiscal policy. Following Hakkio and Rush (1991) a sufficient condition for sustainability is the cointegration of revenue and expenditure; alternatively, following Bohn (2008), a Johansen cointegration of primary balance and public debt. The cointegration analysis of fiscal policy, developed after the Nelson and Plosser (1982) work, search for a

| TABLE 3 | ADF TEST FOR VARIABLE AT DIFFERENCE |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| **Variable**    | **No Intercept; No trend** | **Intercept; No trend** | **Intercept; Trend** | **Unit Root/ Stationary** |
| GDP             | -0.48           | -2.73           | -2.72           | Unit root       |
| Public Debt     | -0.59           | -1.81           | -1.67           | Unit root       |
| State Balance   | -5.67           | -5.7            | -5.64           | Stationary      |
| Primary Balance | -5.54           | -5.45           | -5.4            | Stationary      |
| Interest Spending | -3.33          | -3.42**         | -3.52**         | Stationary      |
| Critical Values (5%) | -1.94         | -2.86           | -3.41           |                |
| Critical Values (1%) | -2.56        | -3.43           | -3.96           |                |

*Note: *, ** are ADF significance at 5%, 1%. Optimal lag is zero given by SIC.*

| TABLE 4 | ADF TEST FOR VARIABLES AT DIFFERENCE IN REAL TERMS |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| **Variable**    | **No Intercept; No trend** | **Intercept; No trend** | **Intercept; Trend** | **Unit Root/ Stationary** |
| Public Debt     | -2.68           | -2.98**         | -3.14*          | Stationary      |
| State Balance   | -6.18           | -6.09           | -6.43           | Stationary      |
| Primary Balance | -7.09           | -7.03           | -6.93           | Stationary      |
| Interest Spending | -3.2          | -3.18**         | -3.72**         | Stationary      |
| Critical Values (5%) | -1.94        | -2.86           | -3.41           |                |
| Critical Values (1%) | -2.56        | -3.43           | -3.96           |                |

*Note: *, ** are ADF significance at 5%, 1%. Optimal lag is zero given by SIC.*
linear stochastic trend in the time series. If we do not consider this trend, the OLS estimation may be biased, inefficient and inconsistent. A method to solve this problem is to difference the variable by one lag, missing one information. Hakkio and Rush (1991) demonstrated following Trehan and Walsh (1988) that intertemporal budget constraint is satisfied whether exist a linear relationship between total revenue and total expenditure of the government budget. Bohn (2008) provides the sufficient condition for the \textit{ad hoc} sustainability if primary balance and debt are cointegrated with a vector \((1, r)\). We estimate with Johansen (1988) cointegration procedure both analysis: the estimation consider all the possible vectors of cointegration and verify whether they are

TABLE 5

BOHN REGRESSION (1)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Prob &gt; F</th>
<th>R-squared</th>
<th>Adj R-Squared</th>
<th>Root MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>446.77</td>
<td>4</td>
<td>111.69</td>
<td>0.00</td>
<td>0.856</td>
<td>0.837</td>
<td>1.557</td>
</tr>
<tr>
<td>Residual</td>
<td>75.17</td>
<td>31</td>
<td>2.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>521.94</td>
<td>35</td>
<td>14.91</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Regression Primary Balance - Debt

| Primary Balance | Coeff. | Std. Err | t   | p>|t| | [95% Conf. Control] |
|-----------------|--------|----------|-----|-------|---------------------|
| Constant        | -15.174| 2.161    | -7.020| 0.000 | -19.581 to -10.767  |
| Debt            | 0.181  | 0.050    | 3.630| 0.001 | 0.079 to 0.283      |
| Interest spending | -0.250 | 0.186    | -1.340| 0.189 | -0.631 to 0.130     |
| Social spending | 58.339 | 44.404   | 1.310| 0.199 | -32.224 to 148.90   |
| GDP             | 0.001  | 0.003    | 0.290| 0.771 | -0.005 to 0.007     |

