Household Portfolios Efficiency in the Presence of Restrictions on Investment Opportunities

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This paper proposes a new test for verifying the mean-variance efficiency of household portfolios. Unlike the standard statistics, the test takes account of two additional aspects: 1) wealth consists of real estate, held in fixed proportions in the short term, as well as financial assets, and 2) it is not possible to assume short positions in several financial assets. Performing the test on Italian households' portfolios as they appear in the SHIW 2000 survey, and treating housing as a fixed asset, we obtain an efficiency much more widespread than with common tests, revealing how inaccurate the standard theory is [JEL Code: C12, D14, G11].

1. Introduction

The scientific community has been showing a remarkable interest in investment efficiency since Markowitz (1952) began to develop portfolio choice theory. Beyond technical aspects, the topic is of undoubted practical usefulness: having a proper test statistic, we can indeed establish whether a portfolio has been built in a suitable way. That test could be particularly useful for who-

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ever by profession must decide how to invest the capital which he has been entrusted: it would be an essential instrument for everyday work of several professional figures such as financial promoters, firm managers or mutual funds managers.

It is obvious that, in order to run an analysis confidently, it is necessary that the test starts from a theoretical model which takes account of all the main characteristics of the reality, even though it can be only a *simplification*, of course. Until now this aspect has been causing the major criticism of the large existing literature: although during the years many important results have been attained in elaborating statistics more and more reliable, the models are not very useful out of research environment. When these tests were developed, indeed, a not negligible purpose was to keep under control the complexity of the underlying algebra: it was consequently assumed that the investor allocates his wealth into a set of financial assets and is subject to no restriction.

These assumptions cause a rough approximation of the reality, enough to bring the validity of the results in question. Running the classical test proposed by Jobson and Korkie (1982), it is usually verified that no portfolio, apart from the risk free one, is compatible with the hypothesis of efficiency. A thorough examination questions whether this inefficiency is due to wrong decisions taken by the investors, or instead to the assumptions of the model; we could reply to the query performing a test which takes account of some restrictions on portfolio composition. Before deriving an appropriate test, it is nevertheless indispensable to specify what chief aspects in portfolio composition are neglected by the traditional analysis; in this paper we are going to focus on the household as an economical agent, but the analysis could also be developed in other situations with just a few changes.

First of all, although the standard models take merely financial wealth into consideration, we know that households choose to allocate their own wealth dividing it not only into financial assets, but also into real ones, such as family concerns, cars, jewels and especially houses. Until now real estate has been neglected by the literature essentially for two reasons: 1) it is not often possible to obtain reliable return time series — for the most important real assets there is not a market as organized as the financial one — and 2) it would not make much sense to treat real assets as financial ones. In order to clarify the second reason, we pay attention to the real asset in which household investment is larger: housing. This asset plays the twofold role of investment and consumption good; in addition it cannot be negotiated as it is liquid, because of not negligible transaction costs[1] which lie heavy on it during the sale and the purchase. It means that, in reply to changes either in household wealth or in market conditions, there will be an immediate variation in financial investment but not in housing. A change in the real asset investment will occur solely when bearing the transaction costs will be advantageous, namely when either there will be a marked difference in household wealth or the market will offer very changed returns. In a one-period model as the one elaborated by Markowitz (1952) and on which we rely, it is correct to treat the real asset investment as fixed: indeed, the analysis concerns exclusively the short term, where it is more likely that transaction costs are significant.

Secondly, the most evident failing in the standard models is that they often conclude the optimal portfolio should hold negative positions in several financial assets (short selling). Nonetheless, because of the nature of some assets (for instance stocks), such a kind of investment is not feasible: short selling is practicable with difficulty for small investors.

The aim of this paper is consequently to develop a statistical test that, in order to evaluate the efficiency of a given portfolio, takes account of the two aspects above-mentioned: *(i)* the presence of durable goods in the set of primitive assets, treating their investment as fixed in the short term, and *(ii)* the impossibility to short-sell some financial assets. Until now the question has been neglected in the literature because of the complexity in acquiring the test, besides the reasons previously mentioned; nevertheless, we show that it is possible to derive a sta-

[1] Although there are also transaction costs related to financial assets, these have a size so small that they can be considered insignificant.

tistic through a modification of the model originally introduced by Basak *et* Al. (2002).

The paper is structured as follows: Section 2 reports a summary of portfolio choice theory in its classical version. Section 3 introduces two variations: the first one takes the durable assets into consideration and the second one, contribution of this essay, also considers the impossibility to make short selling. For each model we emphasize its peculiarities and introduce an appropriate test, stressing its features and shortcomings. In Section 4 the evidence from Italian data (return series and portfolios) is described; we also verify the observance of a theoretical condition required by Flavin and Yamashita (2002) and perform an analysis on the attitude of households towards diversification. Section 5 discusses the results of the empirical analysis and Section 6 concludes, highlighting the topics on which the research should be focused in future.

2. - Savings Allocation in the Standard Analysis

In the standard model originally proposed by Markowitz (1952), the market at time t offers one risk free asset, which assures a certain return r_0, and n risky assets, for which we adopt the following notation, hypothesizing stationarity:

$$\underline{r}_t = \begin{bmatrix} r_{1t} \\ r_{2t} \\ \vdots \\ r_{nt} \end{bmatrix} \sim \left(\underline{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}, \mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & & \sigma_{2n} \\ \vdots & & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \right)$$

where we represent the expected return on the j-eth asset as:

$$\mu_j = E\,[r_{jt}], \qquad j = 1,\ldots,n$$

and the covariance between the returns on the i-eth and j-eth assets as:

$$\sigma_{ij} = E\left[\left(r_{it} - \mu_i\right)\left(r_{jt} - \mu_j\right)\right] \qquad \text{i}, j = 1,\ldots,n$$

Beginning from the time series $\{\underline{r}_t, t = 1,\ldots,T\}$, μ_j and σ_{ij} can be estimated consistently by the corresponding sample moments:

$$\underline{\hat{\mu}}_j = \overline{r}_j = \frac{1}{T}\sum_{t=1}^{T} r_{jt}$$

$$\hat{\sigma}_{ij} = s_{ij} = \frac{1}{T}\sum_{t=1}^{T}\left(r_{it} - \overline{r}_i\right)\left(r_{jt} - \overline{r}_j\right) = \frac{1}{T}\sum_{t=1}^{T} r_{it} r_{jt} - \overline{r}_i \overline{r}_j$$

from which we obtain the vector $\underline{\overline{r}}$ and the matrix $\boldsymbol{S}$, consistent estimators of $\underline{\mu}$ and $\boldsymbol{\Sigma}$.

At time t the economic agent can invest his wealth splitting it into different assets, connecting each of them with a weight w_j, $j = 1,\ldots,n$ (w_0 for the risk free asset). Therefore, his portfolio has the return:

$$r_{pt} = w_0 r_0 + \sum_{j=1}^{n} w_j r_{jt} = r_0 + \underline{w}'\left(\underline{r}_t - r_0 \underline{i}\right) = r_0 + \underline{w}' e_t$$

with: $$\sum_{j=1}^{n} w_j = 1 - w_0$$

and its sample moments are given by:

$$\hat{\mu}_p = w_0 r_0 + \sum_{j=1}^{n} w_j \overline{r}_j = r_0 + \underline{w}'\left(\underline{\overline{r}} - r_0 \underline{i}\right) = r_0 + \underline{w}'\underline{\overline{e}}$$

$$\hat{\sigma}_p^2 = \sum_{j=1}^{n}\sum_{k=1}^{n} w_j w_k s_{jk} = \sum_{j=1}^{n} w_j^2 s_{jj} + 2\sum_{j=1}^{n-1}\sum_{k=j+1}^{n} w_j w_k s_{jk} = \underline{w}'\boldsymbol{S}\underline{w}$$

The model expects that people behave rationally and accordingly assumes that the agent, with the intention of allocating his wealth into the different available assets (so determining $\underline{w}$), wants to obtain his maximum satisfaction at time $t + 1$. At that time his wealth will have changed, according to household's income, consumption and the return on the portfolio chosen at time t.

Nonetheless, since the asset returns at time $t + 1$ are uncertain at the previous instant, the individual must take account of two basic characteristics of his portfolio: expected return and risk (that is, in statistics words, mean and standard deviation of the returns). Thus, we assume that he is interested in maximizing a Von Neumann-Morgenstern expected utility function,

$$EU = \hat{\mu}_p - \gamma\hat{\sigma}_p^2 \qquad (1)$$

where γ is a constant which expresses the absolute risk aversion[2]. Besides simplifying the analysis, this function would represent a suitable approximation of the reality even though it was not correct: whatever the real shape of the utility is, it is possible to achieve the expression *(1)* using a second order Taylor series expansion.

Maximizing his utility, the agent has to make his choice within a set of feasible portfolios, namely which are endowed with moments really achievable through an opportune combination of the primitive assets. The set of attainable portfolios is bound by the so-called efficient frontier, which defines the optimal portfolios, specifically those portfolios that assure the best connection between risk and return.

In the typical situation in which no restrictions are imposed on the vector $\underline{w}$, the efficient frontier takes the simple shape of a straight line,

$$\mu_{p*} = r_0 + \sigma_{p*}\sqrt{\underline{\bar{e}}'\mathbf{S}^{-1}\underline{\bar{e}}}$$

called capital market line; its slope, the well-known Sharpe

[2] Given the definition of utility, it is common to indicate as "mean-variance criterion" the method adopted by economic agents to make their choice.

potential performance, is given by the element inside the square root.

On the basis of this model several tests can be created in order to verify the efficiency of a given portfolio. The statistic historically most utilized, because of its simplicity, has been initially proposed by Jobson and Korkie (1982) and later presented again by Gibbons *et* Al. (1989) in a different light: it basically refers to the Sharpe performances. In order to use it, we must calculate the Sharpe ratio, defined for a generic portfolio as:

$$\hat{S}_p = \frac{\hat{\mu}_p - r_0}{\hat{\sigma}_p} = \frac{\underline{w}'\underline{\bar{e}}}{\sqrt{\underline{w}'\boldsymbol{S}\underline{w}}}$$

The higher the *ratio* is, the more efficient the portfolio appears, because it assures a high return associated with a low risk; the maximum value $\hat{S}_{p*}$ is achieved by the portfolios placed upon the efficient frontier, and coincides with the potential performance.

In order to verify whether a portfolio is efficient, it seems to be reasonable to assess the distance, in a probabilistic way, between the Sharpe ratio of the observed portfolio and the potential performance. If the null hypothesis is $H_0 : S_p = S_{p*}$, the statistic is:

$$JK = T\frac{\hat{S}^2_{p*} - \hat{S}^2_p}{1 + \hat{S}^2_p} = T\frac{\left(\underline{\bar{e}}'\boldsymbol{S}^{-1}\underline{\bar{e}} - \frac{(\underline{w}'\underline{\bar{e}})^2}{\underline{w}'\boldsymbol{S}\underline{w}}\right)}{1 + \frac{(\underline{w}'\underline{\bar{e}})^2}{\underline{w}'\boldsymbol{S}\underline{w}}} \xrightarrow{d} \chi^2_{n-1} \quad (2)$$

3. - Savings Allocation in the Presence of Constraints

The usual analysis assesses the quality of a portfolio comparing its performance with the potential one relative to any portfolio located on the efficient frontier; this line contains a set of portfolios merely based on financial assets which are considered as not restricted.

Nevertheless, the efficient frontier that is estimated this way is not the touchstone for observed portfolios, because no doubt it is not accessible by investors in reality. Indeed, households can also invest in real assets and must face several constraints: because of high transaction costs, in the short term real assets behave as durable goods, so that they stay fixed in the problem of portfolio choice. All the same, real estate affects, through covariances, the decision about investments in financial assets; furthermore, the option of short selling is denied for some other assets. In consequence of both these restrictions, the portfolios which form the real efficient frontier provide performances at most as high as the ones estimated by the model.

For this reason, the investor could still achieve the same portfolio as the one assigned by the standard model, in case the constraints are not binding, but he could also acquire a portfolio less efficient, if at least one of the constraints is binding instead.

In the latter situation we would compare the performance of the observed portfolio with the performance of a portfolio which is not accessible. The standard test interprets it as a lack of efficiency, even though it depends solely on the inadequacy of the hypotheses of the model: consequently we would get an underestimate of the portfolio quality.

In the remainder of this Section we propose two tests which take these aspects into consideration.

3.1 *The Analysis With Real Assets*

Real portfolios actually hold both financial and real assets; in particular, dwelling is the real asset in which most of household wealth is allocated.

Housing can be seen as a good of both consumption and investment. These two aspects could conceptually be kept separate through rental markets for housing, which would allow modifying the consumption level even without purchasing any house. All the same, in our analysis we assume that rental markets are not adequate substitutes for purchasing, because of both high agency costs and an unfavourable taxation structure: as a result, households are forced to invest in housing an amount at least corresponding to their own consumption need.

On the other hand, real estate is not an instrument as negotiable as a financial asset, because marked transaction costs have to be faced whenever it is sold: durable goods are subject to adjustment costs proportional to their value. Hence, the real assets owner decides, before leaving home[3], to expect until tolerating the costs becomes favourable. Grossman and Laroque (1990), elaborating a dynamic model of consumption and portfolio selection in the presence of durable and illiquid goods, by means of numerical simulation estimated that the house sale commonly occurs only after several decades from the purchase; in particular, if a modest transaction cost as 5% of the house value is supposed, they verified that the average time between house purchases is 20 to 30 years.

Therefore, if we focus on the short term, we can reasonably presume that the investments held in real assets are fixed, following Flavin and Yamashita (2002). Nonetheless, we diverge from them because we do not assume that real and financial assets are uncorrelated.

In this framework, let us partition the n risky assets into two subsets with size n_1 and n_2 respectively: the former concerns financial assets, the latter real ones. As a result, the weights vector is:

$$\underline{w} = \begin{bmatrix} \underline{w}_1 \\ \underline{w}_2 \end{bmatrix}$$

and the first two moments of the asset returns are:

[3] We assume that there is only one opportunity to modify the investment in housing: selling the current property and buying another one.

$$\underline{\mu} = \begin{bmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{bmatrix} \qquad \mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{bmatrix}$$

The investor's problem remains to decide how to arrange his financial portfolio, given that the real assets are treated as fixed: Pelizzon and Weber (2003) argue that the optimal financial portfolios are obtained by the equation:

$$\underline{w}_1 = \gamma \mathbf{\Sigma}_{11}^{-1}\left(\underline{\mu}_1 - r_0 \underline{i}\right) - \mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12}\underline{w}_2 \tag{3}$$

where γ is the investor's absolute risk aversion. It is worth specifying that, on the right-hand of equation *(3)*, the first component is exactly the same as the one we would attain in the absence of constraints. The second one, instead, is peculiar to this analysis: it no longer depends on the risk aversion, and can be interpreted as a hedge portfolio against house price risk.

Real estate is held in fixed position in the short term; hence it does not appear explicitly in the problem of portfolio selection. Notwithstanding, it affects investment choice in financial assets both indirectly, via risk aversion, and directly, through its correlations. Thus, the investor has to be efficient with respect to the risky financial assets and chooses the efficient portfolio according to his risk aversion. However, he also exploits the risky financial assets in order to hedge his exposure to the constrained assets. In case that $\mathbf{\Sigma_{12}} = \mathbf{0}$, the problem would be equivalent to the not constrained one: in the absence of correlation between financial and real assets, the latter would no longer be able to exert hedging effects. Hence, there would not be any reason to continue to take them into account. Indeed Flavin and Yamashita (2002) demonstrate that, when the return covariance matrix is block-diagonal, the household chooses how much to invest in financial assets according solely to its risk aversion, independently of its investments in durable goods.

On the basis of this reasoning, in order to assess the efficiency of a portfolio in which real assets are held in fixed proportions, let us examine solely the financial share of the observed portfolio, and move its weights gaining the vector $\underline{v}$ through the sample equation:

$$\underline{v} = \underline{w}_1 + \mathbf{S}_{11}^{-1}\mathbf{S}_{12}\underline{w}_2 \tag{4}$$

Let us now focus on the performance of a financial portfolio, of which weights have been corrected so that they no longer depend on real estate[4]. Defined:

$$\underline{\bar{e}} = (\underline{\bar{r}} - r_0\underline{i}) = \begin{bmatrix} \underline{\bar{e}}_1 \\ \underline{\bar{e}}_2 \end{bmatrix} = \begin{bmatrix} \underline{\bar{r}}_1 - r_0\underline{i} \\ \underline{\bar{r}}_2 - r_0\underline{i} \end{bmatrix}$$

we construct the Sharpe ratios:

$$S(\underline{w}) = \frac{\underline{\bar{e}}'\underline{w}}{\sqrt{\underline{w}'\mathbf{S}\underline{w}}}$$

$$S_1(\underline{w}) = \sqrt{\frac{(\underline{\bar{e}}_1'\underline{v})}{\underline{v}'\mathbf{S}_{11}\underline{v}}}$$

$$S_{2.1}(\underline{w}) = \frac{\underline{\bar{e}}_{2.1}'\underline{w}_2}{\sqrt{\underline{w}_2'\mathbf{S}_{22.1}\underline{w}_2}}$$

where:

$$\underline{\bar{e}}_{2.1} = \underline{\bar{e}}_2 - \mathbf{S}_{21}\mathbf{S}_{11}^{-1}\underline{\bar{e}}_1$$

$$\mathbf{S}_{22.1} = \mathbf{S}_{22} - \mathbf{S}_{21}\mathbf{S}_{11}^{-1}\mathbf{S}_{12}$$

The maximum values are achieved by the potential performances:

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[4] If the observed financial portfolio is optimal, its transformation through *(4)* will give the same shares that we would have in the absence of constraints: indeed see equation *(3)*.

$$S=\sqrt{\underline{\bar{e}}'\mathbf{S}^{-1}\underline{\bar{e}}}\qquad S_1=\sqrt{\underline{\bar{e}}_1'\mathbf{S}_{11}^{-1}\underline{\bar{e}}_1}\qquad S_{2.1}=\sqrt{\underline{\bar{e}}_{2.1}'\mathbf{S}_{22.1}^{-1}\underline{\bar{e}}_{2.1}}$$

Thus, Gouriéroux and Jouneau (1999) propose this test:

$$GJ=T\frac{\left(S_1^2-S_1^2\left(\underline{w}\right)\right)}{1+S_1^2\left(\underline{w}\right)\dfrac{\underline{w}'\mathbf{S}\underline{w}}{\underline{v}'\mathbf{S}_{11}\underline{v}}}=T\frac{\left(\underline{\bar{e}}_1'\mathbf{S}_{11}^{-1}\underline{\bar{e}}_1-\dfrac{\left(\underline{\bar{e}}_1'\underline{v}\right)^2}{\underline{v}'\mathbf{S}_{11}\underline{v}}\right)}{1+\dfrac{\left(\underline{\bar{e}}_1'\underline{v}\right)^2}{\left(\underline{v}'\mathbf{S}_{11}\underline{v}\right)^2}\underline{w}'\mathbf{S}\underline{w}}\xrightarrow{d}\chi^2_{n_1-1}$$

If n_2 = 0 we return to the analysis in the absence of constraints and the GJ test coincides with the JK test of equation *(2)*. Moreover, if $\mathbf{S}_{12} = \mathbf{0}$, the denominator of the test still depends on the investment in real assets. This fact should not surprise: indeed we refer to sample measures. $\mathbf{S}_{12} = \mathbf{0}$ does not necessarily imply that $\mathbf{\Sigma}_{12} = \mathbf{0}$ as well: in plain words, the correlations are estimated equal to 0, but in reality they could be different. Instead, the discussion would change if we knew that $\mathbf{\Sigma}_{12} = \mathbf{0}$: on the basis of equation *(3)*, the optimal portfolio would exactly coincide with the one we obtain in the absence of real assets. Thus, it would be pointless to apply the GJ test: we should use the JK test instead.