Regression Primary Balance - Debt (robust)

| Primary Balance | Coeff. | Std. Err | t   | p>|t| | [95% Conf. Control] |
|-----------------|--------|----------|-----|-------|---------------------|
| Constant        | -15.174| 2.021    | -7.51| 0.000 | -19.295 to -11.052  |
| Debt            | 0.181  | 0.050    | 3.65 | 0.001 | 0.079 to 0.282      |
| Interest spending | -0.250 | 0.179    | -1.40| 0.172 | -0.615 to -0.114    |
| Social spending | 58.339 | 50.717   | 1.15 | 0.259 | -45.099 to 161.777  |
| GDP             | 0.001  | 0.003    | 0.25 | 0.804 | -0.005 to 0.007     |
significant\textsuperscript{14}. We firstly analyze whether exists a long-term relationship between real revenues and real expenditures in Italy. In other words we aims to verify whether revenues and expenditures share a common trend. Once we have controlled that all the variables are I(0) in first differences, so they are I(1) in levels, see Tables 1-4 and Tables 7-8, we show the empirical cointegration tests for total revenues and total expenditure in GDP term. Results in Table 9 rejects a zero vector of cointegration (57.69 (0.00)) and accept on vector of cointegration (11.14 (0.019)),

\textsuperscript{14}To remark that, as argued by QUINTOS C.E. (2005), a cointegration coefficient close to one make the series not stationary in levels integrated of order one. This is a necessary and sufficient condition for sustainability. In the case of $0 \leq \beta \leq 1$, the condition is only sufficient. She concludes that US fiscal policy was sustainable until the early 1980s and then there was a structural break (QUINTOS C.E., 1995).
implying no long-term relationship between revenues and expenditures. The conclusion, therefore, is that Italian fiscal policy was sustainable. We applied the same cointegration analysis for primary balance and public debt in GDP terms. Results in Table 10 rejects the zero cointegration vector (42.81 (0.00)) and accept one vector (9.58 (0.00)). In conclusion the cointegration analysis consider fiscal policy sustainable in the period 1970-2006.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Intercept;</td>
<td>Z(rho)</td>
<td>Level</td>
</tr>
<tr>
<td>Total Revenue</td>
<td>1.48</td>
<td>-5.29</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>No trend</td>
<td>-3.22</td>
<td>0.23</td>
</tr>
<tr>
<td>Total Expenditure</td>
<td>1.59</td>
<td>-6.19</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>-2.71</td>
<td>0.36</td>
</tr>
<tr>
<td>Critical Values (5%)</td>
<td>-1.94</td>
<td>-18.73</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>-2.86</td>
<td></td>
<td>0.14</td>
</tr>
<tr>
<td>Critical Values (1%)</td>
<td>-2.56</td>
<td>-23.90</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>-3.43</td>
<td></td>
<td>0.21</td>
</tr>
</tbody>
</table>

Note: *, ** are ADF significance at 5%, 1%. Optimal lag is zero given by SIC.

Note: *, ** are KPSS significance at 5%, 1%. Optimal lag is zero given by SIC.
### Table 8

**ADF TEST FOR VARIABLES AT DIFFERENCE ONE**

<table>
<thead>
<tr>
<th>Variable</th>
<th>No Intercept; No trend</th>
<th>Intercept; No trend</th>
<th>Intercept; Trend</th>
<th>Unit Root/Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Revenue</td>
<td>-0.99</td>
<td>-3.01</td>
<td>3.82*</td>
<td>Stationary</td>
</tr>
<tr>
<td>Total Expenditure</td>
<td>-0.99</td>
<td>-3.24*</td>
<td>-3.82*</td>
<td>Stationary</td>
</tr>
<tr>
<td>Critical Values (5%)</td>
<td>-1.94</td>
<td>-2.86</td>
<td>-3.41</td>
<td></td>
</tr>
<tr>
<td>Critical Values (1%)</td>
<td>-2.56</td>
<td>-3.43</td>
<td>-3.96</td>
<td></td>
</tr>
</tbody>
</table>