3.2 *The Analysis With Real Assets and Inequality Constraints*

Apart from the presence of durable goods, in order to achieve a better resemblance to the reality we should also insert some constraints on financial investments. Performing traditional tests, it is usually verified that the weights of the optimal portfolios assume extremely long and short positions in investments. Nonetheless, because of the intrinsic characteristics of some assets, it is not possible to acquire short positions: for instance think about stocks. The real world allows short positions only in mortgages and debts.

Furthermore, there could be restrictions on running into debt, or you could be forced to choose how much to invest solely within a range of values: it frequently happens in mutual fund management, for instance. Anyway, we focus on households, and therefore in the empirical analysis we only require the observance of the non-negativity constraints; however, the test discussed in this section is able to deal with the other kinds of restrictions too.

The calculation of the efficient frontier, and consequently the construction of an efficiency test, in the presence of inequality constraints is made difficult by the complexity of the computation: although it is possible to gain an algebraic expression, we cannot utilize it, because it still depends on the constraints, and we do not know *a priori* if they are binding or they are not.

As a result, the only option we have is to derive the frontier numerically; nonetheless, it remains the difficulty to attain an efficiency test and its probability distribution. The test we propose here comes from an adaptation of the work of Basak *et* Al. (2002); their paper focuses on obtaining a statistic able to assess the efficiency of a benchmark in the presence of non-negativity constraints. The test is derived asymptotically by means of the central limit theorem and the asymptotic Delta method, beginning from few weak assumptions. The version discussed in this essay replaces the benchmark with the household's observed portfolio, and extends the constraints definition, so that it can work on our structure.

Together with the weights $\underline{w}$ related to the observed portfolio, we now consider the weights $\underline{\omega}$: they are associated with an efficient portfolio, calculated so that it takes the same expected return as the given portfolio and satisfies several restrictions.

The test compares the variances of the two portfolios, in accordance with equation *(5)*, where the constraints are the most general:

(5) $$\lambda_T = \min_{\underline{\omega}} \left\{ \underline{\omega}' \boldsymbol{S} \underline{\omega} - \underline{w}' \boldsymbol{S} \underline{w} \right\}$$

$$s.t. \begin{cases} \underline{\omega}' \underline{\bar{e}} = \underline{w}' \underline{\bar{e}} \\ \underline{\omega} \geq \underline{a} \\ \underline{\omega} \leq \underline{b} \end{cases}$$

The last two constraints describe the boundaries to which the weights must submit: these constraints can be characterized in any way. Either restriction could be absent as regards some assets; instead, both of them have to appear and coincide concerning real assets. The distribution we will achieve is indeed robust to any characterization of the constraints $\underline{a}$ and $\underline{b}$: the only rule we must respect is:

$$\underline{i}' \underline{a} < 1 < \underline{i}' \underline{b}$$

$$\underline{\bar{e}}' \underline{a} < \underline{w}' \underline{\bar{e}} < \underline{\bar{e}}' \underline{b}$$

In other words it is not possible to perform an analysis, actually somewhat unrealistic, where portfolios consist purely of real estate, and consequently we impose purely equality constraints.

In order to derive the distribution for λ_T, let us notice that equation *(5)* is a non-linear function of both the expected excess returns vector and the sample covariance matrix. Hence, we can use the following procedure: firstly we apply the central limit theorem to derive the joint distribution of the sample moments vector. Lastly, we attain the distribution of equation *(5)* by means of the asymptotic Delta method.

For this purpose we define the vector:

$$\underline{\bar{U}} = \begin{bmatrix} \underline{\bar{e}} = \frac{1}{T} \sum_{t=1}^{T} \underline{e}_t \\ vech\left(\frac{1}{T} \sum_{t=1}^{T} \underline{e}_t \underline{e}_t' \right) \end{bmatrix} = \frac{1}{T} \sum_{t=1}^{T} \underline{U}_t = \frac{1}{T} \sum_{t=1}^{T} \begin{bmatrix} \underline{e}_t \\ vech\left(\underline{e}_t \underline{e}_t' \right) \end{bmatrix}$$

of the first two non-central sample moments of the excess returns; we assume the vector is stationary and ergodic with mean $\underline{\alpha}$ and variance $\mathbf{\Lambda_0}$. Applying the central limit theorem,

$$\sqrt{T}\left(\overline{\underline{U}} - \underline{\alpha}\right) \xrightarrow{d} N\left(\underline{0}, \mathbf{\Lambda}_{\boldsymbol{LP}}\right)$$

where $\mathbf{\Lambda}_{\boldsymbol{LP}}$ is the long-run variance:

$$\mathbf{\Lambda}_{\boldsymbol{LP}} = \mathbf{\Lambda_0} + \sum_{j=1}^{\infty}\left(\mathbf{\Lambda}_{\boldsymbol{j}} + \mathbf{\Lambda}'_{\boldsymbol{j}}\right) = \mathbf{\Lambda_0} + \sum_{j=1}^{\infty}\left(\mathrm{cov}\left(\underline{U}_t, \underline{U}_{t-j}\right) + \mathrm{cov}\left(\underline{U}_t, \underline{U}_{t-j}\right)'\right)$$

Now, notice that λ_T, is a function of $\overline{\underline{U}}$:

$$\lambda_T = g\left(\overline{\underline{V}}\right) = g\left(f\left(\overline{\underline{U}}\right)\right)$$

where

$$\overline{\underline{V}} = \begin{bmatrix} \overline{\underline{e}} = \frac{1}{T}\sum_{t=1}^{T}\underline{e}_t \\ vech(\boldsymbol{S}) = vech\left(\frac{1}{T}\sum_{t=1}^{T}\underline{e}_t\underline{e}'_t - \overline{\underline{e}}\overline{\underline{e}}'\right) \end{bmatrix}$$

Through the Delta method we attain:

$$\sqrt{T}(\lambda_T - \lambda) = \sqrt{T}\left(g\left(f\left(\overline{\underline{U}}_T\right)\right) - g\left(f\left(\underline{\alpha}\right)\right)\right) \xrightarrow{d} N\left(\underline{0}, \frac{\partial g}{\partial f\left(\underline{\alpha}\right)}' \frac{\partial f}{\partial \underline{\alpha}} \mathbf{\Lambda}_{\boldsymbol{LP}} \frac{\partial f}{\partial \alpha}' \frac{\partial g}{\partial f\left(\underline{\alpha}\right)}\right)$$

Now, we must replace the variance with an estimate, substituting each of the matrices with a corresponding sample value. $\mathbf{\Lambda}_{\mathrm{LP}}$ can be estimated using the method suggested by Newey and West (1987):

$$\mathbf{\Lambda}_{\boldsymbol{T}} = \hat{\mathbf{\Lambda}}_{\mathbf{0}} + \sum_{j=1}^{m}\left(1 - \frac{j}{m+1}\right)\left(\hat{\mathbf{\Lambda}}_{\boldsymbol{j}} + \hat{\mathbf{\Lambda}}'_{\boldsymbol{j}}\right)$$

with:

$$\hat{\boldsymbol{\Lambda}}_{\boldsymbol{j}} = \hat{\text{cov}}\left(\underline{U}_t, \underline{U}_{t-j}\right) = \frac{1}{T}\sum_{t=j+1}^{T}\left(\underline{U}_t - \overline{\underline{U}}_T\right)\left(\underline{U}_{t-j} - \overline{\underline{U}}_T\right)'$$

and m number of lags sufficient to entirely capture the autocorrelation. Then we have:

$$\frac{\partial f}{\partial \overline{\underline{U}}} = \begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0}_{n\times\frac{n(n+1)}{2}} \\ \begin{bmatrix} \boldsymbol{K}_1 \\ \vdots \\ \boldsymbol{K}_n \end{bmatrix} & \boldsymbol{I}_{\frac{n(n+1)}{2}} \end{bmatrix}$$

with $\boldsymbol{K}_i$ equal to:

$$\underset{n-i+1\times n}{\boldsymbol{K}_i} = -\begin{bmatrix} \underset{(n-i+1)\times(i-1)}{\boldsymbol{0}} & \begin{bmatrix} \bar{e}_i \\ \vdots \\ \bar{e}_n \end{bmatrix} & \underset{(n-i+1)\times(n-i)}{\boldsymbol{0}} \end{bmatrix} - \bar{e}_i \begin{bmatrix} \underset{(n-i+1)\times(i-1)}{\boldsymbol{0}} & \boldsymbol{I}_{n-i+1} \end{bmatrix}$$

and $\bar{e}_i$ i-eth element of $\bar{e}$. Finally, the last derivative is:

$$\frac{\partial g}{\partial \overline{\underline{V}}}' = \left[\delta\underline{\omega}' \;\; \omega_1^2 \;\; 2\omega_1\omega_2 \;\; \cdots \;\; 2\omega_1\omega_n \;\; \omega_2^2 \;\; 2\omega_2\omega_3 \;\; \cdots \;\; \omega_n^2\right] - \\ + \left[\delta\underline{w}' \;\; w_1^2 \;\; 2w_1w_2 \;\; \cdots \;\; 2w_1w_n \;\; w_2^2 \;\; 2w_2w_3 \;\; \cdots \;\; w_n^2\right]$$

and it also depends on the value taken by the Lagrange multiplier δ associated with the equality constraint between the expected returns on portfolios.

Therefore, in order to verify the null hypothesis we consider the statistical test:

$$BJS = \sqrt{T}\frac{\lambda_T}{\sqrt{\frac{\partial g}{\partial \overline{\underline{V}}}' \frac{\partial f}{\partial \overline{\underline{U}}} \boldsymbol{\Lambda_T} \frac{\partial f}{\partial \overline{\underline{U}}}' \frac{\partial g}{\partial \overline{\underline{V}}}}} \xrightarrow{d} N(0,1)$$

This test has been conceived and derived in a way completely dissimilar from the statistics usually applied to assess portfolio efficiency: this peculiarity leads the test to be noticeable under several aspects, both positive and negative.

We can ascribe at least three good points to our test: first of all, starting with general inequality constraints allows us to use the same statistic every time in order to deal with a wide range of more specific problems, even out of the aim of this paper. Secondly, as we have to assess the efficiency of a portfolio in which the investments are held merely in risk free and real (then fixed) assets but not in financial ones ($\underline{w}_1 = \underline{0}$), the test leads to different results depending on the value taken by w_2: it is natural to expect this feature on the basis of what appears on equation *(3)*[5]. Lastly, our test does not require normality or independence in the return series, and for this reason it is valid in a general context.

However, the test is not free of shortcomings: we highlight two of them. Firstly, it is not always clear how many lags have to be placed in the Newey-West estimate of the long-run variance. We should take a number sufficient to capture a large amount of the autocorrelation between the series $\underline{U}$; nevertheless it is true that, using a number of lags even slightly different, we run the risk of obtaining absolutely contrasting values. In this regard the formula of automatic selection described by Newey and West (1994) could be used. One other defect of the test is that, because it needs the application of numerical methods, it could arise that the algorithm cannot solve equation *(5)* determining an absolute minimum, but it stops on a relative minimum, comparing in this way the efficiency of a given portfolio with the one of another inefficient portfolio: then, there would be the tendency to accept the null hypothesis more than what should be correct.

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[5] Instead, in this situation the GJ test is always concluding the same way, independently of w_2, deciding solely on the basis of the excess-return moments.

4. - The Data Used

In the empirical analysis we examine the Italian household portfolios as of 31st December 2000, acquired by the *Survey of Household Italian Wealth (SHIW)* for the year 2000 performed on behalf of the Bank of Italy; the sample consists of 22,268 individuals belonging to 8.001 households living in 333 towns.

The different savings and investment instruments taken into consideration by the survey are sorted into six categories in order to simplify the analysis: we consider risk free assets, government bonds (divided into short and long term bonds according to their duration), corporate bonds, stocks and housing. We could also deal with other properties apart from the house where people live, but we prefer not to take them into account because the possession of another house is still an occasional occurrence. In Table 1 all the items which compose each of the six groups are listed; because the questionnaire does not query more detailed information about the kind of mutual funds, managed savings and foreign assets held, we choose to distribute them in equal parts among corporate bonds and stocks. The fraction of the overall wealth invested in each asset by the sampled households in 2000 is displayed in the last column. The clearest aspect is that nearly 80% of the holding is invested in housing[6]: it proves that the most appreciable household investment is mistakenly ignored by the common efficiency analyses. Moreover, 10% of the wealth is invested in risk free assets, especially in bank current account deposits; the investments in other assets become, instead, lower and less frequent. They indeed take values between 3% and 5%, except for long term government bonds which even show a negative position. Notice that in our framework only three out of the six groups can assume negative weights by construction: the risk free assets, because of the presence of debts to friends and relatives not living together, the short term government bonds, which contain debts on not durable goods, and just the long term go-

[6] From the same survey we discover that, amongst the real assets, households prefer to invest in housing, where 85% of the overall real wealth is allocated.

TABLE 1

GROUPS ADOPTED IN THE ANALYSIS AND THEIR COMPOSITION*

Group	Composition	Wealth invested (%)
Risk free Assets	Cash	0.30
	Bank current account deposits	7.58
	Bank savings deposits	0.93
	Post office current accounts and deposit books	0.85
	Credits to friends and relatives not living together	0.10
	Debts to friends and relatives not living together (–)	–0.08
	Total	**9.68**
Short team government Bonds	Certificates of deposit	0.32
	Repos	0.33
	BOT (Italian T-Bills)	1.65
	CCT (Italian T-Certificates)	0.70
	CTZ (Italian T-Bonds)	0.08
	Other Italian government bonds	0.04
	Debts on not durable goods (–)	–0.01
	Total	**3.11**
Long term government bonds	Post office savings certificates	0.45
	BTP (Italian T-Bonds)	0.60
	Mortgage (-)	–1.95
	Total	**–0.91**
Corporate Bonds	Italian and foreign corporate bonds	1.10
	Mutual funds (1/2)	3.11
	Managed savings (1/2)	1.47
	Other foreign assets (1/2)	0.27
	Total	**3.51**
Stocks	Italian and foreign stocks	2.45
	Loans to cooperatives	0.07
	Mutual funds (1/2)	3.11
	Managed savings (1/2)	1.47
	Other foreign assets (1/2)	0.27
	Total	**4.96**
Housing	Value of housing if it is household's property	79.65
	Total	**79.65**

* The last column displays the fraction of the overall wealth invested by the households on 31st December 2000.

vernment bonds, which include mortgages. The latter category is the only one which takes substantial proportions: indeed its group is just related to a negative investment.

Then, Italy still shows backwardness in comparison with other developed countries: the typical behaviour is to invest merely in housing and deposit the remaining wealth in a safe bank current account, almost completely disregarding every other financial asset. Although slow, it is however recognizable the tendency to achieve a better portfolio diversification, encouraged by the expansion of the Italian market in the late nineties: indeed, comparing the aggregate portfolios derived from the 1998 and 2000 *SHIW* waves, it is immediately noticeable that, notwithstanding a general consolidation of the positions, in two years the weights associated with the investments in corporate bonds and stocks increased of 30%. Nevertheless, it is true that the event still concerns few households (Table 2): 43% of them have a portfolio composed exclusively of risk free assets and housing; another 17% have solely risk free assets[7]. 33% of the households, instead, invest in a more diversified portfolio, formed by financial assets as well as real and risk free ones.

We pay attention to a point in particular: stocks have features very dissimilar from the other financial assets, which are substantially bonds. Therefore, if we examine how many households are into possession of a diversified portfolio (maybe bad managed), in which the wealth is split into risk free assets, housing and other financial assets, we obtain disappointing results: only 15% of portfolios are diversified. If we focus on regional differences, we observe that the South is the area where there is the greatest backwardness.

It is natural to wonder what the reason for such an attitude is. Obviously, it is not ensured that diversifying leads to an efficient portfolio, but what trouble us is that most of households do not know the existence of many investment assets: Guiso and Jappelli (2000) have taken an interest in this regard. A large num-

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[7] They are the poorest households, who neither have enough money to purchase a property nor give sufficient security for taking out a mortgage.

Table 2

PORTFOLIOS COMPOSITION
FOR THE SAMPLE USED IN THE ANALYSIS*

Portfolios composition	North-West		North-East		Centre		South		Italy	
	no.	%	no.	%	no.	%	no.	%	no.	%
(1) : *RF*	362	19.8	223	14.8	261	17.1	370	16.0	1216	17.0
(2) : *RF* + *S*	27	1.5	16	1.1	22	1.4	7	0.3	72	1.0
(3) : *RF* + *F*	73	4.0	47	3.1	48	3.1	52	2.2	220	3.1
(4) : *RF* + *S* + *F*	102	5.6	83	5.5	47	3.1	29	1.3	261	3.6
(5) : (2) +(3) + (4)	202	11.1	146	9.7	117	7.7	88	3.8	553	7.7
(6) : *RF* + *H*	557	30.5	431	28.6	638	41.8	1432	61.9	3058	42.6
(7) : *RF* + *H* + *S*	39	2.1	55	3.6	37	2.4	28	1.2	159	2.2
(8) : *RF* + *H* + *F*	288	15.8	286	19.0	228	15.0	273	11.8	1075	15.0
(9) : *RF* + *H* + *S* + *F*	376	20.6	367	24.3	244	16.0	124	5.4	1111	15.5
(10): (7) + (8) + (9)	703	38.5	708	47.0	509	33.4	425	18.4	2345	32.7
Total	1824	100.0	1508	100.0	1525	100.0	2315	100.0	7172	100.0

Legend:
RF: risk free assets,
S: stocks,
F: other financial (short/long term government or corporate bonds),
H: housing.

ber of households do not invest in certain assets only because they are unknown, so investment opportunities are abundantly limited. This phenomenon is also borne out by the application of a probit regression (Table 3), where the dependent variable takes the value 1 if the household holds both stocks and other financial assets in its portfolio: it emerges that households are more inclined to diversify when they have high income and education (which can be seen as a proxy for the awareness of the financial instruments) and when the householder is about 50 years old (see Graph 1). In addition the choice depends on an environmental factor, because the propensity to diversify is greater in the northern area.