*Note: *, ** are ADF significance at 5% – 1%. Optimal lag is zero given by SIC.*

### Table 9

**THE COINTEGRATION REVENUE - EXPENDITURE**

<table>
<thead>
<tr>
<th>Trend</th>
<th>N. of obs.</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>= 36</td>
<td>10</td>
</tr>
</tbody>
</table>

**Maximum rank**

<table>
<thead>
<tr>
<th>LR</th>
<th>p-value</th>
<th>value 10%</th>
<th>value 5%</th>
<th>value 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>57.69</td>
<td>17.98</td>
<td>20.16</td>
<td>24.69</td>
</tr>
<tr>
<td>1</td>
<td>11.14</td>
<td>7.60</td>
<td>9.14</td>
<td>12.53</td>
</tr>
</tbody>
</table>

Optimal SIC is 10 lags.

### Table 10

**THE COINTEGRATION PRIMARY BALANCE AND DEBT**

<table>
<thead>
<tr>
<th>Trend</th>
<th>N. of obs.</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>= 36</td>
<td>6</td>
</tr>
</tbody>
</table>

**Maximum rank**

<table>
<thead>
<tr>
<th>LR</th>
<th>p-value</th>
<th>value 10%</th>
<th>value 5%</th>
<th>value 1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>42.8</td>
<td>17.98</td>
<td>20.16</td>
<td>24.69</td>
</tr>
<tr>
<td>1</td>
<td>9.58</td>
<td>7.60</td>
<td>9.14</td>
<td>12.53</td>
</tr>
</tbody>
</table>

Optimal SIC is 6 lags.
7.4 The Chow Test

The Chow test is applied to primary balance, with constant and a break, to detect the presence of a structural break in 1992. Results in Table 11 confirm the presence of a structural break in 1992 giving reasonable support to the systematic change in fiscal policy needed to adopt the Treaty Maastricht by the Italian government. The Graph 5 helps to detect what part of fiscal policy has contributed to the break: the main contribution is given by a lower interest rate, hence lower interest spending, and by lower expenditure with constant revenues. These changes take back fiscal policy on a sustainability path. This result is in favor of the indicators response in spite of that of test.

<table>
<thead>
<tr>
<th>Year 1992</th>
<th>Debt (p-value)</th>
<th>Chow test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-6.47</td>
<td>70.42</td>
</tr>
<tr>
<td></td>
<td>(.000)</td>
<td>(.000)</td>
</tr>
<tr>
<td>Intercept and Trend</td>
<td>2.12 (.359)</td>
<td>35.69</td>
</tr>
<tr>
<td></td>
<td>-.505(.000)</td>
<td>(.000)</td>
</tr>
</tbody>
</table>

Note: For the Chow Test is reported the F-version.

8. - Conclusions

The analysis of sustainability of fiscal policy in the period 1970-2006 is computed using both the indicators and tests techniques. The indicators Primary Gap, Tax Gap and $S_2$, as forward looking instruments, show a non-sustainable path for Italian fiscal policy. At the other side, despite of the Trehan and Walsh (1991) methodology, tests following Bohn (1998) and the cointegration analysis confirm the sustainability response. In order to disentangle the two conflicting results we use the strategy proposed by De Luzenberger and Marini (1992) applying the
Chow-type test to primary balance to capture the existence of a structural break. The positive response of Chow test devolve in favour of indicators results because the presence of systematic change in fiscal policy, occurred in 1992-1993 to adhere to the Maastricht Treaty, was captured only by indicators. This conclusion is mainly supported by the fact that structural break seriously undermine the response of test, not their overall validity. On the other hand, in case of no systematic break, tests response is more reliable because indicators were likely to reflect cyclical factors. These reasons motivate our final response in favour of not sustainability of Italian fiscal policy during the period 1970-2006.