We use semi-annual series covering the period between January 1987 and December 2000. We utilize the series of six-months BOT (T-bills) returns as risk free asset; the return on short term government bonds is derived from an average of the returns

TABLE 3

OUTPUT OF A PROBIT REGRESSION ABOUT DIVERSIFICATION OF THE ITALIAN HOUSEHOLDS' PORFOLIOS*

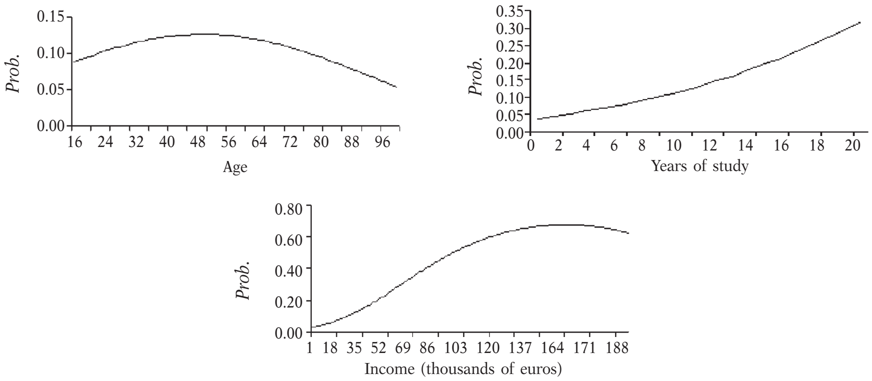
Probit estimates	Observations =	7.793
	LR $chi^2(11)$	1587.66
	Prob > chi^2	0.0000
Log- likelihood –2832.7126	Pseudo R^2	0.2189

Diversifying probability	Coef.	Std. Error.	*z*	*P*>|*z*|	[95% confidence interval]	
Age	0.0184525	0.0089168	2.07	0.039	0.0009759	0.035929
Age^2	–0.0001855	0.0000817	–2.27	0.023	–0.0003456	–0.0000255
Income	0.0291013	0.0017502	16.63	0.000	0.0256709	0.0325318
Income^2	–0.0000908	0.0000103	–8.85	0.000	–0.0001108	–0.0000707
Gender	0.0812667	0.0435399	1.87	0.062	–0.0040699	0.1666032
North-West	0.8081689	0.056036	14.42	0.000	0.6983404	0.9179974
North-East	0.8639581	0.0578883	14.92	0.000	0.7504991	0.9774172
Centre	0.5921904	0.059712	9.92	0.000	0.4751571	0.7092238
Education	0.0636759	0.0049556	12.85	0.000	0.0539632	0.0733886
Employee	–0.081228	0.0547352	–1.48	0.138	–0.1885071	0.026051
Self-employed	–0.103863	0.0670685	–1.55	0.121	–0.2353148	0.0275888
Constant	–3.347021	0.2474657	–13.53	0.000	–3.832045	–2.861997

* The dependent variable takes the value 1 when the portfolio holds both stocks and other financial assets, whatever the investment in housing is. Among the explanatory variables, age is in completed years, income in thousands of euros, gender takes 1 when the householder is male and education is expressed in years of study.

GRAPH 1

PROBABILITY TO INVEST HOUSEHOLD WEALTH IN A DIVERSIFIED PORTFOLIO*



* According to householder's age, his education (years of study) and household wealth (thousands of euros).

on BOT, CCT (T-certificates) and CTZ (T-bonds) listed, weighted with the residual debts. We assign the BTP (T-bonds) monthly average return with residual duration more than one year to long term government bonds; with reference to corporate bonds, we make use of the real return on a sample of fixed rate bonds issued by banks, with duration more than one year and a significant negotiation value. The source for all these series is the Bank of Italy. Stocks returns are instead calculated from the series of the *MSCI Italy Total Return* index, through the method of the simple capitalization; this index measures the market performance, taking account of both the prices and the cash flow produced by the dividend payment.

Our first intention was to deal with monthly series, but it cannot be propounded in view of the characteristics of our real estate data. About housing, indeed, we only have semi-annual price series of the housing market for the Italian provinces, acquired thanks to the helpfulness of the magazine *Consulente Immobiliare*. Thus, we aggregate the values, on the basis of the amount of the resident population, into four wide geographical areas: the North-West (in which there are the three large industrial cities of Milan, Turin and Genoa), the North-East (where there are many medium-sized cities, such as Venice and Bologna), Central Italy (with Florence and Rome), the South and the islands (a largely rural territory, but including Naples, Bari and Palermo). We did not make further aggregation because we do not want to lose the peculiarities of each area. In order to derive the returns we also take imputed rents R_t and maintenance costs M_t into consideration, assuming they are constant for each area all the time:

$$r_{t,H} = \frac{P_{t,H} - P_{t-1,H}}{P_{t-1,H}} + \frac{R_t - M_t}{P_{t-1,H}} \cong \frac{P_{t,H} - P_{t-1,H}}{P_{t-1,H}} + k$$

We estimate k qual to 3.36% on an annual basis, through the 2000 *SHIW* survey; anyhow its choice is immaterial in the analysis in which housing is treated as constrained, as long as k is a fixed number. If we dealt with the housing data as monthly ones, trivially dividing by six the semi-annual returns, we would risk running into bias in the covariance matrix, and consequently in the efficient frontier and in the results of the test founded on it; for this reason we choose to perform an analysis on semi-annual returns, even though we have just few observations.

In Table 4 we display the expected return and the risk on our time series. Stocks differ abundantly from all the other assets: they have the best expected return, but they also show a volatility considerably higher than any other asset has. The remaining financial aggregates, instead, behave in a similar way, although corporate bonds stand out because of the best Sharpe *ratio*. The risk free assets achieve a return quite close to the government and corporate bonds, but they enjoy the complete absence of volatility. Real assets are, instead, different: their returns are averagely lower than the ones on all the financial assets, the risk free included (apart from the North-East), and have a volatility lower only than the stocks. Nonetheless, let us keep in mind that here we are talking about returns attained by a portfolio of properties acquired in dissimilar geographical areas, although close, whilst the investor holds a single house at most, with a different return and a presumably higher risk.

Table 5 displays the correlations between the excess-returns: the tightest connections appear between government and corporate bonds, which followed rather similar trends in the sample period. The correlations between the other assets are, instead, lower and often negative; for durable goods, this result would mean that there is something true in the popular conception on the basis of which house is seen as a hedging good.

It is of great interest for us to assess whether the sample covariance matrix is block-diagonal: if so, the optimal investment in financial assets would be independent of durable goods, provided that the sample moments coincided with the population ones. Hence we verify, in the same way as De Roon *et* Al. (2002), if some coefficients are significant when we regress the housing excess-returns on the financial ones: if so, we would be inclined to believe there is correlation between financial and real assets. Nevertheless, because of the

Table 4

EXPECTED RETURNS AND RISKS
ON FINANCIAL AND REAL ASSETS
(SAMPLE PERIOD: 1987-2000, LENGTH: 6 MONTHS)*

Expected return and risk (28 obs.)	Risk free assets	Short term bonds	Long term bonds	Corporate bonds	Stocks	North-West housing	North-East housing	Centre housing	South housing
Mean (%)	4.0985	4.9044	4.9275	5.0608	6.1172	3.9174	4.3616	3.5763	3.4893
Std. Dev. (%)	0	1.8003	1.6596	1.5161	14.8040	5.5244	4.2550	5.4633	3.7596
Sharpe	×	0.4476	0.4995	0.6347	0.1364	–0.0328	0.0618	–0.0956	–0.1620

* On the last line the Sharpe performance is displayed; it is the *ratio* of the expected excess-returns (expected return on the asset minus return on the risk free asset) to the standard deviation.

Table 5

CORRELATION MATRIX
OF THE EXCESS-RETURNS ON THE ASSESTS
(SAMPLE PERIOD: 1987-2000, LENGHT: 6 MONTHS)*

Correlations of the assets (28 obs.)	Short term bonds	Long term bonds	Corporate bonds	Stocks	North-West housing	North-East housing	Centre housing	South housing
Short term	1.0000							
Long term	0.5566	1.0000						
Corp. bonds	0.2008	0.7265	1.0000					
Stocks	-0.0966	–0.1575	–0.0293	1.0000				
North-West	–0.1029	0.0775	0.3351	–0.0205	1.0000			
Nord-East	0.1054	0.3459	0.4097	–0.2392	(0.607)	1.0000		
Centre	0.0133	0.0531	0.0531	–0.0014	(0.600)	(0.573)	1.0000	
South	–0.1184	0.0047	0.2099	0.0281	(0.488)	(0.424)	(0.578)	1.0000

* In brackets the correlations among the excess-returns on housing in the four geographical areas are reported.

low sample size, the test does not refuse the null hypothesis for all the areas at the usual significance levels; however, the high value achieved by the *p*-value for the central area suggests that at least in this zone there is no evidence against the block-diagonality of the covariance matrix[8]. Anyway, it does not necessary mean that the population matrix is block-diagonal as well: the estimate could just be the result of an erratic component. As we have no particular reason for assuming that $\mathbf{\Sigma_{12}} = \mathbf{0}$, we can implement the tests with fixed real assets for all the areas, Central Italy included.

5. - Portfolio Efficiency

The check on the efficiency is based on a total of 7,172 households, whose portfolios are described in Table 2. From the sample interviewed in the year 2000 we exclude the households who neither declared the amount invested in one or more assets nor wanted to supply information useful to infer it (only 167 observations, equal to 2% of the sample, as a sign of an excellent quality of the questionnaire) and those whose overall wealth is negative because it consists predominantly of debts (41). Moreover, we disregard the households who possess solely cash (621, connected with a very low wealth) because, although their investments would be assessed trivially efficient, we could not talk literally of portfolio since the wealth is not invested in any way.

The nature of the data compels to apply semi-annual time series: as a result, on the one hand we can use homogeneous data, but on the other, the disadvantage to have only 28 observations arises. The effect is that whatever test applied to these observations takes low power; it has been already detected in the test on block-diagonality (Table 6). Low power implies that tests tend to accept the null hypothesis more than what they ought to do, because there is no information sufficient to discriminate between the two hypotheses. Even though in the future we own time series with more observations, probably this problem will not be coped: indeed, in this kind of analysis it is not reasonable to take

[8] This deduction is borne out by the application to monthly returns (although it is of no practical value), where the sample size is higher and there are not low-power problems.

account of a sample period larger than 15-20 years, because presumably the series are no longer regular in consequence of changes in the market conditions. In order to manage partly with this problem we can rely upon a significance level higher than usual: in the remainder, indeed, we consider the results at the levels of 25% and 40% as well as the ones of 1%, 5% and 10%.

The analysis is executed in accordance with all the theoretical models discussed in the previous Sections; we begin with the tests in the absence of constraints, where portfolios consist of either financial assets or these and housing[9]. Despite the low pow-

Table 6

BLOCK-DIAGONALITY TEST*

Block-diagonality of the covariance matrix (28 observations)		North-West housing	North-East housing	Centre housing	South housing
Short term government bonds	Coefficient	–0.5669	–0.2137	–0.2280	–0.4979
	t-statistic	–0.1600	–0.0900	–0.0600	–0.2100
	p-value	0.8710	0.9300	0.9500	0.8390
Long term government bonds	Coefficient	–4.3759	0.5339	0.5890	–2.3572
	t-statistic	–0.9400	0.1700	0.1200	–0.7300
	p-value	0.3550	0.8700	0.9050	0.4750
Corporate bonds	Coefficient	10.3369	4.6526	0.3812	4.9164
	t-statistic	1.9600	1.2700	0.0700	1.3300
	p-value	0.0620	0.2180	0.9460	0.1960
Stocks	Coefficient	–0.0199	–0.0531	0.0016	–0.0016
	t-statistic	–0.3100	–1.1800	0.0200	–0.0400
	p-value	0.7600	0.2510	0.9810	0.9720
Joint significance	*F*-statistic	1.2200	1.6300	0.0200	0.5800
	p-value	0.3293	0.2018	0.9992	0.6780

* Based on the regression of the housing excess-returns on the financial assets excess-returns. Each column describes a different regression; the last but one lines display the value of the F-statistic concerning the joint significance of the explanatory variables coefficients.

[9] Although the latter case is not very important, because including real assets does not make sense whether they are treated as financial assets, it is useful to make a comparison with the tests proposed hereafter, based on the same set of constrained assets.

er, both JK and BJS[10] tests suggest that only the risk free portfolios are efficient, since they are trivially positioned along the efficient frontier. Nevertheless, inserting housing in the portfolio, the efficiency makes worse further: indeed, going by the tangency portfolios (Table 7), which describes the weights of a risky efficient portfolio[11], the weight associated with the real asset takes a negative value, against the logic of the real world.

It is then sensible to wonder if the portfolios, at a rough guess distant from the frontiers in Graph 2, are inefficient either because households have not selected their investments in an optimal way or merely because several crucial hypotheses have been omitted in the underlying model. Therefore, we insert non-negativity constraints on corporate bonds, stocks and housing; we allow, instead, government bonds and risk free assets to assume short positions. The new frontiers, although they reduce the set of portfolios achievable by the

TABLE 7

TANGENCY PORTFOLIOS WITHOUT AND WITH HOUSING

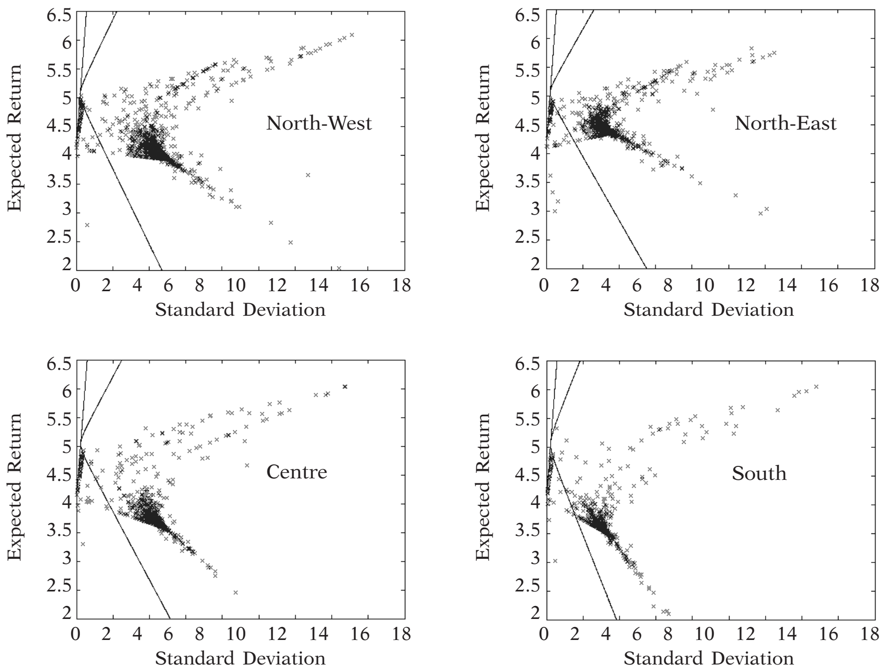
Tangency portfolios	Without housing	With housing (North-West)	With housing (Nord-East)	With housing (Centre)	With housing (Sud e isole)
Short term bonds	47.18	43.29	42.71	47.15	45.61
Long term bonds	–46.92	–50.35	–41.78	–46.78	–49.84
Corporate bonds	99.71	108.55	101.33	99.89	105.79
Stocks	0.02	–0.01	–0.09	0.02	0.02
Housing	×	–1.48	–2.17	–0.29	–1.57
Expected return (%)	5.0498	5.0771	5.0639	5.0539	5.0809
Expected risk (%)	1.5828	0.2289	0.2249	0.2340	0.2352

[10] For the test, here and hereafter, the Newey-West estimate with one lag is considered.

[11] On the basis of the two funds theorem, any efficient portfolio is given by the connection between the risk free portfolio and the risky one, called tangency portfolio. Then the tangency portfolio is the model which portfolios ought to approach in order to be managed in the best way.

Graph 2

ITALIAN HOUSEHOLD PORTFOLIOS COMPARED WITH THE NOT RESTRICTED EFFICIENT FRONTIER*



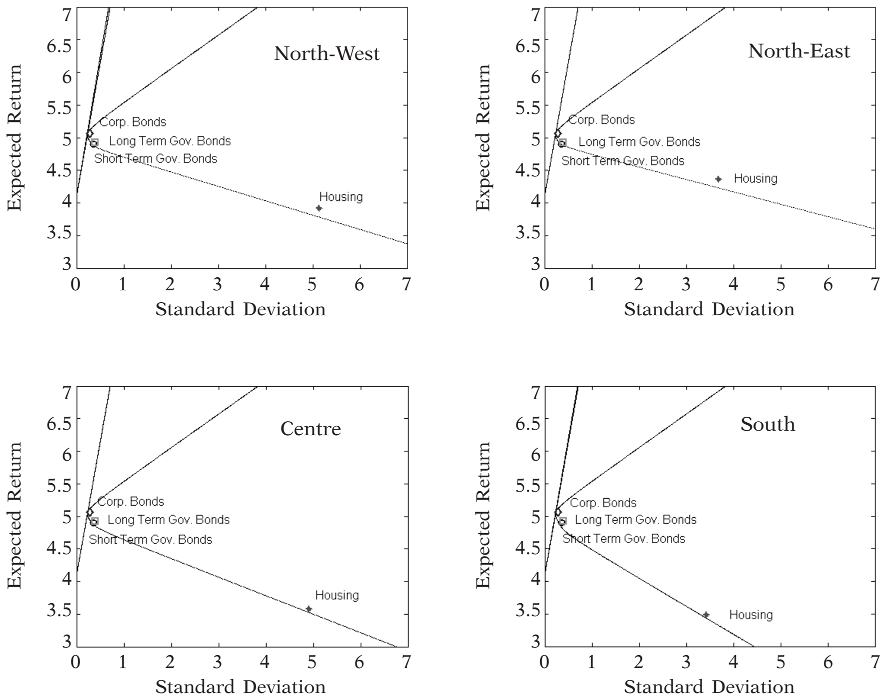
* Portfolios take housing into account.

investor, determine a scant variation, graphically imperceptible (Graph 3). This result is not surprising since in the tangency portfolios the positions held in corporate bonds are abundantly positive and those in stocks and housing are very small-sized. In this context only the BJS test is applicable: we achieve that, besides the risk free portfolios, other 20 ones are significant (8 come from the North-West) at the 10% level, composed of risk free assets and at least one between government and corporate bonds.

However, the greatest efficiency improvements occur when we treat the weight on housing as fixed. Observe Graph 4: for each area we report the efficient frontiers connected with a

GRAPH 3

EFFICIENT FRONTIERS IN THE PRESENCE OF SHORT SALES CONSTRAINTS ON CORPORATE BOND, STOCKS AND HOUSING*



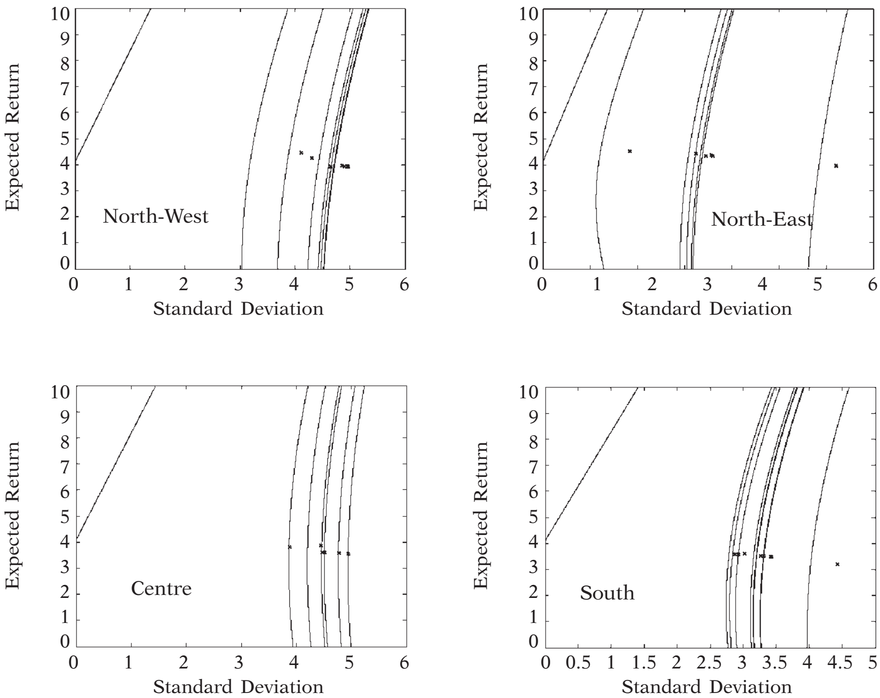
* The locations held by government bonds, corporate bonds and bousing are also drawn, while the stocks are excluded because they would give a perspective distortion. In each situation the constrained and the unconstrained frontiers with risk free asset lay one upon the other.

sample of households randomly selected[12]. All the portfolios are close more to their bounded frontier than to the unrestricted one. The difference in the results borne by the two tests which we can run here (GJ and BJS) depends mainly on the dissimilar basic assumptions: the GJ test presumes that the returns

[12] Each geographical area has a different size in the household sample: because of this, in Graph 4 we allocate seven portfolios to the North-West, six to the North-East and to the Centre, ten to the South.

Graph 4

EFFICIENT FRONTIERS WHEN INVESTMENT IN HOUSING IS FIXED*



* The unconstrained frontier is drawn on the extreme left of each graph.

are independent and identically distributed, whereas the BJS test requires less binding properties like stationarity and ergodicity.

The GJ test reveals (Table 8) that 31-32% of the portfolios are efficient at levels of 40% and 25%; moreover, it emphasizes that the households who do not own their house are efficient in no case, unless we use lower significance levels. The geographical area where most of efficient portfolios are concentrated is the Central Italy (Table 9): here we believe that, besides the risk free assets, all the portfolios holding solely housing and almost all (in only one case the test rejects the null hypothesis) the ones which also include at least one between government and corporate bonds are

TABLE 8

EFFICIENT PORTFOLIOS IN ITALY ON THE BASIS OF THE GJ TEST WITH INVESTMENT IN HOUSING HELD IN FIXED PROPORTIONS*

Significance level →	40%		25%		10%		5%		1%	
Eff. port. in Italy	no.	%	no.	%	no.	%	no.	%	no.	%
(1) : *RF*	1216	100.0	1216	100.0	1216	100.0	1216	100.0	1216	100.0
(2) : *RF* + *S*	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
(3) : *RF* + *F*	0	0.0	0	0.0	5	2.3	5	2.3	7	3.2
(4) : *RF* + *S* + *F*	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
(5) : (2) +(3) + (4)	0	0.0	0	0.0	5	0.9	5	0.9	7	1.3
(6) : *RF* + *H*	638	20.9	638	20.9	2627	85.9	2627	85.9	3058	100.0
(7) : *RF* + *H* + *S*	12	7.5	30	18.9	62	39.0	85	53.5	91	57.2
(8) : *RF* + *H* + *F*	241	22.4	268	24.9	690	64.2	985	91.6	1071	99.6
(9) : *RF* + *H* + *S* + *F*	89	8.0	121	10.9	370	33.3	480	43.2	576	51.8
(10): (7) + (8) + (9)	342	14.6	419	17.9	1122	47.8	1550	66.1	1738	74.1
Total	2196	30.6	2273	31.7	4970	69.3	5398	75.3	6019	83.9

* see note Table 2.

TABLE 9

EFFICIENT PORTFOLIOS IN THE FOUR AREAS ON THE BASIS OF THE GJ TEST WITH INVESTMENT IN HOUSING HELD IN FIXED PROPORTIONS*

Significance level = 40%	North-West		North-East		Centre		South	
Eff. port per area	no.	%	no.	%	no.	%	no.	%
(1) : *RF*	362	100.0	223	100.0	261	100.0	370	100.0
(2) : *RF* + *S*	0	0.0	0	0.0	0	0.0	0	0.0
(3) : *RF* + *F*	0	0.0	0	0.0	0	0.0	0	0.0
(4) : *RF* + *S* + *F*	0	0.0	0	0.0	0	0.0	0	0.0
(5) : (2) +(3) + (4)	0	0.0	0	0.0	0	0.0	0	0.0
(6) : *RF* + *H*	0	0.0	0	0.0	638	100.0	0	0.0
(7) : *RF* + *H* + *S*	0	0.0	0	0.0	12	32.4	0	0.0
(8) : *RF* + *H* + *F*	3	1.0	0	0.0	227	99.6	11	4.0
(9) : *RF* + *H* + *S* + *F*	1	0.3	18	4.9	70	28.7	0	0.0
(10): (7) + (8) + (9)	4	0.6	18	2.5	309	60.7	11	2.6
Total	366	20.1	241	16.0	1208	79.2	381	16.5

* See ,note Table 2.

optimally combined. In Section 4 we proved that at least just in this area the block-diagonality test concludes that the returns on financial assets are uncorrelated with the ones on real estate. However, it is not assured that the optimal portfolios are chosen independently of the investment in property, because we do not know anything about the population correlation; we possess merely an estimate afflicted with error. Therefore, the efficiency test gives emphasis to the supremacy of a «diversification» effect: the possible moderate performance of the portfolio is less considerable if the financial wealth is a small component of the total wealth.

The BJS test highlights a remarkable resemblance to the results of the GJ test at the 40% significance level (Tables 10 and 11), while the contrast appears at lower levels, especially because of the presence of the autocorrelation[13]. Anyway, even

TAV. 10

EFFICIENT PORTFOLIOS IN ITALY ON THE BASIS OF THE BJS TEST WITH INVESTMENT IN HOUSING HELD IN FIXED PROPORTIONS*

Significance level →	40%		25%		10%		5%		1%	
Eff. port. in Italy	no.	%	no.	%	no.	%	no.	%	no.	%
(1) : *RF*	1216	100.0	1216	100.0	1216	100.0	1216	100.0	1216	100.0
(2) : *RF* + *S*	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
(3) : *RF* + *F*	0	0.0	0	0.0	10	4.5	11	5.0	30	13.6
(4) : *RF* + *S* + *F*	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
(5) : (2) + (3) + (4)	0	0.0	0	0.0	10	1.8	11	2.0	30	5.4
(6) : *RF* + *H*	638	20.9	2624	85.8	3058	100.0	3058	100.0	3058	100.0
(7) : *RF* + *H* + *S*	6	3.8	73	45.9	111	69.8	124	78.0	140	88.0
(8) : *RF* + *H* + *F*	227	21.1	584	54.3	1075	100.0	1075	100.0	1075	100.0
(9) : *RF* + *H* + *S* + *F*	33	3.0	402	36.2	729	65.6	844	76.0	959	86.3
(10): (7) + (8) + (9)	266	11.3	1059	45.2	1915	81.7	2043	87.1	2174	92.7
Total	2120	29.6	4899	68.3	6199	86.4	6328	88.2	6478	90.3

* See note Table 2.

[13] We have, indeed, repeated the BJS test using a Newey-West estimate with 0 lags, namely assuming there is absence of autocorrelation: we have obtained that the number of efficient portfolios is fairly close to the results given by the GJ test for every significance level.

TABLE 11

EFFICIENT PORTFOLIOS IN THE FOUR AREAS
ON THE BASIS OF THE BJS TEST WITH INVESTMENT IN
HOUSING HELD IN FIXED PROPORTIONS*

Significance level = 40%	North-West		North-East		Centre		South	
Eff. port. per area	no.	%	no.	%	no.	%	no.	%
(1) : *RF*	362	100.0	223	100.0	261	100.0	370	100.0
(2) : *RF* + *S*	0	0.0	0	0.0	0	0.0	0	0.0
(3) : *RF* + *F*	0	0.0	0	0.0	0	0.0	0	0.0
(4) : *RF* + *S* + *F*	0	0.0	0	0.0	0	0.0	0	0.0
(5) : (2) +(3) + (4)	0	0.0	0	0.0	0	0.0	0	0.0
(6) : *RF* + *H*	0	0.0	0	0.0	638	100.0	0	0.0
(7) : *RF* + *H* + *S*	0	0.0	0	0.0	6	16.2	0	0.0
(8) : *RF* + *H* + *F*	1	0.3	0	0.0	226	99.1	0	0.0
(9) : *RF* + *H* + *S* + *F*	0	0.0	0	0.0	33	13.5	0	0.0
(10): (7) + (8) + (9)	1	0.1	0	0.0	265	52.1	0	0.0
Total	363	19.9	223	14.8	1164	76.3	370	16.0

* see note Table 2.

here it is clear the divergence of efficiency between the financial portfolios and the ones which treat housing as fixed. Taking the 40% level as proper we could conclude there is efficiency only in a large number of portfolios of the households resident in Central Italy; at lower levels, we notice efficiency first in the South, then in the North-West and finally in the North-East.

Finally we add non-negativity constraints on corporate bonds and stocks to fixed durable goods: the BJS test concludes that few portfolios more than before are efficient (Table 12). From Graph 5 we perceive that the frontiers are very similar to the ones in Graph 4, but the line about the South and the islands approaches to portfolios more perceptibly than the others: indeed in this area there are 30 out of 31 new overall efficiencies, pertinent to portfolios with real estate and investments in financial assets (Table 13).

Consequently, with the data at our disposal, 30% of the sampled households' portfolios would be efficient. Most of the 2,151

Table 12

EFFICIENT PORTFOLIOS IN ITALY ON THE BASIS OF THE BJS TEST WITH FIXED INVESTMENT IN HOUSING AND SHORT SALES CONSTRAINTS ON CORPORATE BONDS AND STOCKS*

Significance level →	40%		25%		10%		5%		1%	
Eff. port. in Italy	no.	%	no.	%	no.	%	no.	%	no.	%
(1) : *RF*	1216	100.0	1216	100.0	1216	100.0	1216	100.0	1216	100.0
(2) : *RF* + *S*	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
(3) : *RF* + *F*	0	0.0	0	0.0	20	9.1	20	9.1	36	16.4
(4) : *RF* + *S* + *F*	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
(5) : (2) +(3) + (4)	0	0.0	0	0.0	20	3.6	20	3.6	36	6.5
(6) : *RF* + *H*	638	20.9	2628	85.9	3058	100.0	3058	100.0	3058	100.0
(7) : *RF* + *H* + *S*	6	3.8	76	47.8	119	74.8	132	83.0	143	89.9
(8) : *RF* + *H* + *F*	256	23.8	848	78.9	1075	100.0	1075	100.0	1075	100.0
(9) : *RF* + *H* + *S* + *F*	35	3.2	450	40.5	804	72.4	889	80.0	985	88.7
(10): (7) + (8) + (9)	297	12.7	1374	58.6	1998	85.2	2096	89.4	2203	93.9
Total	2151	30.0	5218	72.8	6292	87.7	6390	89.1	6513	90.8

* See note Table 2.

Table 13

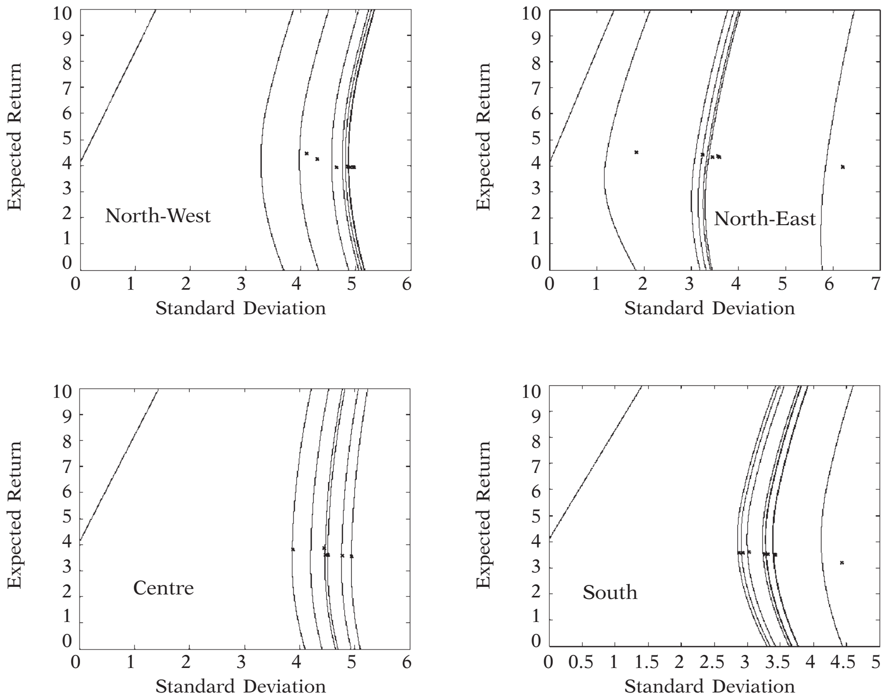
EFFICIENT PORTFOLIOS IN THE FOUR AREAS ON THE BASIS OF THE BJS TEST WITH FIXED INVESTMENT IN HOUSING AND SHORT SALES CONSTRAINTS IN CORPORATE BONDS AND STOCKS*

Signficance level = 40%	North-West		North-East		Centre		South	
Eff. port. per area	no.	%	no.	%	no.	%	no.	%
(1) : *RF*	362	100.0	223	100.0	261	100.0	370	100.0
(2) : *RF* + *S*	0	0.0	0	0.0	0	0.0	0	0.0
(3) : *RF* + *F*	0	0.0	0	0.0	0	0.0	0	0.0
(4) : *RF* + *S* + *F*	0	0.0	0	0.0	0	0.0	0	0.0
(5) : (2) + (3) + (4)	0	0.0	0	0.0	0	0.0	0	0.0
(6) : *RF* + *H*	0	0.0	0	0.0	638	100.0	0	0.0
(7) : *RF* + *H* + *S*	0	0.0	0	0.0	6	16.2	0	0.0
(8) : *RF* + *H* + *F*	1	0.3	0	0.0	226	99.1	29	10.6
(9) : *RF* + *H* + *S* + *F*	0	0.0	0	0.0	34	13.9	1	0.8
(10): (7) + (8) + (9)	1	0.1	0	0.0	266	52.3	30	7.1
Total	363	19.9	223	14.8	1165	76.4	400	17.3

* See note Table 2.

GRAPH 5

EFFICIENT FRONTIERS WHEN INVESTMENT IN HOUSING IS FIXED AND SHORT SALES CONSTRAINTS ARE SET ON CORPORATE BONDS AND STOCKS*



* The unconstrained frontier is drawn on the extreme left of each Graph.

efficient portfolios (86%) hold no risky financial assets, but they are composed solely of either risk free assets or housing and risk free assets. Instead, if we consider only the remaining portfolios, the percentage of efficient portfolios decreases to 6%: in this case there are only one efficient portfolio in the North-West and none in the North-East.

The analysis with monthly returns, although it depends on a covariance matrix consisting of heterogeneous values, would give similar results anyway, just slightly more favourable to the South and the islands.

6. - Conclusions

This paper has introduced a new test to assess the efficiency of a given portfolio. The test takes account of several restrictions present in the reality, important enough to make the efficient frontier dissimilar to the ones which are usually considered in literature. Amongst the feasible assets we set durable goods as well as financial assets, and treat the former as illiquid owing to high transaction costs which must be faced during purchase and sale; the financial assets are, instead, subject to inequality constraints. The statistical test here presented is inspired by the work of Basak *et* Al. (2002); it is derived numerically, since the presence of inequality constraints denies the possibility of making an algebraic form explicit, and its distribution is obtained through asymptotic approximation.

Our efficiency test, as the other tests discussed in this essay, has been applied to Italian household portfolios as they appeared on 31st December 2000. Given the particular kind of investors, housing has been analyzed as a real good, while non-negativity constraints have been set in several financial assets. We have used semi-annual data covering the period 1987 — 2000 as return series. The low sample size (only 28 observations) causes problems in the power of the tests; in order to obviate them, at least partly, we have chosen to focus on high significance levels.

The tests which do not take account of the presence of constraints have classified solely the risk free portfolios as efficient. If we set non-negativity constraints on corporate bonds, stocks and housing, the efficiency improvement is slight whereas the results greatly change when housing is treated as illiquid asset: in this case most of portfolios in the Central Italy have been evaluated efficient. Then, if we add non-negativity constraints on corporate bonds and stocks, once again the betterment is not remarkable, and it has been entirely concentrated in the South.

Thus, the double effect of inserting both a real asset and several constraints would reveal, with the data at our disposal, that 30% of Italian household portfolios are efficient against 17% achieved by the traditional analyses; almost all the efficient portfolios come from the Central Italy.

A promising development of this research could consist in elaborating a change of the test that, although it moves from the same assumptions and follows the same procedure closely, takes account of the twofold effect provided by expected return and risk at the same time. Instead, the new test here discussed compares exclusively the variances, treating the expected return as fixed. From the investor's point of view the strategy appears sensible: he wonders, given that he wants to keep his current expected return, how much the risk he is assuming is higher than the optimal one. However, in this way it is estimated identical the inefficiency of two portfolios which, having a risk equally far from the minimum, are provided with different expected returns: it does not attach importance to the dimension of the investment.

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